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ROUGH OPERATORS ON L^x DEFINED BY A SOFT SET

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Abstract. In [2], the notion of a soft set in **L**-set theory is defined, several operators for **L**-soft set theory are introduced, and the rough operators on the set of all **L**-soft sets induced by the rough operators on L^x are investigated. As a continuation, in this paper, rough operators on L^x are studied and discussed their properties.

1. Introduction

Rough set theory was proposed firstly by Pawlak [8] for analyzing data and reasoning about it. The rough set theory can be considered as a tool to reduce the input dimensionality and to deal with vagueness and uncertainty in data sets. There has been a rapid growth in interest in this theory and its applications especially in research areas.

The notion of L-set was introduced in [4] as a generalization of [9]. This paper discusses the basic concepts of rough set theory and point out some rough set-based research directions to L-set theory and applications.

For this research, inspired by [2], the generalization of soft sets in fuzzy settings is studied. This enables us to see clearly the motivations for developing L-set theory, its basic components and appropriate applications, leading to an appreciation for these theories.

2. Preliminaries

Definition 2.1. [7] A pair (f, E) is called a soft set (over U) if and only if f is a mapping or E into the set of all subsets of the set U.

From now on, we will use definitions and operations about soft sets which are more suitable for pure mathematics based on study of [7].

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Definition 2.2. [7] A soft set f on the universe U is defined by the set of ordered pairs

$$f = \Big\{\big(e, f(e)\big) : e \in E\Big\}$$

where $f : E \to \mathcal{P}(U)$ such that $f(e) = \emptyset$ if $e \in E \setminus A$ then $f = f_A$.

Note that the set of all soft sets over U will be denoted by $\mathbb{S}$.

Definition 2.3. [2] (L-sets)
For a universe set X, an L-set in X is a mapping $\widetilde{A} : X \to L$. $\widetilde{A}(x)$ indicates the truth degree of "x belongs to $\widetilde{A}$". We use the symbol L^x to denote the set of all L-sets in X.

Definition 2.4. [5] (Fuzzy Soft Sets)

Let X be a universe and E a set of attributes. Then the pair $\widetilde{(X, E)}$ denotes the collection of all fuzzy soft sets on X with attributes from E and is called a fuzzy soft class.

Definition 2.5. [8] (Rough Sets)

Let X be a universe and E a set of attributes. Then the pair $\widetilde{(X, E)}$ denotes the collection of all fuzzy soft sets on X with attributes from E and is called a fuzzy soft class.

Definition 2.6. [2] (L-soft set)
A pair (F, A) is called an L-soft set over X, if $A \subseteq E$ and $F \to L^X$, denoted by $\theta = (F, A)$.

Definition 2.7. [2] Let $LS(X)$ be the set of all L-soft sets over X. Suppose $(F, A), (G, B) \in LS(X)$ are two L-soft sets. Then the union of (F, A) and (G, B) is an L-soft set (H, C), where $C = A \cup B$, and for $t \in C$,

$$H(t) = \begin{cases} F(t) & \text{if } t \in A - B \\ G(t) & \text{if } t \in B - A \\ F(t) \vee G(t) & \text{if } t \in A \cap B \end{cases}$$

and written as $(F, A) \tilde{\bigcup} (G, B) = (H, C)$.

In [5], Maji et al. also defined the intersection of two fuzzy soft sets, i.e., suppose $(F, A), (G, B) \in LS(X)$ are two L-soft sets, then the intersection of (F, A) and (G, B) is also an L-soft set (K, D), where $D = A \cap B$, and for $t \in D$, $K(t) = F(t)$ or $G(t)$ (as both are the same L-set).

But generally $F(t) = G(t)$ does not hold, and $A \cap B$ may be a empty set. So Ali et al. introduced a new definition that is called the restricted intersection in [1].

Definition 2.8. [2] Suppose $(F, A), (G, B) \in LS(X)$ are two L-soft sets. Then the extended intersection of (F, A) and (G, B) is also an L-soft set (J, C), where $C = A \cup B$, and for $t \in C$,

$$J(t) = \begin{cases} F(t) & \text{if } t \in A - B \\ G(t) & \text{if } t \in B - A \\ F(t) \wedge G(t) & \text{if } t \in A \cap B \end{cases}$$

and written as $(F, A) \sqcap (G, B) = (J, C)$.

Definition 2.9. [2] The complement of an L-soft set (F, A) is denoted by $(F, A)^c$, and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to L^X$, for every $-t \in \rceil A$,

$$F^c(-t) = F^*(t) = F(t) \to 0.$$

Finally, the operators OR,AND on $LS(X)$ can be defined:

Definition 2.10. [2] Suppose $(F, A), (G, B) \in LS(X)$, $(F; A)$ AND $(G; B)$ is an L-soft set, denoted by $(F, A)\tilde{\wedge}(G, B) = (H, C)$, where $C = A \times B$, for every $t_1 \in A, t_2 \in B, H(t_1, t_2) = F(t_1) \wedge G(t_2)$.

(F, A) OR (G, B) is an L-soft set, denoted by $(F, A)\tilde{\vee}(G, B) = (K, D)$, where $D = A \times B$, for every $t_1 \in A; t_2 \in B, K(t_1, t_2) = F(t_1) \vee G(t_2)$.

3. Rough Operators on L^X Defined by a Soft Set

In the section, suppose X is a universe set, L^X is the set of all L-sets in X, E is a collection of all possible parameters with respect to X. Apart form definitions and theorems are numbered, known concepts are mentioned in the text along with the reference [2].

Definition 3.1. [2] Suppose $\theta = (F, A)$ is an L-soft set, two L-rough operators N and H are defined as follows: for every $\tilde{A} \in L^X; x \in X$,

$$N(\tilde{A})(x) = \bigvee_{t \in A} F(t)(x) \otimes S(F(t), \tilde{A}),$$

$$H(\tilde{A})(x) = \bigvee_{t \in A} F(t)(x) \otimes \rho(F(t), \tilde{A}).$$

If $N(\tilde{A}) = H(\tilde{A})$, then $\tilde{A}$ is called a definable L-set, otherwise, $\tilde{A}$ is called an undefinable L-set. $(N(\tilde{A}), H(\tilde{A}))$ is referred to as a pair of L-rough set.

Proposition 1. [2]
1. $H(\tilde{0}_X) = \tilde{0}_X$,
2. $S(\tilde{A}, \tilde{B}) \leq S(N(\tilde{A}), N(\tilde{B}))$,
3. $S(\tilde{A}, \tilde{B}) \leq S(H(\tilde{A}), \tilde{B})$.

Remark 3.2. (F, A) is called a full soft set, if $\bigvee_{t \in A} F(t) = \tilde{1}_X$. i.e., $\{F(t) | t \in A\}$ is a cover of L^X. In the case, we have $N(\tilde{1}_X) = \tilde{1}_X$.

In [1], the set of all lower approximations and the set of all upper approximations form complete lattices. Algebraic structure of P and Q is introduced in [2] can be seen in the next proposition:

Proposition 2. [2] Suppose $\{\tilde{A}_i | i \in I\} \subseteq L^X$. Then
1. $\bigvee_{i \in I} N(\tilde{A}_i) \subseteq N(\bigvee_{i \in I} \tilde{A}_i)$,
2. $N(\bigwedge_{i \in I} \tilde{A}_i) \subseteq \bigwedge_{i \in I} N(\tilde{A}_i)$,
3. $H(\bigwedge_{i \in I} \tilde{A}_i) \subseteq \bigwedge_{i \in I} H(\tilde{A}_i)$.

L satisfies idempotency, if it satisfies $a \otimes a = a$, for every $a \in L$ (See [1]).

Proposition 3. [2] Suppose L satisfies idempotency. Then for $\tilde{A}, \tilde{B} \in L^X$,

$$H(\tilde{A}) \otimes H(\tilde{B}) = H(\tilde{A} \otimes \tilde{B}).$$

Proposition 4. [2] Suppose $Q = \{H(\tilde{A}) | \tilde{A} \in L^X\}$ is closed for $\bigcup$, that is, $\mathcal{U}$ is an L-set in Q. Then $\bigcup \mathcal{U} \in Q$.

Remark 3.3. [2] By the above propositions, we know that if L satisfies idempotency, Q is an L-open topology on X (See [1]). Unfortunately, P does not form an L-topology on X.

In fact, Q is a semilattice with respect to $\bigcup$, and the minimal element is 0_X.

Conclusion

This paper discusses the basic concepts of rough set theory and point out some rough set-based research directions to L-set theory and applications. In addition to this, two rough operators on L^X by an L-soft set is studied and discussed some of their properties by propositions.

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