5TH IFSCOM2018 PROCEEDING BOOK
ISBN: 978-605-68670-0-2

IFSCOM2018
5TH IFS AND CONTEMPORARY MATHEMATICS CONFERENCE
SEPTEMBER, 05-09, 2018, KAHRAMANMARAS, TURKEY
pp: 93-97

A STUDY ON BIPOLAR SOFT TOPOLOGICAL SPACES

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ABSTRACT. In this present study, some properties of bipolar soft closed sets are introduced and the concept of closure, interior, basis and subspaces which are the building blocks of classical topology are defined on bipolar soft topological spaces. In addition, examples have been presented so that the subject can be better understood.

1. INTRODUCTION

Introducing fuzzy set [11], intiutionistic fuzzy set[1], soft set[6] and etc. theories which contribute to solution of problems such as decision making and uncertainity. A lot of researcher has been done on these theories[2, 3, 7, 10]

In the past years, M. Shabir & M. Naz [9] and F. Karaaslan & S. Karatas [4] differently defined bipolar soft set. Obviously, bipolar soft sets satisfied more sharp results than soft sets. Therefore the concept of bipolar soft topology has a great importance.

In this study, we define a short notation for writing simplicity in the application of bipolar soft sets and investigate the relationship between the soft topological spaces and the bipolar soft topological spaces. Moreover, we define the notion of bipolar soft closure, bipolar soft interior, bipolar soft basis, bipolar soft subspace. The basis theorems of these notations are provided and supported with examples.

2. PRELIMINARY

In this section, we will give some preliminary information about bipolar soft sets and bipolar soft topological spaces. Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X and $A, B, C \subseteq E$.

2000 *Mathematics Subject Classification.* 54A40, 54E55, 54D10.
Key words and phrases. bipolar soft set, bipolar soft topology, bipolar soft topological spaces, bipolar soft open(close), bipolar soft interior, bipolar soft basis.

Definition 2.1. [5] Let $E = \{e_1,\ e_2,\ e_3, ..., e_n\}$ be a set of parameters. The not set of E denoted by $\neg E$ is defined by $\neg E = \{\neg e_1,\ \neg e_2,\ \neg e_3, ..., \neg e_n\}$ where for all i, $\neg e_i = not\ e_i$.

Definition 2.2. [9] A triplet (F, G, A) is called a bipolar soft set over X, where F and G are mappings, $F : A \to P(X)$ and $G : \neg A \to P(X)$ such that $F(e) \cap G(\neg e) = \emptyset$ for all $e \in A$ and $\neg e \in \neg A$.

Definition 2.3. [9] For two bipolar soft sets (F_1, G_1, A) and (F_2, G_2, B) over X, (F_1, G_1, A) is called a bipolar soft subset of (F_2, G_2, B) if

(1) $A \subseteq B$ and
(2) $F_1(e) \subseteq F_2(e)$ and $G_2(\neg e) \subseteq G_1(\neg e)$ for all $e \in A$.
This relationship is denoted by $(F_1, G_1, A) \widetilde{\subseteq} (F_2, G_2, B)$. (F_1, G_1, A) and (F_2, G_2, B) are said to be equal if (F_1, G_1, A) is a bipolar soft subset of (F_2, G_2, B) and (F_2, G_2, A) is a bipolar soft subset of (F_1, G_1, B).

Definition 2.4. [9] Bipolar soft complement of a bipolar soft set (F, G, A) over X is denoted by $(F, G, A)^c$ and is defined by $(F, G, A)^c = (F^c, G^c, A)$ where $F^c : A \to P(X)$ and $G^c : \neg A \to P(X)$ are given by $F^c(e) = G(\neg e)$ and $G^c(\neg e) = F(e)$ for all $e \in A$ and $\neg e \in \neg A$.

Definition 2.5. [9] Bipolar soft union of two bipolar soft sets (F_1, G_1, A) and (F_2, G_2, B) over X is the bipolar soft set (H, I, C) over X where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F_1(e), & \text{if } e \in A - B, \\ F_2(e), & \text{if } e \in B - A, \\ F_1(e) \cup F_2(e), & \text{if } e \in A \cap B. \end{cases}$$

$$I(\neg e) = \begin{cases} G_1(\neg e), & \text{if } \neg e \in (\neg A) - (\neg B)\,, \\ G_2(\neg e), & \text{if } \neg e \in (\neg B) - (\neg A)\,, \\ G_1(\neg e) \cap G_2(\neg e), & \text{if } \neg e \in (\neg A) \cap (\neg B)\,. \end{cases}$$

It is denoted by $(F_1, G_1, A) \widetilde{\cup} (F_2, G_2, B) = (H, I, C)$.

Definition 2.6. [9] Bipolar soft intersection of two bipolar soft sets (F_1, G_1, A) and (F_2, G_2, B) over X is the bipolar soft set (H, I, C) over X where $C = A \cup B$ is non-empty and for all $e \in C$,

$$H(e) = F_1(e) \cap F_2(e) \text{ and } I(\neg e) = G_1(\neg e) \cup G_2(\neg e).$$

It is denoted by $(F_1, G_1, A) \widetilde{\cap} (F_2, G_2, B) = (H, I, C)$.

Definition 2.7. [9] Let (F_1, G_1, A) and (F_2, G_2, B) be two bipolar soft sets over X. Then,

(1) $\left((F_1, G_1, A) \widetilde{\cup} (F_2, G_2, B)\right)^c = (F_1, G_1, A)^c \widetilde{\cap} (F_2, G_2, B)^c$,
(2) $\left((F_1, G_1, A) \widetilde{\cap} (F_2, G_2, B)\right)^c = (F_1, G_1, A)^c \widetilde{\cup} (F_2, G_2, B)^c$.

Definition 2.8. [9] A bipolar soft set (F, G, A) over X is said to be relative null bipolar soft set, denoted by $\left(\Phi, \widetilde{X}, A\right)$, if for all $e \in A$, $F(e) = \emptyset$ and for all $\neg e \in \neg A$, $G(\neg e) = X$.

The relative null bipolar soft set with respect to the universe set of parameters E is called a NULL bipolar soft set over X and is denoted by $\left(\Phi, \widetilde{X}, E\right)$.

Definition 2.9. [9] A bipolar soft set (F, G, A) over X is said to be relative absolute bipolar soft set, denoted by $\left(\widetilde{X}, \Phi, A\right)$, if for all $e \in A$, $F(e) = X$ and for all $\neg e \in \neg A$, $G(\neg e) = \emptyset$.

The relative absolute bipolar soft set with respect to the universe set of parameters E is called a ABSOLUTE bipolar soft set over X and is denoted by $\left(\widetilde{X}, \Phi, E\right)$.

Definition 2.10. [8] Let $\widetilde{\widetilde{\tau}}$ be the collection of bipolar soft sets over X with E as the set of parameters. Then $\widetilde{\widetilde{\tau}}$ is said to be a bipolar soft topology over X if

(1) $\left(\Phi, \widetilde{X}, E\right)$ and $\left(\widetilde{X}, \Phi, E\right)$ belong to $\widetilde{\widetilde{\tau}}$
(2) the bipolar soft union of any number of bipolar soft sets in $\widetilde{\widetilde{\tau}}$ belongs to $\widetilde{\widetilde{\tau}}$
(3) the bipolar soft intersection of finite number of bipolar soft sets in $\widetilde{\widetilde{\tau}}$ belongs to $\widetilde{\widetilde{\tau}}$.

Then $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ is called a bipolar soft topological space over X .

Definition 2.11. [8] Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a bipolar soft topological space over X, then the members of $\widetilde{\widetilde{\tau}}$ are said to be bipolar soft open sets in X.

Definition 2.12. [8] Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a bipolar soft topological space over X. A bipolar soft set (F, G, E) over X is said to be a bipolar soft closed set in X, if its bipolar soft complemet $(F, G, E)^c$ belongs to $\widetilde{\widetilde{\tau}}$.

Definition 2.13. [8] Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a bipolar soft topological space over X. A bipolar soft set (F, G, E) over X is said to be a bipolar soft clopen set in X, if it is both a bipolar soft closed set and a bipolar soft open set over X.

3. The Main Results

Definition 3.1. Let (F, G, A) be a bipolar soft set over X. The presentation of

$$(F, G, A) = \{(e,\ F(e),\ G(\neg e)) : e \in A \subseteq E,\ \neg e \in \neg A \subseteq \neg E \text{ and } F(e),\ G(\neg e) \in P(X)\}$$

is said to be a short expansion of bipolar soft set (F, G, A).

Theorem 3.2. *Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a BSTS over X. Then*

(1) $\left(\Phi, \widetilde{X}, E\right), \left(\widetilde{X}, \Phi, E\right)$ *are bipolar soft closed sets over X,*
(2) *Arbitrary bipolar soft interscetions of the bipolar soft closed sets are bipolar soft closed set over X,*
(3) *Finite bipolar soft unions of the bipolar soft closed sets are bipolar soft closed set over X.*

Proof. 1. Since $\left(\Phi, \widetilde{X}, E\right)^c = \left(\widetilde{X}, \Phi, E\right) \in \widetilde{\widetilde{\tau}}$ and $\left(\widetilde{X}, \Phi, E\right)^c = \left(\Phi, \widetilde{X}, E\right) \in \widetilde{\widetilde{\tau}}$, then $\left(\Phi, \widetilde{X}, E\right), \left(\widetilde{X}, \Phi, E\right)$ are bipolar soft closed sets over X.

2. Let $\{(F_i, G_i, E)\}_{i \in I}$ be a family of bipolar soft closed sets over X. Then

$$\left(\widetilde{\bigcap_{i \in I}}(F_i, G_i, E)\right)^c = \widetilde{\bigcup_{i \in I}}(F_i, G_i, E)^c \in \widetilde{\widetilde{\tau}}.$$

Therefore, $\underset{i\in I}{\widetilde{\cap}}(F_i, G_i, E)$ is a bipolar soft closed set over X.

3. Let $\{(F_i, G_i, E)\}_{i=\overline{1,n}}$ be a finite family of bipolar soft closed sets over X. Then

$$\left(\overset{n}{\underset{i=1}{\widetilde{\cup}}}(F_i, G_i, E)\right)^c = \overset{n}{\underset{i=1}{\widetilde{\cap}}}(F_i, G_i, E)^c \in \widetilde{\widetilde{\tau}}.$$

Thus, $\overset{n}{\underset{i=1}{\widetilde{\cup}}}(F_i, G_i, E)$ is a bipolar soft closed set over X. □

Definition 3.3. Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a $BSTS$ over X and (F, G, E) be a bipolar soft set over X. Then the bipolar soft closure of (F, G, E), denoted by $\overline{(F, G, E)}$, is the bipolar soft intersection of all bipolar soft closed super sets of (F, G, E).

Obviously, $\overline{(F, G, E)}$ is the smallest bipolar soft closed set over X that containing (F, G, E).

Definition 3.4. Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a $BSTS$ over X and (F, G, E) be a bipolar soft set over X. Then the bipolar soft interior of (F, G, E), denoted by $(F, G, E)^\circ$, is the bipolar soft union of all bipolar soft open subsets of (F, G, E).

Obviously, $(F, G, E)^\circ$ is the biggest bipolar soft open set over X that is contained by (F, G, E).

Theorem 3.5. *Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a $BSTS$ over X, (F, G, E) be a bipolar soft sets over X. Then $\left[\overline{(F, G, E)}\right]^c = \left[(F, G, E)^c\right]^\circ$.*

Proof. From the definitions of a bipolar soft closure and a bipolar soft interior, we have

$$\left[\overline{(F, G, E)}\right]^c = \left(\widetilde{\bigcap_{\substack{(F_i, G_i, E)\widetilde{\supseteq}(F, G, E)\\ (F_i, G_i, E)^c \in \widetilde{\widetilde{\tau}}}}} (F_i, G_i, E)\right)^c = \widetilde{\bigcup}(F_i, G_i, E)^c = \left[(F, G, E)^c\right]^\circ.$$

□

Definition 3.6. Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a $BSTS$ over X and $\widetilde{\widetilde{B}} \subseteq \widetilde{\widetilde{\tau}}$. $\widetilde{\widetilde{B}}$ is said to be a bipolar soft basis for the bipolar soft topology $\widetilde{\widetilde{\tau}}$ if every element of $\widetilde{\widetilde{\tau}}$ can be written as the bipolar soft union of elements of $\widetilde{\widetilde{B}}$.

Definition 3.7. Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a $BSTS$ over X and $(F, G, E)\widetilde{\subseteq}\left(\widetilde{X}, \Phi, E\right)$. Then the collection

$$\widetilde{\widetilde{\tau}}_{(F,G,E)} = \left\{(F, G, E)\widetilde{\cap}(F_i, G_i, E) : (F_i, G_i, E) \in \widetilde{\widetilde{\tau}} \text{ for } i \in I\right\}$$

is called a bipolar soft subspace topology on (F, G, E) and $\left(X_{(F,G,E)}, \widetilde{\widetilde{\tau}}_{(F,G,E)}, E, \neg E\right)$ is called a bipolar soft topological subspace of $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$.

4. Conclusion

In this paper, we introduced some properties of bipolar soft topological spaces and the relationships between soft topological spaces and bipolar soft topological spaces. We hope that, the results of this study may help to next studies for many researchers.

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