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A DITOPOLOGICAL FUZZY STRUCTURAL VIEW OF HUTTON SPACES

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Abstract. We summarize our investigations on a generalization of the notion of Hutton texture to include the non-complemented case and the notion of Hutton space to *Hutton dispace* by not postulating an order-reversing involution and replacing the topology with a ditopology as they were conducted in more detail in [24]. Thus, many of the point-free concepts defined in the category of ditopological texture spaces will go over to the category whose objects are Hutton dispaces via some equivalence functors. In patticular the reader is referred to [24] for proofs of the stated results and detailed references to the literature.

Keywords : Hutton Algebra, Texture, Hutton Space, Equivalence Functor, Ditopology, Full Subcategory, Adjoint Functor, Almost Plain Texture, Co-adjoint Functor.

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1. Introduction

The notion of *texture* was introduced by Lawrence M. Brown as a point-based setting for the study of complement-free mathematical concepts, besides crisp sets, fuzzy sets, L-valued sets and intuitionistic sets.

Additionally, it was shown that the *ditopological texture spaces* [2] do indeed provide a unified setting for the study of topologies on Hutton algebras (fuzzy lattices) on the one hand, and of bitopological and topological spaces on the other.

In this study, we shall generalize the notion of Hutton texture introduced in [5] to include the non-complemented case and the notion of Hutton space to Hutton dispace by not postulating an order-reversing involution and replacing the topology with a ditopology.

Briefly, the principal aim of this paper is to adapt the notion of Hutton space to the context of ditopological texture spaces.

We recall from [23] the notion of almost plain texture, which is weaker than that of nearly plain texture introduced in [21]. Specifically, from [23, Lemma 2.5 (1)], $(S, \mathcal{S})$ is almost plain if and only if it is **dfTex**-isomorphic to a plain texture. It follows that if the texture $(S, \mathcal{S})$ is plain then the Hutton texture of $(\mathcal{S}, \subseteq)$ is almost plain, because it is known to be **dfTex**-isomorphic to $(S, \mathcal{S})$. Hence, in particular, the Hutton texture of $(\mathcal{R}, \subseteq)$ is almost plain. Moreover, the **dfTex**-isomorphism between $(\mathbb{R}, \mathcal{R})$ and the Hutton texture of $(\mathcal{R}, \subseteq)$ extends to a **dfDitop**-isomorphism between these textures under their natural ditopologies.

The layout of the paper is as follows. In Section 2 some necessary background material is recalled. Section 3 is devoted to the theory of Hutton dispaces as a ditopological fuzzy structural counterpart of Hutton spaces.

Incidentally, the reader is referred to [**?**] for terms from lattice theory not defined here and our general reference for category theory is [1].

2. Background

In this section we recall some basic definitions and results that will enable the casual reader to follow the general ideas presented here. The more dedicated reader is referred to the various references for further details. Some standard references for ditopological texture spaces are [4, 5, 6, 7, 8]. Some additional references which lie in the general area of this paper are [2, 9, 11].

Textures and ditopological texture spaces

If S is a set then by a *texturing of* S we mean a subset $\mathcal{S} \subseteq \mathcal{P}(S)$ which is a point-separating, complete, completely distributive lattice containing S and $\emptyset$, and for which meet coincides with intersection and finite join with union. The pair $(S, \mathcal{S})$ is then called a *texture*. Unless stated otherwise we confine our attention to the sets in the texturing $\mathcal{S}$, which is regarded as a generalization of the powerset of S rather than as an analogue of a collection of open or closed subsets. In particular for $s \in S$ we set $P_s = \bigcap\{A \in \mathcal{S} \mid s \in A\} \in \mathcal{S}$, $Q_s = \bigvee\{A \in \mathcal{S} \mid s \notin A\} \in \mathcal{S}$. These so-called p-sets and q-sets are useful in defining textural concepts. A particularly useful result is

$$A \not\subseteq B \iff \exists\, s \in S \text{ with } A \not\subseteq Q_s \text{ and } P_s \not\subseteq B \tag{2.1}$$

for all $A, B \in \mathcal{S}$. This result depends on, in fact is equivalent to, the complete distributivity of $\mathcal{S}$.

We note that each p-set is a molecule (non-zero join-irreducible element) of $\mathcal{S}$. If all the molecules have this form the texture is said to be simple.

We present some basic examples and the notion of product texture for future reference.

Examples 2.1.

(1) The *discrete texture* $(X, \mathcal{P}(X))$ on the set X. For $x \in X$, $P_x = \{x\}$, $Q_x = X \setminus \{x\}$.

(2) The texture $(L, \mathcal{L})$, where $L = (0, 1]$ and $\mathcal{L} = \{(0, r] \mid 0 \le r \le 1\}$. For $r \in L$, $P_r = Q_r = (0, r]$.

(3) The *unit interval texture* $(\mathbb{I}, \mathcal{I})$, $\mathbb{I} = [0, 1]$, $\mathcal{I} = \{[0, r) \mid r \in \mathbb{I}\} \cup \{[0, r] \mid r \in \mathbb{I}\}$. For $r \in \mathbb{I}$, $P_r = [0, r]$ and $Q_r = [0, r)$.

(4) The *real texture* $(\mathbb{R}, \mathcal{R})$, where $\mathbb{R}$ is the set of real numbers and $\mathcal{R} = \{(-\infty, r), (-\infty, r] \mid r \in \mathbb{R}\} \cup \{\emptyset, \mathbb{R}\}$, the set of lower sets of $(\mathbb{R}, \leq)$. For $r \in \mathbb{R}$, $P_r = (-\infty, r]$ and $Q_r = (-\infty, r)$.

(5) The *product texture* $(S \times T, \mathcal{S} \otimes \mathcal{T})$ of textures $(S, \mathcal{S})$ and $(T, \mathcal{T})$. The product texturing $\mathcal{S} \otimes \mathcal{T}$ of $S \times T$ consists of arbitrary intersections of sets of the form

$$(A \times T) \cup (S \times B), \ A \in \mathcal{S} \text{ and } B \in \mathcal{T}.$$

For $(s, t) \in S \times T$, $P_{(s,t)} = P_s \times P_t$ and $Q_{(s,t)} = (Q_s \times T) \cup (S \times Q_t)$.

Setting $\mathcal{S}^c = \{S \setminus A \mid A \in \mathcal{S}\}$, it is clear that $(S, \mathcal{S})$ is a texture if and only if $(S, \mathcal{S}^c)$ is a T_0 topological space whose lattice of open (or closed) sets is completely distributive, that is a C (or core) space (see, for example, [12]). This gives a direct link between textures and C-spaces and discussed in greater detail in [3]. Indeed, if $\omega = \omega_S$ is the interior relation of $(S, \mathcal{S}^c)$ it is shown in [3] that

$$s_1 \, \omega_S \, s_2 \iff P_{s_2} \not\subseteq Q_{s_1},$$

a relation of particular interest in the study of textures. A point function between the base sets of two textures that preserves the interior relations is called *ω-preserving*. The category of textures and ω-preserving functions is denoted by **ifTex**. It is interesting to compare ω_S with the specialization (pre-) order $s_1 \leq s_2 \iff s_1 \in \overline{\{s_2\}}$ of $(S, \mathcal{S}^c)$, which is a partial order in this case. Since we clearly have $\overline{\{s_2\}} = P_{s_2}$ we obtain $s_1 \leq s_2 \iff P_{s_1} \subseteq P_{s_2}$, so $\omega_S \subseteq \leq$. The opposite inclusion holds if and only if ω_S is reflexive, which is a characteristic property of plain textures.

Direlations and difunctions between textures are considered in detail in [6]. An analysis in the context of C-spaces is given in [3]. Direlations represent an appropriate generalization of the classical notion of relation and since they also play an important role here, we summarize the main definitions and results for the convenience of the reader.

Let $(S, \mathcal{S})$, $(T, \mathcal{T})$ be textures. We consider the product $(S \times T, \mathcal{P}(S) \otimes \mathcal{T})$ of the textures $(S, \mathcal{P}(S))$ and $(T, \mathcal{T})$, denoting the p-sets and q-sets by $\overline{P}_{(s,t)}$, $\overline{Q}_{(s,t)}$, respectively.

$r \in \mathcal{P}(S) \otimes \mathcal{T}$ is called a *relation from* $(S, \mathcal{S})$ *to* $(T, \mathcal{T})$ if it satisfies

R1 $r \not\subseteq \overline{Q}_{(s,t)}, P_{s'} \not\subseteq Q_s \implies r \not\subseteq \overline{Q}_{(s',t)}$.

R2 $r \not\subseteq \overline{Q}_{(s,t)} \implies \exists s' \in S$ such that $P_s \not\subseteq Q_{s'}$ and $r \not\subseteq \overline{Q}_{(s',t)}$.

$R \in \mathcal{P}(S) \otimes \mathcal{T}$ is called a *corelation from* $(S, \mathcal{S})$ *to* $(T, \mathcal{T})$ if it satisfies

CR1 $\overline{P}_{(s,t)} \not\subseteq R, P_s \not\subseteq Q_{s'} \implies \overline{P}_{(s',t)} \not\subseteq R$.

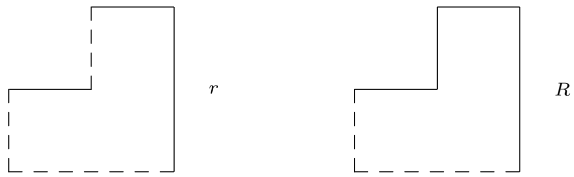
CR2 $\overline{P}_{(s,t)} \not\subseteq R \implies \exists s' \in S$ such that $P_{s'} \not\subseteq Q_s$ and $\overline{P}_{(s',t)} \not\subseteq R$.

A *direlation* is now a pair (r, R) consisting of a relation r and a corelation R.

If one considers, for example, the discrete textures $(S, \mathcal{P}(S))$, $(T, \mathcal{P}(T))$ and $r \in \mathcal{P}(S) \otimes \mathcal{P}(T) = \mathcal{P}(S \times T)$ then $r \not\subseteq \overline{Q}_{(s,t)}$ is equivalent to $(s, t) \in r$ and $P_s \not\subseteq Q_{s'} \iff s = s'$ so *R1* is automatically satisfied. Likewise *R2* is satisfied. Hence an arbitrary relation in the classical sense from S to T is a relation in the above sense for discrete textures. Likewise, any such relation is also a corelation since $\overline{P}_{(s,t)} \not\subseteq R$ is equivalent to $(s, t) \notin R$.

As a second example we may mention the following direlation on the texture $(L, \mathcal{L})$ of Examples 2.1 (2) given in [3, Example 3.17], which is actually an equivalence:

$$r = \{(s,t) \mid 0 < t \leq \tfrac{1}{2} \text{ or } \tfrac{1}{2} < s \leq 1\}, \text{ and}$$
$$R = \{(s,t) \mid 0 < t \leq \tfrac{1}{2} \text{ or } \tfrac{1}{2} \leq s \leq 1\}.$$



Given a direlation (r, R) from $(S, \mathcal{S})$ to $(T, \mathcal{T})$, $(r, R)^{\leftarrow} = (R^{\leftarrow}, r^{\leftarrow})$; $r^{\leftarrow} = \bigcap\{\overline{Q}_{(t,s)} \mid r \not\subseteq \overline{Q}_{(s,t)}\}$, $R^{\leftarrow} = \bigvee\{\overline{P}_{(t,s)} \mid \overline{P}_{(s,t)} \not\subseteq R\}$, is a direlation from $(T, \mathcal{T})$ to $(S, \mathcal{S})$ called the *inverse* of (r, R). Clearly, the inverse operation is idempotent.

A composition $(d, D) \circ (c, C) = (d \circ c, D \circ C)$ of direlations $(S, \mathcal{S}) \xrightarrow{(c,C)} (T, \mathcal{T}) \xrightarrow{(d,D)} (U, \mathcal{U})$ is defined by

$$d \circ c = \bigvee\{\overline{P}_{(s,u)} \mid \exists\, t \in T \text{ with } c \not\subseteq \overline{Q}_{(s,t)} \text{ and } d \not\subseteq \overline{Q}_{(t,u)}\},$$
$$D \circ C = \bigcap\{\overline{Q}_{(s,u)} \mid \exists\, t \in T \text{ with } \overline{P}_{(s,t)} \not\subseteq C \text{ and } \overline{P}_{(t,u)} \not\subseteq D\},$$

see [6, Definition 2.13]. Composition is associative with identity (i_S, I_S) on $(S, \mathcal{S})$ given by

$$i_{(S,\mathcal{S})} = \bigvee\{\overline{P}_{(s,s)} \mid s \in S\}, \; I_{(S,\mathcal{S})} = \bigcap\{\overline{Q}_{(s,s)} \mid s \in S\}.$$

Generally, the inverse defined above is not an inverse for the operation of composition.

For textures $(S, \mathcal{S})$, $(T, \mathcal{T})$, direlation $(r, R) : (S, \mathcal{S}) \to (T, \mathcal{T})$ and $A \in \mathcal{S}$, the A-section of r and the A-section of R are respectively $r^{\rightarrow}A = \bigcap\{Q_t \mid \forall\, s,\ r \not\subseteq \overline{Q}_{(s,t)} \implies A \subseteq Q_s\}$, $R^{\rightarrow}A = \bigvee\{P_t \mid \forall\, s,\ \overline{P}_{(s,t)} \not\subseteq R \implies P_s \subseteq A\}$.

For $B \in \mathcal{T}$ the B-presection $r^{\leftarrow}B$ of r is the B-section of the corelation $r^{\leftarrow}$ while the B-presection $R^{\leftarrow}B$ of R is the B-section of the relation $R^{\leftarrow}$. That is, $r^{\leftarrow}B = (r^{\leftarrow})^{\rightarrow}B$ and $R^{\leftarrow}B = (R^{\leftarrow})^{\rightarrow}B$. Sections of r and presections of R preserve arbitrary joins, while sections of R and presections of r preserve arbitrary intersections [6, Corollary 2.12].

A *difunction* $(f, F) : (S, \mathcal{S}) \to (T, \mathcal{T})$ is a direlation which satisfies

DF1 For $s, s' \in S$, $P_s \not\subseteq Q_{s'} \implies \exists\, t \in T$ with $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s',t)} \not\subseteq F$.

DF2 For $t, t' \in T$ and $s \in S$, $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s,t')} \not\subseteq F \implies P_{t'} \not\subseteq Q_t$.

It is known that difunctions between discrete textures have the form (φ, φ'), where φ is a function between the base sets and φ' its set complement [6].

It is characteristic of a difunction $(f, F) : (S, \mathcal{S}) \to (T, \mathcal{T})$ that $f^{\leftarrow}B = F^{\leftarrow}B$ for all $B \in \mathcal{T}$ [6, Theorem 2.24]. This common "inverse image" therefore preserves both arbitrary joins and arbitrary intersections. The identity direlation is a difunction, and the composition of difunctions is a difunction. Hence textures and difunctions form a category which is denoted by **dfTex**. Conversely, given a mapping $\theta : \mathcal{T} \to \mathcal{S}$ preserving arbitrary joins and intersections there exists a unique difunction $(f^{\theta}, F^{\theta}) : (S, \mathcal{S}) \to (T, \mathcal{T})$ satisfying $(f^{\theta})^{\leftarrow}B = \theta(B) = (F^{\theta})^{\leftarrow}B$ for all $B \in \mathcal{T}$ [7, Proposition 4.1]. This generalizes the classical point free representation of ordinary (point) functions. Clearly the difunction corresponding to the identity mapping on $\mathcal{S}$ is the identity difunction on $(S, \mathcal{S})$. We note that on the other hand, the image $f^{\to}A$ and the co-image $F^{\to}A$ of $A \in \mathcal{S}$ under a difunction (f, F) are generally unequal. The reader is referred to [17, 18, 19] for a general discussion of powerset operators.

Since a texture is a point-based construct it is also natural to consider point functions between the base sets. Indeed, our earlier work on real dicompactness involves such functions in a non-trivial manner. To obtain a connection with difunctions a minimal condition is that the point function be ω-preserving (called condition (a) in some earlier references). By [6, Lemma 3.4], any ω-preserving point function $\varphi : S \to T$ gives rise to a difunction $(f_\varphi, F_\varphi) : (S, \mathcal{S}) \to (T, \mathcal{T})$, $f_\varphi = \bigvee\{\overline{P}_{(s,t)} \mid \exists u \in S \text{ satisfying } P_s \not\subseteq Q_u \text{ and } P_{\varphi(u)} \not\subseteq Q_t\}$, $F_\varphi = \bigcap\{\overline{Q}_{(s,t)} \mid \exists u \in S \text{ satisfying } P_u \not\subseteq Q_s \text{ and } P_t \not\subseteq Q_{\varphi(u)}\}$, for which $f_\varphi^{\leftarrow}B = \varphi^{\leftarrow}B = F_\varphi^{\leftarrow}B$ where

$$\varphi^{\leftarrow}B = \bigvee\{P_u \mid \varphi(u) \in B\} = \bigcap\{Q_v \mid \varphi(v) \notin B\}.$$

The pointed definition of difunction is clearly useful in establishing this and similar connections, and this is one of the reasons why we often prefer to work with difunctions rather than the corresponding complete lattice homomorphisms between the texturings. A difunction (f, F) is said to be *representable* if there exists an ω-preserving point function $\varphi : S \to T$ with $(f, F) = (f_\varphi, F_\varphi)$.

In general a difunction need not be representable, and a representable difunction need not be uniquely representable. However, for certain special classes of textures we can be more precise.

The texture $(S, \mathcal{S})$ is called *plain* if $\mathcal{S}$ is closed under arbitrary unions, equivalently if the corresponding C-space is an Alexandroff-discrete [10] or A-space [12], or if the interior relation ω_S is reflexive. For plain textures all difunctions are uniquely representable and we have an isomorphism between **ifPTex** and **dfPTex**, where "P" denotes a restriction to plain textures. The structure of plain textures is particularly simple (see, for example, [20]), and in particular we have $f_\varphi^{\leftarrow}B = \varphi^{-1}(B) = F_\varphi^{\leftarrow}B$ for an ω-preserving point function φ between plain textures. Note that we use the classical notation for the backward (and forward) powerset operator [17, 18, 19] in this and similar cases to emphasize it is given by the familiar formula for the discrete case.

The more general class of nearly plain textures was introduced in [21]. If $(S, \mathcal{S})$ is a texture we call $s \in S$ a "plain point" if $s\,\omega_S\,s$. Hence, $(S, \mathcal{S})$ is plain if and only if every point of S is plain. The texture $(S, \mathcal{S})$ is called *nearly plain* if given $s \in S$ there exists a plain point $w \in S$ satisfying $Q_s = Q_w$. This plain point w is necessarily unique, and setting $\varphi_p(s) = w$ gives a mapping from S to the set S_p of plain points. The texturing $\mathcal{S}$ on S induces a plain texturing $\mathcal{S}_p$ on S_p in a natural way. Moreover, $\varphi_p : S \to S_p$ is ω-preserving and so induces a difunction $(f_p, F_p) = (f_{\varphi_p}, F_{\varphi_p}) : (S, \mathcal{S}) \to (S_p, \mathcal{S}_p)$. This difunction is known to be a **dfTex**-isomorphism [21, Proposition 2.7]. The proof of this proposition shows that the inverse of this isomorphism is also representable, namely by the inclusion function $\epsilon = \varphi_p|_{S_p} : S_p \to S$. It follows that every difunction between nearly plain textures is (not necessarily uniquely) representable [21, Theorem 2.10].

We have already mentioned the weaker notion of *almost plain texture* in the introduction. If $(S, \mathcal{S})$ is almost plain then by [23, Lemma 2.5] we still have the plain subtexture $(S_p, \mathcal{S}_p)$, the inclusion function $\epsilon : S_p \to S$ remains ω-preserving and the corresponding difunction $(f_\epsilon, F_\epsilon) : (S_p, \mathcal{S}_p) \to (S, \mathcal{S})$ is still a **dfTex**-isomorphism. By [23, Lemma 2.8], the difference is that the inverse of (f_ϵ, F_ϵ) will not be representable unless $(S, \mathcal{S})$ is actually nearly plain.

In general a texturing $\mathcal{S}$ need not be closed under the operation of taking the set complement, so in the context of a texture $(S, \mathcal{S})$ the notion of topology is replaced by that of dichotomous topology. A *dichotomous topology*, or *ditopology* for short, on a texture $(S, \mathcal{S})$, is a pair (τ, κ) of subsets of $\mathcal{S}$, where the set τ of *open sets* satisfies

(1) $S, \emptyset \in \tau$,
(2) $G_1, G_2 \in \tau \implies G_1 \cap G_2 \in \tau$ and
(3) $G_i \in \tau,\ i \in I \implies \bigvee_i G_i \in \tau$,

and the set κ of *closed sets* satisfies

(1) $S, \emptyset \in \kappa$,
(2) $K_1, K_2 \in \kappa \implies K_1 \cup K_2 \in \kappa$ and
(3) $K_i \in \kappa,\ i \in I \implies \bigcap K_i \in \kappa$.

Hence a ditopology is essentially a "topology" for which there is no *a priori* relation between the open and closed sets. A canonical example, that will play an important role here, is the real ditopological space $(\mathbb{R}, \mathcal{R}, \tau_{\mathbb{R}}, \kappa_{\mathbb{R}})$. Here $(\mathbb{R}, \mathcal{R})$ is the real texture (see Examples 2.1 (4)), $\tau_{\mathbb{R}} = \{(-\infty, r) \mid r \in \mathbb{R}\} \cup \{\mathbb{R}, \emptyset\}$ and $\kappa_{\mathbb{R}} = \{(-\infty, r] \mid r \in \mathbb{R}\} \cup \{\mathbb{R}, \emptyset\}$.

In a ditopology, equal emphasis is placed on the open and closed sets. This particularly affects our choice of separation axioms, which are discussed in detail in [8]. In particular by [8, Theorem 4.17] the ditopological texture space $(S, \mathcal{S}, \tau, \kappa)$ is called bi-T_2 if given $s, s' \in S$ with $Q_s \not\subseteq Q_{s'}$ there exists $H \in \tau, K \in \kappa$ with $H \subseteq K$, $P_s \not\subseteq K$ and $H \not\subseteq Q_{s'}$. This effectively represents the Hausdorff axiom in this context. It should be noted that here and later the prefix "bi" is often used to indicate that two dual *properties* hold - in this case T_2 and co-T_2 - whereas the prefix "di" generally indicates a *construct* with two parts first introduced in a textural context, such as a ditopology.

We also recall that an ω-preserving point function, or a difunction, is *bicontinuous* if the inverse image of every open set is open and that of every closed set is closed. We denote by **ifDitop** the construct of ditopological texture spaces and bicontinuous **ifTex**-morphisms, by **dfDitop** the category of ditopological texture spaces and bicontinuous **dfTex**-morphisms.

If $(S, \mathcal{S}, \tau, \kappa)$ is a ditopological texture space and $(S, \mathcal{S})$ is almost plain (in particular, nearly plain or plain), then the induced ditopology (τ_p, κ_p) on S_p gives us the plain ditopological texture space $(S_p, \mathcal{S}_p, \tau_p, \kappa_p)$ which is **dfDitop**-isomorphic to $(S, \mathcal{S}, \tau, \kappa)$ by [23, Lemma 2.10]. As in [3] we will often refer to a **dfDitop**-isomorphism as a *dihomeomorphism*, which is short for difunctional homeomorphism.

Hutton algebras, spaces and textures

By a Hutton algebra (fuzzy lattice) is generally meant a complete, completely distributive lattice $\mathbb{L}$ with an order-reversing involution $'$. We denote by $0_{\mathbb{L}}$ the smallest and by $1_{\mathbb{L}}$ the largest element of $\mathbb{L}$. A *Hutton algebra morphism* $\theta : (\mathbb{L}_1, \prime_1) \to (\mathbb{L}_2, \prime_2)$ is a function $\theta : \mathbb{L}_2 \to \mathbb{L}_1$ which preserves arbitrary meets, joins and the order-reversing involution.

Definition 2.2. $\tau \subseteq \mathbb{L}$ is called a *topology* on $\mathbb{L}$ if it satisfies:

(1) $0_{\mathbb{L}}, 1_{\mathbb{L}} \in \tau$,
(2) $t_1, t_2 \in \tau \implies t_1 \wedge t_2 \in \tau$,
(3) $t_j \in \tau,\ j \in J, \implies \bigvee_{j \in J} t_j \in \tau$.

The triple $(\mathbb{L}, \prime, \tau)$ is called a *Hutton space*. A *Hutton space morphism* is a Hutton algebra morphism that satisfies $\theta[\tau_2] \subseteq \tau_1$. The category of Hutton spaces and Hutton space morphisms will be denoted by **H**.

Given a Hutton algebra $(\mathbb{L}, \prime)$ we denote the set of molecules (non-zero join-irreducible elements) in $\mathbb{L}$ by $M_{\mathbb{L}}$, and for $\alpha \in \mathbb{L}$ we let $\widehat{\alpha} = \{\mu \in M_{\mathbb{L}} \mid \mu \leq \alpha\}$, $\mathcal{M}_{\mathbb{L}} = \{\widehat{\alpha} \mid \alpha \in \mathbb{L}\}$ and $\lambda(\widehat{\alpha}) = \widehat{\alpha'}$, $\alpha \in \mathbb{L}$. Then in [5] it is noted that $(\mathcal{M}_{\mathbb{L}}, \lambda)$ is a Hutton algebra isomorphic to $(\mathbb{L}, \prime)$, and that $(M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}})$ is a simple texture. Moreover $\lambda : \mathcal{M}_{\mathbb{L}} \to \mathcal{M}_{\mathbb{L}}$ is an inclusion reversing involution, whence $(M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}}, \lambda)$ is a complemented texture. It is called the *Hutton texture* of $(\mathbb{L}, \prime)$. Conversely, every simple complemented texture may be obtained as the Hutton texture of a suitable Hutton algebra.

If $(\mathbb{L}, \prime, \tau)$ is a Hutton space then $(M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}}, \lambda, \widehat{\tau}, \lambda(\widehat{\tau}))$ is the corresponding *Hutton complemented ditopological texture space*. Let $\theta : (\mathbb{L}_1, \prime_1, \tau_1) \to (\mathbb{L}_2, \prime_2, \tau_2)$ be a Hutton space morphism, that is a Hutton algebra morphism satisfying $\theta[\tau_2] \subseteq \tau_1$. In view of the isomorphism between $(\mathbb{L}_k, \prime_k)$ and $(\mathcal{M}_{\mathbb{L}_k}, \subseteq)$, $k = 1, 2$, we have a corresponding mapping $\widehat{\theta} : \mathcal{M}_{\mathbb{L}_2} \to \mathcal{M}_{\mathbb{L}_1}$ preserving the complementations, arbitrary intersections and joins. Hence by [7, Proposition 4.1], we have a unique difunction $(f^{\widehat{\theta}}, F^{\widehat{\theta}}) : (M_{\mathbb{L}_1}, \mathcal{M}_{\mathbb{L}_1}) \to (M_{\mathbb{L}_2}, \mathcal{M}_{\mathbb{L}_2})$ satisfying

$$(f^{\widehat{\theta}})^{\leftarrow} \widehat{\alpha_2} = \widehat{\theta(\alpha_2)} = (F^{\widehat{\theta}})^{\leftarrow} \widehat{\alpha_2}$$

for all $\alpha_2 \in \mathbb{L}_2$. It transpires that

$$(f^{\widehat{\theta}}, F^{\widehat{\theta}}) : (M_{\mathbb{L}_1}, \mathcal{M}_{\mathbb{L}_1}, \lambda_1, \widehat{\tau_1}, \lambda_1(\widehat{\tau_1})) \to (M_{\mathbb{L}_2}, \mathcal{M}_{\mathbb{L}_2}, \lambda_2, \widehat{\tau_2}, \lambda_2(\widehat{\tau_2}))$$

is a **dfDitop**-morphism that is also complemented. The above correspondence is used in [7] to show that **cdfSDitop** is equivalent to **H**, where **cdfSDitop** is the category of simple complemented ditopological texture spaces and complemented bicontinuous difunctions. We have no need to consider complemented textures in this paper and so do not enter into details.

3. Hutton Dispaces

In this section we will have no use for the order-reversing involution postulated for Hutton spaces, so we work with a generalization. We call a *Hutton dispace* a complete, completely distributive lattice $\mathbb{L}$ on which are defined a topology τ satisfying Definition 2.2, and a cotopology κ satisfying the dual conditions:

(1) $0_{\mathbb{L}}, 1_{\mathbb{L}} \in \kappa$,
(2) $k_1, k_2 \in \kappa \implies k_1 \vee k_2 \in \kappa$,
(3) $k_j \in \kappa,\ j \in J, \implies \bigwedge_{j \in J} k_j \in \kappa$.

Given two Hutton dispaces $(\mathbb{L}_i, \tau_i, \kappa_i)$, $i = 1, 2$, a mapping $\varphi : \mathbb{L}_2 \to \mathbb{L}_1$ preserving arbitrary meets and joins, $\varphi[\tau_2] \subseteq \tau_1$ and $\varphi[\kappa_2] \subseteq \kappa_1$ is regarded as a morphism from $(\mathbb{L}_1, \tau_1, \kappa_1)$ to $(\mathbb{L}_2, \tau_2, \kappa_2)$, so defining the category **diH** of Hutton dispaces and bicontinuous mappings between them.

A classical Hutton space $(\mathbb{L}, {}', \tau)$ may be regarded as a Hutton dispace by setting $\kappa = \tau'$, so we may think of **H** as a (non-full) subcategory of **diH**.

We note that if $(S, \mathcal{S}, \tau, \kappa)$ is a ditopological texture space then $(\mathcal{S}, \tau, \kappa)$ is a Hutton dispace. Moreover, given a bicontinuous difunction $(f, F) : (S_1, \mathcal{S}_1, \tau_1, \kappa_1) \to (S_2, \mathcal{S}_2, \tau_2, \kappa_2)$, the mapping $\theta_{(f,F)} : \mathcal{S}_2 \to \mathcal{S}_1$ defined in [7] by $\theta_{(f,F)}(B) = f^{\leftarrow}B = F^{\leftarrow}B$, $B \in \mathcal{S}_2$, is clearly a **diH**-morphism. Hence we have a functor $\mathfrak{E} :$ **dfDitop** $\to$ **diH** defined by

$$\mathfrak{E}((S_1, \mathcal{S}_1, \tau_1, \kappa_1) \xrightarrow{(f,F)} (S_2, \mathcal{S}_2, \tau_2, \kappa_2)) = (\mathcal{S}_1, \tau_1, \kappa_1) \xrightarrow{\theta_{(f,F)}} (\mathcal{S}_2, \tau_2, \kappa_2).$$

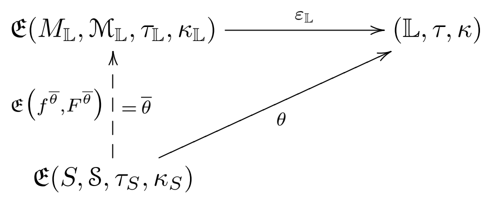
In the opposite direction, if $(\mathbb{L}, \tau, \kappa) \in \mathrm{Ob}\,\mathbf{diH}$ and $(M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}})$ is the Hutton texture corresponding to $\mathbb{L}$, then we may define $\tau_{\mathbb{L}} = \{\widehat{\alpha} \mid \alpha \in \tau\}$, $\kappa_{\mathbb{L}} = \{\widehat{\alpha} \mid \alpha \in \kappa\}$, and clearly $(\mathcal{M}_{\mathbb{L}}, \tau_{\mathbb{L}}, \kappa_{\mathbb{L}})$ is a Hutton dispace isomorphic to $(\mathbb{L}, \tau, \kappa)$ in the category **diH**. Corresponding to this we have the *Hutton ditopological texture space* $(M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}}, \tau_{\mathbb{L}}, \kappa_{\mathbb{L}})$ of $(\mathbb{L}, \tau, \kappa)$. A mapping $\theta : \mathbb{L}_2 \to \mathbb{L}_1$ preserving arbitrary meets and joins gives rise to the mapping $\widehat{\theta} : \mathcal{M}_{\mathbb{L}_2} \to \mathcal{M}_{\mathbb{L}_1}$, $\widehat{\theta}(\widehat{\alpha}) = \widehat{\theta(\alpha)}$, which also preserves arbitrary meets and joins, and hence by [7, Proposition 4.1] we obtain the difunction $(f^{\widehat{\theta}}, F^{\widehat{\theta}}) : (M_{\mathbb{L}_1}, \mathcal{M}_{\mathbb{L}_1}) \to (M_{\mathbb{L}_2}, \mathcal{M}_{\mathbb{L}_2})$, which is characterized by

$$\big(f^{\widehat{\theta}}\big)^{\leftarrow}\widehat{\alpha} = \widehat{\theta(\alpha)} = \big(F^{\widehat{\theta}}\big)^{\leftarrow}\widehat{\alpha}.$$

Moreover, if θ is bicontinuous then $(f^{\widehat{\theta}}, F^{\widehat{\theta}})$ becomes a **diH**-morphism, so we have a functor $\mathfrak{H} :$ **diH** $\to$ **dfDitop** defined by

$$\mathfrak{H}((\mathbb{L}_1, \tau_1, \kappa_1) \xrightarrow{\theta} (\mathbb{L}_2, \tau_2, \kappa_2)) = (M_{\mathbb{L}_1}, \mathcal{M}_{\mathbb{L}_1}, \tau_{\mathbb{L}_1}, \kappa_{\mathbb{L}_1}) \xrightarrow{(f^{\widehat{\theta}}, F^{\widehat{\theta}})} (M_{\mathbb{L}_2}, \mathcal{M}_{\mathbb{L}_2}, \tau_{\mathbb{L}_2}, \kappa_{\mathbb{L}_2}).$$

We denote by $\varepsilon_{\mathbb{L}}$ the lattice isomorphism $\alpha \mapsto \widehat{\alpha}$ from $\mathbb{L}$ to $\mathcal{M}_{\mathbb{L}}$. This gives us the **diH**-isomorphism $\varepsilon_{\mathbb{L}} : \mathfrak{E}(M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}}, \tau_{\mathbb{L}}, \kappa_{\mathbb{L}}) \to (\mathbb{L}, \tau, \kappa)$. We claim that the $\mathfrak{E}$-costructured arrow $\big((M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}}, \tau_{\mathbb{L}}, \kappa_{\mathbb{L}}), \varepsilon_{\mathbb{L}}\big)$ with codomain $(\mathbb{L}, \tau, \kappa)$ is $\mathfrak{E}$-co-universal for $(\mathbb{L}, \tau, \kappa)$. Take $(S, \mathcal{S}, \tau_S, \kappa_S) \in Ob\,\mathbf{dfDitop}$ and let $\theta : \mathfrak{E}(S, \mathcal{S}, \tau_S, \kappa_S) \to (\mathbb{L}, \tau, \kappa)$ be a **diH**-morphism. Define $\overline{\theta} : \mathcal{M}_{\mathbb{L}} \to \mathcal{S}$ by $\overline{\theta}(\widehat{\alpha}) = \theta(\alpha)$, $\alpha \in \mathbb{L}$, where θ is the corresponding mapping $\mathbb{L} \to \mathcal{S}$. Clearly the equality $\overline{\theta} \circ \varepsilon_{\mathbb{L}} = \theta$ holds between these mappings, and since θ preserves arbitrary meets and joins so does $\overline{\theta}$. Hence, $(f^{\overline{\theta}}, F^{\overline{\theta}})$ is the (clearly unique) **dfDitop**-morphism from $(S, \mathcal{S}, \tau_S, \kappa_S)$ to $(M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}}, \tau_{\mathbb{L}}, \kappa_{\mathbb{L}})$ for which the following diagram is commutative.



This establishes our claim and we deduce that $\mathfrak{E}$ is co-adjoint. Moreover, since $(M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}}, \tau_{\mathbb{L}}, \kappa_{\mathbb{L}}) = \mathfrak{H}(\mathbb{L}, \tau, \kappa)$ we see that $\mathfrak{H}$ is the corresponding adjoint (see [1]). We have already noted that $\varepsilon = (\varepsilon_{\mathbb{L}}) : \mathfrak{E} \circ \mathfrak{H} \to \mathrm{id}_{\mathbf{diH}}$ is a natural isomorphism, and the corresponding natural transformation $\eta = (\eta_S) : \mathrm{id}_{\mathbf{dfDitop}} \to \mathfrak{H} \circ \mathfrak{E}$ is given by

$\eta_S = (f^{\delta_S}, F^{\delta_S})$, where $\delta_S : \mathcal{M}_\mathcal{S} \to \mathcal{S}$ is the bijection $\delta_S(\widehat{A}) = A$, $A \in \mathcal{S}$. It follows that η is also a natural isomorphism, whence $\mathfrak{H}$ and $\mathfrak{E}$ are equivalences between **dfDitop** and **diH**. We note in passing that the difunction $(f^{\delta_S}, F^{\delta_S})$ is actually representable, namely by the ω-preserving mapping $s \mapsto P_s$, $s \in S$. We omit the straightforward but somewhat tedious proof of this result.

In view of the equivalence between **dfDitop** and **diH** shown above, many of the point free concepts defined in **dfDitop** will carry over to **diH**. Throughout the remainder of this section, therefore, $(\mathbb{L}, \tau, \kappa)$ will be a Hutton dispace.

Now, let us consider the status of the subsets $M_\mathbb{L}$, $(M_\mathbb{L})_p$ of $\mathbb{L}$.

Lemma 3.1.

(1) $M_\mathbb{L} = \{\lambda \in \mathbb{L} \mid \lambda \neq 0_\mathbb{L}$ *and* $\lambda \leq \alpha \vee \beta,\ \alpha, \beta \in \mathbb{L} \implies \lambda \leq \alpha$ *or* $\lambda \leq \beta\}$.
(2) $(M_\mathbb{L})_p = \{\lambda \in \mathbb{L} \mid \lambda \not\leq \bigvee\{\mu \in \mathbb{L} \mid \lambda \not\leq \mu\}\}$.

Proof. (1) is just the definition of $M_\mathbb{L}$, so we prove (2). For $\lambda \in M_\mathbb{L}$ we have $\lambda \in (M_\mathbb{L})_p$ if and only if $P_\lambda \not\subseteq Q_\lambda$, hence if and only if

$$\widehat{\lambda} \not\subseteq \bigvee\{\widehat{\mu} \mid \lambda \notin \widehat{\mu}\} = \bigvee\{\widehat{\mu} \mid \lambda \not\leq \mu\} = \bigvee\{\widehat{\mu \mid \lambda \not\leq} \mu\},$$

and hence if and only if $\lambda \not\leq \bigvee\{\mu \mid \lambda \not\leq \mu\}$. It remains only to note that if conversely $\lambda \in \mathbb{L}$ satisfies this inequality then clearly $\lambda \in M_\mathbb{L}$. □

The following notation will simplify the expression of conditions of this type.

Definition 3.2. For $\lambda \in M_\mathbb{L}$ we let

$$q_\lambda = \bigvee\{\mu \in \mathbb{L} \mid \lambda \not\leq \mu\}.$$

This permits us to write $(M_\mathbb{L})_p = \{\lambda \in M_\mathbb{L} \mid \lambda \not\leq q_\lambda\}$.

Since P_λ in $\mathcal{M}_\mathbb{L}$ is represented by λ, and Q_λ by q_λ, we also note the useful fact

$$\alpha \not\leq \beta \iff \exists \lambda \in M_\mathbb{L} \text{ with } \alpha \not\leq q_\lambda \text{ and } \lambda \not\leq \beta$$

for all $\alpha, \beta \in \mathbb{L}$, which corresponds to (2.1).

Corollary 3.3. $(M_\mathbb{L}, \mathcal{M}_\mathbb{L})$ *is almost plain if and only if* $\mathbb{L}$ *satisfies the condition*

$$\lambda, \mu \in M_\mathbb{L},\ \lambda \not\leq q_\mu \implies \exists\, \gamma \in (M_\mathbb{L})_p \textit{ with } \lambda \not\leq q_\gamma,\ \gamma \not\leq q_\mu.$$

Proof. Immediate by [23, Lemma 2.5 (2)]. □

For convenience we will call $\mathbb{L}$ *almost plain* in case it satisfies the condition of Corollary 3.3, that is if and only if the texture $(M_\mathbb{L}, \mathcal{M}_\mathbb{L})$ is almost plain. Clearly, almost plainness of $\mathbb{L}$ is preserved under **diH**-isomorphisms.

On the other hand, plainness and near plainness depend on the point structure, and are not intrinsic properties of $\mathbb{L}$ in general. However, for Hutton dispaces of the form $\mathfrak{E}(S, \mathcal{S}, \tau, \kappa) = (\mathcal{S}, \tau, \kappa)$ we may recover the set S as the largest element of $\mathcal{S}$, and then it is meaningful to call $(\mathcal{S}, \tau, \kappa)$ plain (nearly plain) in case $(S, \mathcal{S})$ is plain (nearly plain). In particular, if $\mathbb{L}$ is almost plain then we are guaranteed that $((\mathcal{M}_\mathbb{L})_p, (\tau_\mathbb{L})_p, (\kappa_\mathbb{L})_p)$ is a plain Hutton dispace in this sense.

Let **diApH** denote the full subcategory of **diH** whose objects are almost plain. We define a functor $\mathfrak{J}$: **diApH** $\to$ **Top** which generalizes the functor of the same name from **ifNpDitop** to **Top** given in [21, Theorem 4.4]. First we note the following result.

Lemma 3.4. *Let $(S_k, \mathcal{S}_k)$ be plain textures, suppose that the mapping $\theta : \mathcal{S}_2 \to \mathcal{S}_1$ preserves arbitrary intersections and joins and that the point function $\psi : S_1 \to S_2$ is ω-preserving. Then the following are equivalent:*

(1) $(f^\theta, F^\theta) = (f_\psi, F_\psi)$.
(2) *For all $s \in S_1$ we have $P_s \subseteq \theta(C) \iff P_{\psi(s)} \subseteq C, \forall C \in \mathcal{S}_2$.*
(3) *For all $s \in S_1$ we have $\theta(Q_{\psi(s)}) \subseteq Q_s$ and $P_s \subseteq \theta(P_{\psi(s)})$.*

Proof. If (1) holds then, by the discussion preceding [6, Proposition 3.7], ψ is characterized by $f^\theta \not\subseteq \overline{Q}_{(s,\psi(s))}$ and $\overline{P}_{(s,\psi(s))} \not\subseteq F^\theta$. Now (2) follows by applying the formulae for f^θ, F^θ in [7, Proposition 4.1]. The converse is proved likewise, and (2) $\iff$ (3) is straightforward. □

Let $(\mathbb{L}_k, \tau_k, \kappa_k)$, $k = 1, 2$ be almost plain Hutton dispaces, $\theta : (\mathbb{L}_1, \tau_1, \kappa_1) \to (\mathbb{L}_2, \tau_2, \kappa_2)$ a **diH**-morphism. Corresponding to the mapping $\theta : \mathbb{L}_2 \to \mathbb{L}_1$ we have $\widehat{\theta} : \mathcal{M}_{\mathbb{L}_2} \to \mathcal{M}_{\mathbb{L}_1}$ given by $\widehat{\theta}(\widehat{\alpha}) = \widehat{\theta(\alpha)}$, and then we have $\widehat{\theta}^p : (\mathcal{M}_{\mathbb{L}_2})_p \to (\mathcal{M}_{\mathbb{L}_1})_p$ given by $\widehat{\theta}^p(\widehat{\alpha} \cap (M_{\mathbb{L}_2})_p) = \widehat{\theta}(\widehat{\alpha}) \cap (M_{\mathbb{L}_1})_p$. It is straightforward to verify that $\widehat{\theta}^p$ preserves arbitrary joins and intersections since $\widehat{\theta}$ does. For convenience we will write $\widehat{\alpha} \cap (M_{\mathbb{L}})_p$ as $\widehat{\alpha}^p$ for $\alpha \in \mathbb{L}$, so $\widehat{\theta}^p(\widehat{\alpha}^p) = \widehat{\theta(\alpha)}^p$.

Since the textures $(M_{\mathbb{L}_1})_p$, $(M_{\mathbb{L}_2})_p$ are plain we have an ω-preserving point function $\psi : (M_{\mathbb{L}_1})_p \to (M_{\mathbb{L}_2})_p$ satisfying $\big(f^{\widehat{\theta}^p}, F^{\widehat{\theta}^p}\big) = (f_\psi, F_\psi)$. Now we have:

Lemma 3.5. *With the notation as above,*

$$\psi^{-1}\big(\widehat{\alpha}^p\big) = \widehat{\theta(\alpha)}^p = \widehat{\theta}^p\big(\widehat{\alpha}^p\big)$$

for all $\alpha \in \mathbb{L}_2$.

Proof. Since an ω-preserving point function between plain textures satisfies the hypotheses of [6, Lemma 3.9] we have

$$\psi^{-1}(\widehat{\alpha}^p) = f_\psi^{\leftarrow} \widehat{\alpha}^p = \big(f^{\widehat{\theta}^p}\big)^{\leftarrow} \widehat{\alpha}^p = \widehat{\theta}^p\big(\widehat{\alpha}^p\big)$$

for all $\alpha \in \mathbb{L}_2$, as required. Alternatively, this equality may be verified directly using the relations in Lemma 3.4 (3). □

The following result now provides the counterpart of [21, Theorem 4.4].

Theorem 3.6. *The mapping $\mathfrak{J} : \mathbf{diApH} \to \mathbf{Top}$ defined by*

$$\mathfrak{J}\big((\mathbb{L}_1, \tau_1, \kappa_1) \xrightarrow{\theta} (\mathbb{L}_2, \tau_2, \kappa_2)\big) = \big((M_{\mathbb{L}_1})_p, \mathcal{J}_{\tau_{\mathbb{L}_1}\kappa_{\mathbb{L}_1}}\big) \xrightarrow{\psi} \big((M_{\mathbb{L}_2})_p, \mathcal{J}_{\tau_{\mathbb{L}_2}\kappa_{\mathbb{L}_2}}\big),$$

where ψ corresponds to θ as above and $\mathcal{J}_{\tau_{\mathbb{L}_k}\kappa_{\mathbb{L}_k}}$ denotes the joint topology of $(\tau_{\mathbb{L}_k}, \kappa_{\mathbb{L}_k})$, $k = 1, 2$, is an adjoint functor.

Proof. To verify that $\mathfrak{J}$ is a functor it suffices to check that ψ is continuous whenever θ is a **diH**-morphism. By [21] a subbase for the joint topology of $(\tau_{\mathbb{L}_k}, \kappa_{\mathbb{L}_k})$ on $(M_{\mathbb{L}_k})_p$ is $\{\widehat{\gamma}^p \mid \gamma \in \tau_{\mathbb{L}_k}\} \cup \{(M_{\mathbb{L}_k})_p \setminus \widehat{\varphi}^p \mid \varphi \in \kappa_{\mathbb{L}_k}\}$, $k = 1, 2$. Now for $\gamma \in \tau_{\mathbb{L}_2}$ we have by Lemma 3.5 that $\psi^{-1}(\widehat{\gamma}^p) = \widehat{\theta(\gamma)}^p \in \mathcal{J}_{\tau_{\mathbb{L}_1}\kappa_{\mathbb{L}_1}}$, and for $\varphi \in \kappa_{\mathbb{L}_2}$, $\psi^{-1}\big((M_{\mathbb{L}_2})_p \setminus \widehat{\varphi}^p\big) = (M_{\mathbb{L}_1})_p \setminus \psi^{-1}(\widehat{\varphi}^p) = (M_{\mathbb{L}_1})_p \setminus \widehat{\theta(\varphi)}^p \in \mathcal{J}_{\tau_{\mathbb{L}_1}\kappa_{\mathbb{L}_1}}$, which establishes the continuity of ψ.

To show $\mathfrak{J}$ is an adjoint take $(X, \mathcal{T}) \in \mathrm{Ob}\,\mathbf{Top}$ and consider $(\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c) \in \mathrm{Ob}\,\mathbf{diApH}$, where $\mathcal{T}^c = \{X \setminus G \mid G \in \mathcal{T}\}$. Since the molecules in $(\mathcal{P}(X), \subseteq)$ are the singletons we have $M_{\mathcal{P}(X)} = \{\{x\} \mid x \in X\}$ and $\mathcal{M}_{\mathcal{P}(X)} = \{\widehat{A} \mid A \in \mathcal{P}(X)\}$, where

$\widehat{A} = \{\{x\} \mid x \in A\}$. Clearly $(M_{\mathcal{P}(X)}, \mathcal{M}_{\mathcal{P}(X)})$ is plain and $\mathfrak{J}\big(\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c\big) = \big(\widehat{X}, \widehat{\mathcal{T}}\big)$, where $\widehat{\mathcal{T}} = \{\widehat{G} \mid G \in \mathcal{T}\}$. The mapping $s : X \to \widehat{X}$ defined by $x \mapsto s(x) = \{x\}$ is a homeomorphism between $(X, \mathcal{T})$ and $(\widehat{X}, \widehat{\mathcal{T}})$, so we may consider the $\mathfrak{J}$-structured arrow $\big(s, (\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c)\big)$ with domain $(X, \mathcal{T})$. To prove the universal property of $\big(s, (\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c)\big)$ take $(\mathbb{L}, \tau, \kappa) \in \mathrm{Ob}\,\mathbf{diApH}$ and a **Top** morphism $\varphi : (X, \mathcal{T}) \to \mathfrak{J}(\mathbb{L}, \tau, \kappa)$. Clearly the only **Top**-morphism $\overline{\varphi} : \big(\widehat{X}, \widehat{\mathcal{T}}\big) \to \big((M_{\mathbb{L}})_p, \mathcal{J}_{\tau_{\mathbb{L}}\kappa_{\mathbb{L}}}\big)$ making the diagram below commutative is given by setting $\overline{\varphi}(\{x\}) = \varphi(x)$, so we must prove the existence of a unique **diApH**-morphism $\theta : \big(\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c\big) \to (\mathbb{L}, \tau, \kappa)$ for which $\mathfrak{J}(\theta) = \overline{\varphi}$.

$$\begin{array}{ccc} (X, \mathcal{T}) & \xrightarrow{\quad s \quad} & \mathfrak{J}(\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c) = (\widehat{X}, \widehat{\mathcal{T}}) \\ & \searrow^{\varphi} & \Big\downarrow \mathfrak{J}(\theta) = \overline{\varphi} \\ & & \mathfrak{J}(\mathbb{L}, \tau, \kappa) = ((M_{\mathbb{L}})_p, \mathcal{J}_{\tau_{\mathbb{L}}\kappa_{\mathbb{L}}}) \end{array}$$

Now consider

$$(X, \mathcal{P}(X)) \xrightarrow{(f_s, F_s)} (\widehat{X}, \mathcal{P}(\widehat{X})) \xrightarrow{(f_{\overline{\varphi}}, F_{\overline{\varphi}})} ((M_{\mathbb{L}})_p, (\mathcal{M}_{\mathbb{L}})_p) \xrightarrow{(f_\epsilon, F_\epsilon)} (M_{\mathbb{L}}, \mathcal{M}_{\mathbb{L}}),$$

and the corresponding mapping $\theta_1 : \mathcal{M}_{\mathbb{L}} \to \mathcal{P}(X)$ which preserves arbitrary meets and joins. Letting $\theta(\alpha) = \theta_1\big(\widehat{\alpha}\big)$ we have $\theta : \mathbb{L} \to \mathcal{P}(X)$ preserving arbitrary meets and joins. It is not difficult to show that $\mathfrak{J}(\theta) = \overline{\varphi}$, and that this is the only such mapping. It remains to show that θ is a **diApH**-morphism.

Clearly it is sufficient to show that $\overline{\varphi}$ is $\big(\widehat{\mathcal{T}}, \widehat{\mathcal{T}}^c\big)$–$\big((\tau_{\mathbb{L}})_p, (\kappa_{\mathbb{L}})_p\big)$-bicontinuous. But

$$G \in (\tau_{\mathbb{L}})_p \implies G \in \mathcal{J}_{\tau_{\mathbb{L}}\kappa_{\mathbb{L}}} \implies \overline{\varphi}^{-1}(G) \in \widehat{\mathcal{T}}$$

and

$$F \in (\kappa_{\mathbb{L}})_p \implies (M_{\mathbb{L}})_p \setminus F \in \mathcal{J}_{\tau_{\mathbb{L}}\kappa_{\mathbb{L}}} \implies \overline{\varphi}^{-1}((M_{\mathbb{L}})_p \setminus F) \in \widehat{\mathcal{T}} \implies \overline{\varphi}^{-1}(F) \in \widehat{\mathcal{T}}^c,$$

which establishes bicontinuity and completes the proof. □

The proof of Theorem 3.6 leads to the following:

Corollary 3.7. *The functor* $\mathfrak{T} : \mathbf{Top} \to \mathbf{diApH}$ *given by*

$$\mathfrak{T}\big((X, \mathcal{T}) \xrightarrow{\varphi} (Y, \mathcal{V})\big) = \big(\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c\big) \xrightarrow{\overleftarrow{\varphi}} \big(\mathcal{P}(Y), \mathcal{V}, \mathcal{V}^c\big),$$

where $\overleftarrow{\varphi} : \mathcal{P}(Y) \to \mathcal{P}(X)$ *is defined by* $\overleftarrow{\varphi}(A) = \varphi^{-1}(A)$, *is the co-adjoint of* $\mathfrak{J}$. □

Clearly the functor $\mathfrak{T}$ is faithfull and injective on objects, hence an embedding. It is also full. For let $\theta : \big(\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c\big) \to \big(\mathcal{P}(Y), \mathcal{V}, \mathcal{V}^c\big)$ be a **diApH**-morphism. Then the mapping $\theta : \mathcal{P}(Y) \to \mathcal{P}(X)$ preserves arbitrary unions and intersection, hence gives a difunction $(f^\theta, F^\theta) : (X, \mathcal{P}(X)) \to (Y, \mathcal{P}(Y))$ satisfying $(f^\theta)^{\leftarrow} A = \theta(A) = (F^\theta)^{\leftarrow} A$ for all $A \in \mathcal{P}(Y)$. Since the textures are plain we have a (necessarily ω-preserving) function $\varphi : X \to Y$ satisfying $(f^\theta, F^\theta) = (f_\varphi, F_\varphi)$ for which $f_\varphi^{\leftarrow} A = \varphi^{-1} A = F_\varphi^{\leftarrow} A$ for all $A \in \mathcal{P}(Y)$, whence $\overleftarrow{\varphi} = \theta$ and φ is clearly continuous and hence a **Top**-morphism. It follows that the category **Top** is isomorphic to a full coreflective subcategory of **diApH**.

We will say that $(\mathbb{L}, \tau, \kappa)$ satisfies a given separation axiom if the corresponding Hutton ditopological texture space satisfies that axiom. Since the axioms in [8] have point free characterizations it is possible to express these axioms directly in terms of $(\mathbb{L}, \tau, \kappa)$. We leave the details to the interested reader. In parallel with [21, Proposition 4.6] we have that if an almost plain Hutton dispace $(\mathbb{L}, \tau, \kappa)$ is bi-T_2, then the corresponding joint topological space $((M_{\mathbb{L}})_p, \mathcal{J}_{\tau_{\mathbb{L}}\kappa_{\mathbb{L}}})$ is Hausdorff. In general the converse is false, but certainly if $(X, \mathcal{T})$ is Hausdorff then $\big(\mathcal{P}(X), \mathcal{T}, \mathcal{T}^c\big)$ is bi-T_2.

References

[1] J. Adámek, H. Herrlich and G. E. Strecker, *Abstract and Concrete Categories* (John Wiley & Sons, Inc., New York, 1990).

[2] A. Altay and M. Diker *A Wallman-Type Compactification of Texture spaces*, Fuzzy Sets and Systems, **157** (2006), 2683–2705.

[3] L. M. Brown, *Quotients of textures and of ditopological texture spaces*, Topology Proceedings **29** (2) (2005), 337–368.

[4] L. M. Brown and M. Diker, *Ditopological texture spaces and intuitionistic sets*, Fuzzy Sets and Systems **98** (1998), 217–224.

[5] L. M. Brown and R. Ertürk, *Fuzzy sets as texture spaces, I. Representation theorems*, Fuzzy Sets and Systems **110** (2) (2000), 227–236.

[6] L. M. Brown, R. Ertürk and Ş. Dost, *Ditopological texture spaces and fuzzy topology, I. Basic Concepts*, Fuzzy Sets and Systems **147** (2) (2004), 171–199.

[7] L. M. Brown, R. Ertürk and Ş. Dost, *Ditopological texture spaces and fuzzy topology, II. Topological Considerations*, Fuzzy Sets and Systems **147** (2) (2004), 201–231.

[8] L. M. Brown, R. Ertürk and Ş. Dost, *Ditopological texture spaces and fuzzy topology, III. Separation Axioms*, Fuzzy Sets and Systems **157** (14) (2006), 1886–1912.

[9] L. M. Brown and M. M. Gohar, *Compactness in ditopological texture spaces*, Hacettepe J. Math. Stat. **38** (1) (2009), 21–43.

[10] G. C. L. Brümmer, *Functorial transitive quasi-uniformities*, Categorical Topology (Toledo, Ohio, 1983), Sigma Ser. Pure Math. **5**, Heldermann Verlag, Berlin (1984), 163–184.

[11] M. Diker, *One-point compactifications of ditopological texture spaces*, Fuzzy Sets and Systems, **147** (2) (2004), 233–248.

[12] M. Erné, *The ABC of order and topology*, in: Herrlich, H. and Porst, H.-E. (eds.), *Category Theory at Work*, Heldermann Verlag, Berlin, 1991, 57–83.

[13] L. Gillman and M. Jerison, *Rings of Continuous Functions* (D. Van Nostrand, Company, Inc., Princeton, New Jersey 1960).

[14] S. Özçağ, L. M. Brown, *Di-uniform texture spaces*, Appl. Gen. Topology **4** (1) (2003), 157–192.

[15] S. Özçağ, L. M. Brown, *A textural view of the distinction between uniformities and quasi uniformities*, Topology and its Applications **153** (2006), 3294–3307.

[16] S. Özçağ, L. M. Brown and B. Krsteska, *Di-uniformities and Hutton uniformities*, Fuzzy Sets and Systems **195** (2012), 58–74.

[17] S. E. Rodabaugh, *Powerset operator based foundation for point-set lattice-theoretic (poslat) fuzzy set theories and topologies*, Quaest. Math. **20** (3) (1997), 463–530.

[18] S. E. Rodabaugh, *Powerset Operator Foundations for Poslat Fuzzy Set Theories and Topologies*, Mathematics of Fuzzy Sets: Logic, Topology and Measure Theory (U. Höhle and S. E. Rodabaugh, eds), The Handbooks of Fuzzy Sets Series **3** (Kluwer Academic Publishers, Dordrecht, 1999).

[19] S. E. Rodabaugh, *Relationship of algebraic theories to powerset theories and fuzzy topological theories for lattice-valued mathematics*, Int. J. Math. Math. Sci. **2007** (2007), 1–71.

[20] İ. U. Tiryaki and L. M. Brown, *Plain ditopological texture spaces*, Topology and its Applications **158** (15) (2011), 2005–2015.

[21] F. Yıldız and L. M. Brown, *Categories of dicompact bi-T_2 texture spaces and a Banach-Stone theorem*, Quaestiones Mathematicae **30** (2007), 167–192.

[22] F. Yıldız and L. M. Brown, *Real dicompact textures*, Topology and its Applications **156** (11) (2009), 1970–1984.

[23] F. Yıldız and L. M. Brown, *Real dicompactifications of ditopological texture spaces*, Topology and its Applications **156** (18) (2009), 3041–3051.

[24] F. Yıldız and L. M. Brown, Extended real dicompactness and an application to Hutton spaces, Fuzzy Sets and Systems 227 (2013), 74–95.
[25] L. A. Zadeh, *Fuzzy sets*, Inform. Control **8** (1965), 338–353.

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