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BINOMIAL TRANSFORMS OF THE MORGAN-VOYCE SEQUENCES

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ABSTRACT. In this study, we apply binomial, k-binomial transforms to the Morgan-Voyce sequences. Also, the Binet formula, generating function of these transforms are presented and proved. We investigate some interesting properties between the obtained new sequences and the classical Morgan-Voyce sequences. Moreover, we introduce on infinite triangle consist of the terms of Morgan-Voyce sequences and their binomial, k-binomial transforms.

1. INTRODUCTION

Number sequences such as Fibonacci, Lucas, Pell, Jacobsthal, Narayana and Morgan-Voyce have been long interested mathematicians for their intrinsic theory and applications. It is known that there are a lot of article in the literature that concern about the number sequences. Morgan-Voyce [4] defined the two sequences $\{V_{k,n}\}$ and $\{M_{k,n}\}$ by

$$V_{k,n} = (2+k)\, V_{k,n-1} - V_{k,n-2}, \qquad V_{k,0} = 1, V_{k,1} = 1, \ n \geq 2 \tag{1.1}$$

and

$$M_{k,n} = (2+k)\, M_{k,n-1} - M_{k,n-2}, \qquad M_{k,0} = 0, \ M_{k,1} = 1, \ n \geq 2 \tag{1.2}$$

where k is a parameter that we assume to be in $\mathbb{Z}$. The first few values of these sequences are

$$\begin{aligned} \{V_{k,n}\} &= \left\{1, \ 1, \ 1+k, \ k^2+3k+1, ...\right\} \\ \{M_{k,n}\} &= \left\{0, \ 1, \ 2+k, \ k^2+4k+3, ...\right\}. \end{aligned}$$

In addition, some matrices based transforms can be introduced for a given sequence. Some of these transforms are the binomial transform, the rising and falling binomial transforms [5]. Many researchers studied on these transforms to these number sequences. Falcon and Plaza applied the binomial transform to the Fibonacci sequences [2]. Bhadouria et al. investigated binomial transform of Lucas

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sequences [1]. Yılmaz and Taşkara studied the binomial transform to Padovan and Perrin matrix sequences [7].

In [6], the binomial transform B of the sequence $X = \{x_0, x_1, x_2, ...\}$, which is denoted by $B(X) = \{b_n\}$ and defined by

$$b_n =_{i=0}^{n} \left(\begin{array}{c} n \\ i \end{array} \right) x_i.$$

Note that k is any positive integer number for main results in this paper.

2. Main Results

2.1. Binomial transforms of the Morgan-Voyce Sequences. In the section, we will present binomial transform Morgan-Voyce sequences. Also, the Binet formula, generating function of these transforms are found using recurrence relation.

Definition 2.1. Let $\{V_{k,n}\}$ and $\{M_{k,n}\}$ be the Morgan-Voyce sequences. The binomial transforms of these sequences are defined as follows:

i. $B_k = \{b_{k,n}\}$ the binomial transform of Morgan-Voyce $\{V_{k,n}\}$ is

$$b_{k,n} =_{i=0}^{n} \left(\begin{array}{c} n \\ i \end{array} \right) V_{k,i} \tag{2.1}$$

ii. $C_k = \{c_{k,n}\}$ the binomial transform of Morgan-Voyce $\{M_{k,n}\}$ is

$$c_{k,n} =_{i=0}^{n} \left(\begin{array}{c} n \\ i \end{array} \right) M_{k,i}. \tag{2.2}$$

From Definition 2.1, it is easy to obtain that $B_1 = \{1, 2, 5, 15, 50, \ldots\}$ is binomial transform of the classical Morgan-Voyce $\{V_{1,n}\} = \{1, 1, 2, 5, 13, 34, \ldots\}$ and $C_1 = \{0, 1, 5, 20, 75, 275 \ldots\}$ is binomial transform of the classical Morgan- Voyce $\{M_{1,n}\} = \{0, 1, 3, 8, 21, 55, \ldots\}$. A few binomial transform of Morgan-Voyce sequences $\{V_{k,n}\}, \{M_{k,n}\}$.

$B_1 = \{1, 2, 5, 15, 50, 175, \ldots..\}$
$B_2 = \{1, 2, 6, 24, 108, 504, \ldots..\}$
$B_3 = \{1, 2, 7, 35, 196, 1127, \ldots..\}$
$B_4 = \{1, 2, 8, 48, 320, 2176, \ldots..\}$
$C_1 = \{0, 1, 5, 20, 75, 275, 100, \ldots\}$
$C_2 = \{0, 1, 6, 30, 144, 684, 3240, \ldots\}$
$C_3 = \{0, 1, 7, 42, 245, 1421, 8232, ...\}$
$C_4 = \{0, 1, 8, 56, 384, 2624, 17920, \ldots\}$

From Defination 2.1, we can obtain the following Lemma we need to main theorem of this paper.

Lemma 2.2. *The binomial transforms of Morgan-Voyce sequences $\{V_{k,n}\}$ and $\{M_{k,n}\}$ sequences are held:*

i.

$$b_{k,n+1} =_{i=0}^{n} \left(\begin{array}{c} n \\ i \end{array} \right) (V_{k,i+1} + V_{k,i}) \tag{2.3}$$

ii.

$$c_{k,n+1} =_{i=0}^{n} \left(\begin{array}{c} n \\ i \end{array} \right) (M_{k,i+1} + M_{k,i}) \tag{2.4}$$

Proof. **i.** By using equation (2.1) and the well known binomial equality $\binom{n+1}{i} = \binom{n}{i} + \binom{n}{i-1}$. So, we obtain

$$
\begin{aligned}
b_{k,n+1} &= {}_{i=1}^{n+1} \binom{n+1}{i} V_{k,i} + V_{k,0} \\
&= {}_{i=1}^{n+1} \binom{n}{i} V_{k,i} + {}_{i=1}^{n+1} \binom{n}{i-1} V_{k,i} + V_{k,0} \\
&= {}_{i=0}^{n} \binom{n}{i} V_{k,i} + {}_{i=0}^{n} \binom{n}{i} V_{k,i+1} \\
&= {}_{i=0}^{n} \binom{n}{i} (V_{k,i+1} + V_{k,i})
\end{aligned}
$$

where $\binom{n}{n+1} = 0$. This is desired result.

Proof of (ii) can be obtained in the same manner with (i). □

Now we present the main Theorem of this paper.

Theorem 2.3. *For $n > 0$,*

i. *recurrence relation of binomial transform of Morgan-Voyce sequence, $B_k = \{b_{k,n}\}$ is*

(2.5) $$b_{k,n+1} = (k+4)\, b_{k,n} - (k+4)\, b_{k,n-1}$$

with initial conditions $b_{k,0} = 1$ and $b_{k,1} = 2$,

ii. *recurrence relation of binomial transform of Morgan-Voyce sequence, $C_k = \{c_{k,n}\}$ is*

(2.6) $$c_{k,n+1} = (k+4)\, c_{k,n} - (k+4)\, c_{k,n-1}$$

with initial conditions $c_{k,0} = 0$ and $c_{k,1} = 1$.

Proof. **i.** From Lemma 2.2 and equation (1.1), we have

$$
\begin{aligned}
b_{k,n+1} &= {}_{i=0}^{n} \binom{n}{i} (V_{k,i+1} + V_{k,i}) \\
&= V_{k,0} + V_{k,1} + {}_{i=1}^{n} \binom{n}{i} ((k+2)\, V_{k,i} - V_{k,i-1} + V_{k,i}) \\
&= (k+3) {}_{i=1}^{n} \binom{n}{i} V_{k,i} - {}_{i=1}^{n} \binom{n}{i} V_{k,i-1} + 2 \\
&= (k+3) {}_{i=0}^{n} \binom{n}{i} V_{k,i} - {}_{i=1}^{n} \binom{n}{i} V_{k,i-1} - k - 1
\end{aligned}
$$

and so,

(2.7) $$b_{k,n+1} = (k+3)\, b_{k,n} - {}_{i=1}^{n} \binom{n}{i} V_{k,i-1} - k - 1$$

By using that $\left(\begin{array}{c} n \\ n+1 \end{array}\right) = 0$, we have from equation (2.7)

$$\begin{aligned} b_{k,n} &= (k+3)\, b_{k,n-1} - {}_{i=1}^{n-1} \left(\begin{array}{c} n-1 \\ i \end{array}\right) V_{k,i-1} - k - 1 \\ &= (k+4)\, b_{k,n-1} - \left[{}_{i=0}^{n-1} \left(\begin{array}{c} n-1 \\ i \end{array}\right) V_{k,i} + {}_{i=1}^{n-1} \left(\begin{array}{c} n-1 \\ i \end{array}\right) V_{k,i-1} \right] - k - 1 \\ &= (k+4)\, b_{k,n-1} - \left[{}_{i=1}^{n} \left(\begin{array}{c} n-1 \\ i-1 \end{array}\right) V_{k,i-1} + {}_{i=1}^{n} \left(\begin{array}{c} n-1 \\ i \end{array}\right) V_{k,i-1} \right] - k - 1 \\ &= (k+4)\, b_{k,n-1} - {}_{i=1}^{n} \left(\begin{array}{c} n \\ i \end{array}\right) V_{k,i-1} - k - 1. \end{aligned}$$

So, we have

$$b_{k,n} = (k+4)\, b_{k,n-1} - {}_{i=1}^{n} \left(\begin{array}{c} n \\ i \end{array}\right) V_{k,i-1} - k - 1 \tag{2.8}$$

From equations (2.7) and (2.8), we obtain

$$\begin{aligned} b_{k,n+1} - (k+3)b_{k,n} + k + 1 &= b_{k,n} - (k+4)b_{k,n-1} + k + 1 \\ b_{k,n+1} &= (k+4)b_{k,n} - (k+4)b_{k,n-1} \end{aligned}$$

which is desired result.

ii. Proof of (ii) can be obtained in the same manner with (i). □

2.1.1. *Binet's formula for the binomal transforms of the Morgan-Voyce sequences.* The characteristic equation of sequences $\{b_{k,n}\}$ and $\{c_{k,n}\}$ in equations (2.5) and (2.6) is $\lambda^2 - (k+4)\lambda + k + 4 = 0$. Let λ_1 and λ_2 be roots of this equation.

Theorem 2.4. ***i.*** *The n. term of the binomial transform B_k of Morgan-Voyce sequences $\{V_{k,n}\}$ is given by*

$$b_{k,n} = \frac{(2-\lambda_2)\,\lambda_1^n - (2-\lambda_1)\,\lambda_2^n}{\lambda_1 - \lambda_2} \tag{2.9}$$

where $\lambda_1 = \frac{k+4+\sqrt{k^2+4k}}{2}$ *and* $\lambda_2 = \frac{k+4-\sqrt{k^2+4k}}{2}$.

ii. *The n. term of the binomial transform C_k of Morgan-Voyce sequences $\{M_{k,n}\}$ sequence is given by*

$$c_{k,n} = \frac{\lambda_1^n - \lambda_2^n}{\lambda_1 - \lambda_2} \tag{2.10}$$

where $\lambda_1 = \frac{k+4+\sqrt{k^2+4k}}{2}$ *and* $\lambda_2 = \frac{k+4-\sqrt{k^2+4k}}{2}$.

Proof. **i.** The characteristic equation of $b_{k,n+1} = (k+4)\, b_{k,n} - (k+4)\, b_{k,n-1}$ recurrence formula is $\lambda^2 - (k+4)\lambda + k + 4 = 0$. The solutions of this equation are λ_1 and λ_2. The general term of binomial transforms of the B_k sequence may be expressed in the form $b_{k,n} = A\lambda_1^n + B\lambda_2^n$ for some cofficients A and B. For $n = 0$ and $n = 1$, it is obtained

$$A = \frac{2-\lambda_2}{\lambda_1 - \lambda_2} \qquad \text{and} \qquad B = \frac{\lambda_1 - 2}{\lambda_1 - \lambda_2}.$$

So, we obtain desired formula. Also, the roots λ_1 and λ_2 verifies the relation such as

$$\begin{aligned} \lambda_1 \lambda_2 &= k+4 \\ \lambda_1 + \lambda_2 &= k+4 \\ \lambda_1 - \lambda_2 &= \sqrt{k^2+4k} \end{aligned}$$

ii. Proof of (ii) can be obtained in the same manner with (i). □

2.1.2. *Generating function for the binomial transforms of the Morgan-Voyce sequences.* In this section, it is given the generating function of the binomial transforms B_k and C_k of the Morgan-Voyce sequences .

A generating function $g(x)$ is a formal power series

$$g(x) =_{n=0}^{\infty} a_n x^n$$

whose coefficients give the sequence $\{a_0, a_1, \ldots\}$. Given a generating function is the analytic expression for the n. term in the corresponding series.

First we will consider binomial transforms B_k of Morgan-Voyce sequence $\{V_{k,n}\}$. Let the binomial transform of the Morgan-Voyce numbers are cofficient of a potential series centered at the origin and consider the corresponding analytic function $b_k(x)$ such that

$$b_k(x) = b_{k,0} + b_{k,1}x + b_{k,2}x^2 + \cdots$$

Then we can write

$$\begin{aligned} (k+4)b_k(x)x &= (k+4)b_{k,0}x + (k+4)b_{k,1}x^2 + (k+4)b_{k,2}x^3 + \cdots \\ (k+4)b_k(x)x^2 &= (k+4)b_{k,0}x^2 + (k+4)b_{k,1}x^3 + (k+4)b_{k,2}x^4 + \cdots \end{aligned}$$

From the above last equations, we obtain

$$\begin{aligned} b_k(x) - (k+4)b_k(x)x + (k+4)b_k(x)x^2 &= b_{k,0} + b_{k,1}x - (k+4)b_{k,0}x \\ \left(1 - (k+4)x + (k+4)x^2\right) b_k(x) &= 1 + 2x - (k+4)x \end{aligned}$$

where $b_{k,i} = (k+4)\, b_{k,i-1} - (k+4)\, b_{k,i-2}$, $b_{k,0} = 1$ and $b_{k,1} = 2$ from equation (2.5). So, the generating function of the binomial transform $B_k = \{b_{k,n}\}$ of the Morgan-Voyce sequence $\{V_{k,n}\}$ is

$$b_k(x) = \frac{1-(k+2)x}{1-(k+4)x+(k+4)x^2}. \tag{2.11}$$

Similarly to the binomial transforms B_k of Morgan-Voyce sequence $\{V_{k,n}\}$, the generating function of the binomial transform $C_k = \{c_{k,n}\}$ of the Morgan-Voyce sequence $\{M_{k,n}\}$ is

$$c_k(x) = \frac{x}{1-(k+4)x+(k+4)x^2}. \tag{2.12}$$

2.1.3. *2.1.3 Triangles of the binomial transforms of the Morgan-Voyce sequences.* In this section, we introduce an infinite triangle consist of the elements of Morgan-Voyce sequences and their binomial transforms. For each integer k, let be T_k an infinite triangle of numbers by using the following rule:

i. The left diagonal of the triangle consists of the elements of the Morgan-Voyce numbers,

ii. Any number off the left diagonal is the sum of the number to its left and number diagonally above it to the left.

Then, the sequence on the right diagonal is the binomial transforms of the Morgan-Voyce sequences.

For example, the triangle T_1 for the Morgan-Voyce $\{V_{1,n}\}$ and its binomial transform is following:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 2 & & & \\ & & 2 & & 3 & & 5 & & \\ & 5 & & 7 & & 10 & & 15 & \\ 13 & & 18 & & 25 & & 35 & & 50 \end{array}$$

Not that the antidioganal sequences $\{t_{1,n}\}$ of this triangle verify the $t_{1,n+1} = 3t_{1,n} - t_{1,n-1}$ relation as the Morgan-Voyce sequence $\{V_{1,n}\}$ while the all diagonal sequenecs hold $t_{1,n+1} = 5t_{1,n} - 5t_{1,n-1}$. That is the sequence on the left diagonal $\{1, 1, 2, 5, 13, 34, \ldots\}$ is the Morgan-Voyce sequence $\{V_{1,n}\}$ and sequence on the right diagonal $\{1, 2, 5, 15, 50, 175, \ldots..\}$ is the binomial transform of the Morgan-Voyce sequence $B_1 = \{b_{1,n}\}$ which was defined in the previous section. So, every antidiogonal sequence $\{t_{k,n}\}$ of this triangle verify the relation as the Morgan-Voyce sequence $\{V_{k,n}\}$.

2.2. **The k-Binomial Transforms of the Morgan-Voyce Sequences.**

Definition 2.5. $r_{k,n}$ and $s_{k,n}$ are defined by

$$r_{k,n} =_{i=0}^{n} \binom{n}{i} k^n V_{k,i} \quad \text{for } k \neq 0 \text{ or } n \neq 0 \tag{2.13}$$

and

$$s_{k,n} =_{i=0}^{n} \binom{n}{i} k^n M_{k,i} \quad \text{for } k \neq 0 \text{ or } n \neq 0 \tag{2.14}$$

where $r_{k,n} = s_{k,n} = 0$ if $k = 0$ and $n = 0$. The sequence $R_k = \{r_{k,n}\}$ is k-binomial tranform of the Morgan-Voyce sequnce $\{V_{k,n}\}$ and $S_k = \{s_{k,n}\}$ is k-binomial tranform of the Morgan-Voyce sequnce $\{M_{k,n}\}$.

From Definition 2.5, we can write the a few k-binomial tranforms R_k and S_k of the Morgan-Voyce sequnces $\{V_{k,n}\}$ and $\{M_{k,n}\}$ as following:

$$\begin{aligned} R_1 &= \{1, 2, 5, 15, 50, 175, \ldots\} \\ R_2 &= \{1, 4, 24, 192, 1728, \ldots\} \\ R_3 &= \{1, 6, 63, 945, 15876, \ldots\} \\ R_4 &= \{1, 8, 128, 3072, 81920, \ldots\} \\ S_1 &= \{0, 1, 5, 20, 75, \ldots\} \\ S_2 &= \{0, 2, 24, 240, 2304, \ldots\} \\ S_3 &= \{0, 3, 63, 1134, 19845, \ldots\} \\ S_4 &= \{0, 4, 128, 3584, 98304, \ldots\} \end{aligned}$$

We note that the 1-binomial transform R_1 equals the binomial transform B_1 and 1-binomial transform S_1 equals the binomial transform C_1.

Theorem 2.6. *For $n > 0$,*

i. ***i.*** *recurrence relation of k-binomial transform of Morgan-Voyce sequence, $R_k = \{r_{k,n}\}$ is*

$$r_{k,n+1} = \left(k^2 + 4k\right) r_{k,n} - \left(k^3 + 4k^2\right) r_{k,n-1} \tag{2.15}$$

with initial conditions $r_{k,0} = 1$ and $r_{k,1} = 2k$,

ii. *recurrence relation of binomial transform of Morgan-Voyce sequence, $S_k = \{s_{k,n}\}$ is*

$$s_{k,n+1} = \left(k^2 + 4k\right) s_{k,n} - \left(k^3 + 4k^2\right) s_{k,n-1} \tag{2.16}$$

with initial conditions $s_{k,0} = 0$ and $s_{k,1} = k$.

Proof. **i.** If we combine the equations (2.1) and (2.13), we can write

$$r_{k,n} = k^n b_{k,n}.$$

Then

$$\begin{aligned} r_{k,n+1} &= k^{n+1} b_{k,n+1} \\ &= k^{n+1}\left((k+4)\, b_{k,n} - (k+4)\, b_{k,n-1}\right) \\ &= \left(k^2 + 4k\right) k^n b_{k,n} - \left(k^3 + 4k^2\right) k^{n-1} b_{k,n-1} \end{aligned}$$

and we get

$$r_{k,n+1} = \left(k^2 + 4k\right) r_{k,n} - \left(k^3 + 4k^2\right) r_{k,n-1}$$

which is the required result.

ii. Proof of (ii) can be obtained in the same manner with (i). □

2.2.1. *Binet's formula, generating function and Triangles for the k-binomal transform of the Morgan-Voyce sequences.* The characteristic equation of sequences $\{r_{k,n}\}$ and $\{s_{k,n}\}$ in equations (2.15) and (2.16) is $\gamma^2 - (k^2 + 4k)\gamma + k^3 + 4k^2 = 0$. Let γ_1 and γ_2 be roots of this equation.

Theorem 2.7. ***i.*** *The n. term of the k-binomial transform R_k of Morgan-Voyce sequences $\{V_{k,n}\}$ is given by*

$$r_{k,n} = \frac{(2-\gamma_2)\,\gamma_1^n - (2-\gamma_1)\,\gamma_2^n}{\gamma_1 - \gamma_2}$$

where $\gamma_1 = \frac{k+4+\sqrt{k^2+4k}}{2}k$ and $\gamma_2 = \frac{k+4-\sqrt{k^2+4k}}{2}k$.

ii. *The n. term of the k-binomial transform S_k of Morgan-Voyce sequences $\{M_{k,n}\}$ is given by*

$$s_{k,n} = \frac{\gamma_1^n - \gamma_2^n}{\gamma_1 - \gamma_2}$$

where $\gamma_1 = \frac{k+4+\sqrt{k^2+4k}}{2}k$ and $\gamma_2 = \frac{k+4-\sqrt{k^2+4k}}{2}k$.

Proof. Proofs of (i) and (ii) are same as that of binomial transforms of Morgan-Voyce sequences, in Theorem 2.4. Also, the roots λ_1 and λ_2 verifies the relation such as

$$\begin{aligned} \gamma_1\gamma_2 &= k^3 + 4k^2 \\ \gamma_1 + \gamma_2 &= k^2 + 4k \\ \gamma_1 - \gamma_2 &= k\sqrt{k^2+4k}. \end{aligned}$$

□

Now we present generating functions of the k-binomial transform $R_k = \{r_{k,n}\}$ and $S_k = \{s_{k,n}\}$ of Morgan-Voyce sequences $\{V_{k,n}\}$ and $\{M_{k,n}\}$. Similarly to that for binomial transforms equations (2.11) and (2.12), we can obtain the generating functions for k-binomial transforms are

$$r_k(x) = \frac{1-(k^2+2k)x}{1-(k^2+4k)x+(k^3+4k^2)x^2}$$

and

$$s_k(x) = \frac{kx}{1-(k^2+4k)x+(k^3+4k^2)x^2}.$$

We introduce an infinite triangle consist of the elements of Morgan-Voyce sequences and their k-binomial transforms. The left diagonal consists of the elements of the Morgan-Voyce numbers and any number off the left diagonal is k-times the sum of the number to its left and number diagonally above it to the left. Then the right diagonal is the k-binomial transforms R_k and S_k of $\{V_{k,n}\}$ and $\{M_{k,n}\}$.

For example, the following triangle for 2-Morgan Voyce sequence and 2-binomial transforms:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 4 & & & \\ & & 3 & & 8 & & 24 & & \\ & 11 & & 28 & & 72 & & 192 & \\ 41 & & 104 & & 264 & & 672 & & 1728 \end{array}$$

Not that the right diagonal $\{1, 4, 24, 192, 1728, ...\}$ is 2-binomial transforms R_2 of the 2-Morgan Voyce sequence $\{V_{2,n}\}$, as in the above definition. The triangle for the k-Morgan Voyce sequence $\{V_{k,n}\}$ and its k-binomial transforms can be obtained in the same manner with above definition.

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