5TH IFSCOM2018 PROCEEDING BOOK
ISBN: 978-605-68670-0-2

IFSCOM2018
5TH IFS AND CONTEMPORARY MATHEMATICS CONFERENCE
SEPTEMBER, 05-09, 2018, KAHRAMANMARAS, TURKEY
pp: 88-92

ON THE SIGMA INDEX OF MONOGENIC SEMIGROUP GRAPHS

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ABSTRACT. In [9], Das et.al. considered the monogenic semi group S_M with zero having $\{0, x, x^2, ..., x^n\}$. Also they defined undirected graph $\Gamma(S_M)$ associated with S_M whose vertices are the non-zero elements $x, x^2, ..., x^n$ and any two different vertices x^i and x^j are adjacent if $i + j > n$ (for $1 \leq i, j \leq n$).

In this paper we present sigma index monogenic semigroup graphs $\Gamma(S_M)$. Also we calculate forgotten (Zagreb) index of $\Gamma(S_M)$.

1. Introduction

Algebraic structures are very important for graph theory. Each commutative ring R can be described by means of a simple graph $\Gamma(R)$. There are many studies in the literature about zero-divisor graphs [3, 4, 5, 11, 12]. In a recently study Das, Akgunes and Cevik [9] the graph $\Gamma(S_M)$ is defined. The authors considered the finite multiplicative monogenic semigroup (with zero) $S_M = \{0, x, x^2, ..., x^n\}$. The vertices of this graph obtained S_M monogenic semigroup are the non-zero elements $\{x, x^2, ..., x^n\}$ and any two distinct vertices x^i and x^j are adjacent with the rule $x^i.x^j = 0$ if and only if $i + j > n$ $(1 \leq i, j \leq n)$.

In [9], later on giving the definition of $\Gamma(\mathcal{S}_M)$, it has been presented some certain results for the diameter, girth, maximum and minimum degree, cromatic number, clique number, degree sequence, irregularity index and dominating number for it.

In [2], to indicate the importance of this graph and by considering *First Zagreb index, Second Zagreb index, Randić index, geometric-arithmetic index* and *Atom-bond Connectivity index*, we will express some other certain results over $\Gamma(\mathcal{S}_M)$.

Topological indices are defined and used in many areas(physical, chemical, pharmacological, pharmaceutical, biological, etc.) to study several properties of different

2000 *Mathematics Subject Classification.* 05C10; 05C12; 15A18; 15A36.

Key words and phrases. Monogenic semigroup, Zero-divisor graph,sigma index, graph parameters.

objects such as atoms and molecules. Several topological indices have been defined and studied by mathematicians and chemists [7, 10, 17, 18].

Two of the most important degree–based topological graph indices are the first and second Zagreb indices:

$$M_1(G) = \sum_{i \in V} d_i^2 \quad \text{and} \quad M_2(G) = \sum_{ij \in E(G)} d_i d_j$$

respectively. These were introduced in the 1970s [[14, 16]].
One additional Zagreb–type index is the forgotten index [[13]].

$$F = F(G) = \sum_{i \in V} d_i^3$$

Also, recently, Gutman et all. [[15]] defined a new graph invariant *Sigma Index* as

$$\sigma = \sigma(G) = \sum_{uv \in E(G)} (d_u - d_v)^2.$$

In this paper we calculate F and σ for $\Gamma(S_M)$.

2. Indices results over $\Gamma(\mathcal{S}_M)$

In this main section, we will present some theorems and some lemmas over $\Gamma(\mathcal{S}_M)$ in terms of special graph indices that studied previously for some other graphs.

Firstly we give an algoritm about all neighborhood of $\Gamma(\mathcal{S}_M)$.

Lemma 2.1. [2]

To make some simplification in our calculations and so results, let us present the following algorithm about adjacency on $\Gamma(\mathcal{S}_M)$. By the definition, the neigbors of the vertices can be defined as in the following steps:

I_n : *The vertex x^n is adjacent to the all vertices x^{i_1} $(1 \leq i_1 \leq n-1)$ except itself.*

I_{n-1} : *The vertex x^{n-1} is adjacent to the all vertices x^{i_2} $(2 \leq i_2 \leq n-2)$ except itself and to the vertex x^n (by the first step I_n).*

I_{n-2} : *The vertex x^{n-2} is adjacent to the all vertices x^{i_3} $(3 \leq i_3 \leq n-3)$, the vertex x^n (by the first step I_n) and the vertex x^{n-1} (by the second step I_{n-1}).*

By keep going these steps in this algorithm, we have two possibilities at the final two stages depending on the n is even or odd:

<u>n is even</u>:

$I_{\frac{n}{2}+2}$: *The vertex $x^{\frac{n}{2}+2}$ is adjacent not only to the vertices $x^{\frac{n}{2}-1}$, $x^{\frac{n}{2}}$ and $x^{\frac{n}{2}+1}$ but also adjacent to the vertices x^n (by I_n), x^{n-1} (by I_{n-1}), x^{n-2} (by I_{n-2}),…, $x^{\frac{n}{2}+3}$ (by $I_{\frac{n}{2}+3}$).*

$I_{\frac{n}{2}+1}$: *The vertex* $x^{\frac{n}{2}+1}$ *is adjacent not only to the single vertex* $x^{\frac{n}{2}}$ *but also adjacent to the vertices* x^n *(by* I_n*),* x^{n-1} *(by* I_{n-1}*),* x^{n-2} *(by* I_{n-2}*), ...,* $x^{\frac{n}{2}+2}$ *(by* $I_{\frac{n}{2}+2}$*).*

<u>*n is odd*</u>*:*

$I_{\frac{n+1}{2}+2}$: *The vertex* $x^{\frac{n+1}{2}+2}$ *is adjacent not only to the vertices* $x^{\frac{n+1}{2}-2}$, $x^{\frac{n+1}{2}-1}$, $x^{\frac{n+1}{2}}$ *and* $x^{\frac{n+1}{2}+1}$ *but also adjacent to the vertices* x^n *(by* I_n*),* x^{n-1} *(by* I_{n-1}*),* x^{n-2} *(by* I_{n-2}*), ...,* $x^{\frac{n+1}{2}+3}$ *(by* $I_{\frac{n+1}{2}+3}$*).*

$I_{\frac{n+1}{2}+1}$: *The vertex* $x^{\frac{n+1}{2}+1}$ *is adjacent not only to the vertices* $x^{\frac{n+1}{2}-1}$ *and* $x^{\frac{n+1}{2}}$ *but also adjacent to the vertices* x^n *(by* I_n*),* x^{n-1} *(by* I_{n-1}*),* x^{n-2} *(by* I_{n-2}*), ...,* $x^{\frac{n+1}{2}+2}$ *(by* $I_{\frac{n+1}{2}+2}$*).*

In a usual way, let $d_1, d_2, \cdots, d_n$ denote the degrees of the vertices $x, x^2, \cdots, x^n$ in $\Gamma(\mathcal{S}_M)$, respectively. At this point we note that there exist so many studies regarding the degrees of vertices under different names such as *degree sequence* (we may refer [1, 9] and the references cited in them for a list of studies about this subject). In fact the following lemma is directly related with this subject which assures there exists an ordering $d_1 \leq d_2 \leq \cdots \leq d_n$ among degrees. Although the proof of this lemma can be found in [9], one can also see easily by considering the above effective algorithm.

Lemma 2.2 ([2])**.**

$$\left.\begin{aligned} d_1 = 1,\ d_2 = 2,\ \cdots,\ d_{\lfloor \frac{n}{2} \rfloor} = \left\lfloor \tfrac{n}{2} \right\rfloor,\ d_{\lfloor \frac{n}{2} \rfloor + 1} = \left\lfloor \tfrac{n}{2} \right\rfloor, \\ d_{\lfloor \frac{n}{2} \rfloor + 2} = \left\lfloor \tfrac{n}{2} \right\rfloor + 1,\ \cdots,\ d_n = n-1. \end{aligned}\right\} \tag{2.1}$$

Now we give a theorem and a lemma which are the playing an important role in our main results

Theorem 2.3 ([2])**.** *The second Zagreb index of the graph* $\Gamma(\mathcal{S}_M)$ *is*

$$M_2(\Gamma(\mathcal{S}_M)) = \begin{cases} \frac{5n^4-8n^3+10n^2-4n}{48} & ; \quad \text{if } n \text{ is even} \\ \frac{5n^4-8n^3-2n^2+8n-3}{48} & ; \quad \text{if } n \text{ is odd} \end{cases} .$$

Lemma 2.4 ([15])**.** *For any simple connected graph G*

$$\sigma(G) = F(G) - 2M_2(G)\,.$$

Now we will give first main theorem.

Theorem 2.5. *The forgotten Zagreb index of the graph* $\Gamma(\mathcal{S}_M)$ *is*

$$F(\Gamma(\mathcal{S}_M)) = \begin{cases} \frac{1}{8}n^2\left(2n^2-3n+2\right) & ; \quad \text{if } n \text{ is even} \\ \frac{1}{8}\left(n-1\right)^2\left(2n^2+n-1\right) & ; \quad \text{if } n \text{ is odd} \end{cases} .$$

Proof. In the proof, we will mainly consider (2.1). By the definition of forgotten Zagreb index,

$$\begin{aligned} F(\Gamma(\mathcal{S}_M)) &= \sum_{x^i \in V} d_i^3 = 1^3 + 2^3 + \cdots + \underbrace{\left(\left\lfloor \frac{n}{2} \right\rfloor\right)^3 + \left(\left\lfloor \frac{n}{2} \right\rfloor\right)^3} + \left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right)^3 + \cdots + (n-1)^3 \\ &= 1^3 + 2^3 + \cdots + \left(\left\lfloor \frac{n}{2} \right\rfloor\right)^3 + \left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right)^3 + \cdots + (n-1)^3 + \left(\left\lfloor \frac{n}{2} \right\rfloor\right)^3 \qquad (2.2) \\ &= \left(\frac{(n-1)n}{2}\right)^2 + \left(\left\lfloor \frac{n}{2} \right\rfloor\right)^3 . \end{aligned}$$

Now depends on the status of n in $\lfloor \frac{n}{2} \rfloor$, the result as desired. $\left(\text{Because } \lfloor \frac{n}{2} \rfloor = \begin{cases} \frac{n}{2} & ; \quad \text{if } n \text{ is even} \\ \frac{n-1}{2} & ; \quad \text{if } n \text{ is odd} \end{cases} . \right)$ □

Now we will give second main theorem.

Theorem 2.6. *The sigma index of the graph $\Gamma(\mathcal{S}_M)$ is*

$$\sigma(\Gamma(\mathcal{S}_M)) = \begin{cases} = \frac{1}{24} n(n-1)(n-2)(n+2) & ; \quad \text{if } n \text{ is even} \\ = \frac{1}{24} n(n-1)^2 (n+1) & ; \quad \text{if } n \text{ is odd} \end{cases} .$$

Proof. The main goal in here to characterize $\sigma = \sigma(G) = \sum\limits_{uv \in E(G)} (d_u - d_v)^2$ depending on the total number of degrees. We will consider definition of sigma index, (2.3), (2.4) and (2.5) and we calculate the sigma index of the graph $\Gamma(\mathcal{S}_M)$.

<u>*Assume that n is even*</u>:

$$\begin{aligned} \sigma(\Gamma(\mathcal{S}_M)) &= F(\Gamma(\mathcal{S}_M)) - 2M_2(\Gamma(\mathcal{S}_M)) \\ &= \frac{1}{8} n^2 \left(2n^2 - 3n + 2\right) - 2\frac{5n^4 - 8n^3 + 10n^2 - 4n}{48} \\ &= \frac{1}{24} n(n-1)(n-2)(n+2) . \end{aligned}$$

<u>*Assume that n is odd*</u>:

Following same steps as in n is even case, we obtain

$$\begin{aligned} \sigma(\Gamma(\mathcal{S}_M)) &= F(\Gamma(\mathcal{S}_M)) - 2M_2(\Gamma(\mathcal{S}_M)) \\ &= \frac{1}{8} (n-1)^2 \left(2n^2 + n - 1\right) - 2\frac{5n^4 - 8n^3 - 2n^2 + 8n - 3}{48} \\ &= \frac{1}{24} n(n-1)^2 (n+1) \end{aligned}$$

Hence the result. □

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