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APPLICATION OF IMPLICIT RELATIONS TO FIXED POINT THEOREMS IN COMPLETE SOFT QUASI METRIC SPACE

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ABSTRACT

The concept of soft quasi-metric space, according to soft element and some of its properties are given very recently by Bilgili Gungor [2](In review). And an implicit contraction mapping via soft real numbers inspired from the article of Popa and Patriciu [12] is defined by Bilgili Gungor [3](In review). In this paper some examples of implicit contraction mappings via soft real numbers are given and then fixed point theorems are proved by using these implicit relations.

1. Introduction and Preliminaries

Throughout this paper, we follow the notations, used in [[5]] and [[6]]. For the sake of completeness, we recall some basic definitions, notations and results.

Definition 1.1. *([5]) A mapping $d : SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ is said to be a soft metric on $\widetilde{X}$ if d satisfies the following conditions:*
(M1) $d(\widetilde{x}, \widetilde{y})\widetilde{\geq}\bar{0}$, for all $\widetilde{x}, \widetilde{y}\hat{\in}\widetilde{X}$.
(M2) $d(\widetilde{x}, \widetilde{y}) = \bar{0}$ if and only if $\widetilde{x} = \widetilde{y}$,
(M3) $d(\widetilde{x}, \widetilde{y}) = d(\widetilde{y}, \widetilde{x})$, for all $\widetilde{x}, \widetilde{y}\hat{\in}X$,
(M4) $d(\widetilde{x}, \widetilde{y})\widetilde{\leq}d(\widetilde{x}, \widetilde{z}) + d(\widetilde{z}, \widetilde{y})$, for all $\widetilde{x}, \widetilde{y}, \widetilde{z}\hat{\in}X$.
The soft set $\widetilde{X}$ with a soft metric d on $\widetilde{X}$ is said to be a soft metric space and is denoted by $(\widetilde{X}, d)$.

Definition 1.2. *([5]) Let $(\widetilde{x_n})$ be a sequence of soft elements in $(\widetilde{X}, d)$. The sequences $(\widetilde{x_n})$ is said to be convergent in $(\widetilde{X}, d)$, if there is a soft element $\widetilde{x}\hat{\in}\widetilde{X}$ such that $d(\widetilde{x_n}, \widetilde{x}) \to \bar{0}$ as $n \to \infty$.*
A sequence $(\widetilde{x_n})$ of soft elements in $(\widetilde{X}, d)$ is said to be Cauchy sequence in $\widetilde{X}$, if for every $\widetilde{\epsilon}\widetilde{\geq}\bar{0}$, there is a natural number m such that $d(\widetilde{x_i}, \widetilde{x_j})\widetilde{\leq}\widetilde{\epsilon}$, whenever $i, j \geq m$.

Definition 1.3. *([5]) A soft metric space $(\widetilde{X}, d)$ is said to be complete if every Cauchy sequence in $\widetilde{X}$ converges to some soft element of $\widetilde{X}$.*

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Definition 1.4. *([6]) Let X be a nonempty set and E be the nonempty set of parameters. A mapping $\widetilde{G} : SE(\widetilde{X}) \times SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ is said to be a soft generalized metric or soft G-metric on $\widetilde{X}$, if $\widetilde{G}$ satisfies the following conditions:*
$(\widetilde{G}_1)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \bar{0}$, *if* $\widetilde{x} = \widetilde{y} = \widetilde{z}$,
$(\widetilde{G}_2)$ $\widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y}) \widetilde{>} \bar{0}$, *for all* $\widetilde{x}, \widetilde{y} \in SE(\widetilde{X})$ *with* $\widetilde{x} \neq \widetilde{y}$
$(\widetilde{G}_3)$ $\widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y}) \widetilde{\leq} \widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z})$, *for all* $\widetilde{x}, \widetilde{y}, \widetilde{z} \in SE(\widetilde{X})$ *with* $\widetilde{y} \neq \widetilde{z}$,
$(\widetilde{G}_4)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \widetilde{G}(\widetilde{x}, \widetilde{z}, \widetilde{y}) = \widetilde{G}(\widetilde{y}, \widetilde{z}, \widetilde{x}) = ...$
$(\widetilde{G}_5)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) \widetilde{\leq} \widetilde{G}(\widetilde{x}, \widetilde{a}, \widetilde{a}) + \widetilde{G}(\widetilde{a}, \widetilde{y}, \widetilde{z})$, *for all* $x, y, z, a \in SE(\widetilde{X})$.
The soft set $\widetilde{X}$ with a soft G-metric $\widetilde{G}$ on $\widetilde{X}$ is said to be a soft G-metric space and is denoted by $(\widetilde{X}, \widetilde{G}, E)$.

Proposition 1.1. *([6]) For any soft metric d on $\widetilde{X}$, we can construct a soft G-metric by the following mappings $\widetilde{G}_s$ and $\widetilde{G}_m$:*
(1) $\widetilde{G}_s(d)(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \frac{\bar{1}}{3}(d(\widetilde{x}, \widetilde{y}) + d(\widetilde{y}, \widetilde{z}) + d(\widetilde{x}, \widetilde{z}))$,
(2) $\widetilde{G}_m(d)(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \max\{d(\widetilde{x}, \widetilde{y}) + d(\widetilde{y}, \widetilde{z}) + d(\widetilde{x}, \widetilde{z})\}$,

Proposition 1.2. *([6]) For any soft G-metric $\widetilde{G}$ on $\widetilde{X}$, we can construct a soft metric $d_{\widetilde{G}}$ on $\widetilde{X}$ defined by*

$$d_{\widetilde{G}}(\widetilde{x}, \widetilde{y}) = \widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{y}) + \widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y}). \tag{1}$$

Definition 1.5. *([6]) $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $(\widetilde{x_n})$ be a sequence of soft elements in $\widetilde{X}$. The sequence $(\widetilde{x_n})$ is said to be soft G-convergent at $\widetilde{x}$ in $\widetilde{X}$, if for every $\widetilde{\epsilon} \widetilde{\geq} \bar{0}$, chosen arbitrarily, there exists a natural number $N = N(\widetilde{\epsilon})$ such that $\bar{0} \widetilde{\leq} \widetilde{G}(\widetilde{x_n}, \widetilde{x_n}, \widetilde{x}) \widetilde{<} \widetilde{\epsilon}$, whenever $n \geq N$, i.e., $n \geq N \Rightarrow (\widetilde{x_n}) \in B_{\widetilde{G}}(\widetilde{x}, \widetilde{\epsilon})$.*
We denote this by $(\widetilde{x_n}) \to \widetilde{x}$ as $n \to \infty$ or by $\lim_{n\to\infty}(\widetilde{x_n}) = \widetilde{x}$.

Proposition 1.3. *([6]) Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space, for a sequence $(\widetilde{x_n})$ in $\widetilde{X}$ and soft element $\widetilde{x}$, then the followings are equivalent:*
(1) $(\widetilde{x_n})$ is soft G-convergent to $\widetilde{x}$,
(2) $d_{\widetilde{G}}(\widetilde{x_n}, \widetilde{x}) \to \bar{0}$ as $n \to \infty$,
(3) $\widetilde{G}(\widetilde{x_n}, \widetilde{x_n}, \widetilde{x}) \to \bar{0}$ as $n \to \infty$,
(4) $\widetilde{G}(\widetilde{x_n}, \widetilde{x}, \widetilde{x}) \to \bar{0}$ as $n \to \infty$,
(5) $\widetilde{G}(\widetilde{x_n}, \widetilde{x_m}, \widetilde{x}) \to \bar{0}$ as $n, m \to \infty$.

Definition 1.6. *([6]) A soft G-metric space $(\widetilde{X}, \widetilde{G}, E)$ is symmetric if*
$(\widetilde{G}_6)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{y}) = \widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y})$ *for all* $x, y \in SE(\widetilde{X})$.

Definition 1.7. *([7]) Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $(\widetilde{x_n})$ be a sequence of soft elements in $\widetilde{X}$.*
The sequence $(\widetilde{x_n})$ is said to be soft G-Cauchy, if for every $\widetilde{\epsilon} \widetilde{\geq} \bar{0}$, chosen arbitrarily, there exists a natural number k such that $\widetilde{G}(\widetilde{x_n}, \widetilde{x_m}, \widetilde{x_l}) \widetilde{<} \widetilde{\epsilon}$, whenever $n, m, l \geq k$.
A soft G-metric space $(\widetilde{X}, \widetilde{G}, E)$ is said to be soft G-complete, if every soft G-Cauchy sequence in $(\widetilde{X}, \widetilde{G}, E)$ is soft G-convergent in $(\widetilde{X}, \widetilde{G}, E)$.

Proposition 1.4. *([7]) Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $(\widetilde{x_n})$ be a sequence of soft elements in $\widetilde{X}$. Then the followings are equivalent:*
(1) the sequence $(\widetilde{x_n})$ is soft G-Cauchy,
(2) for every $\widetilde{\epsilon} \widetilde{\geq} \bar{0}$, there exists a natural number k such that $\widetilde{G}(\widetilde{x_n}, \widetilde{x_m}, \widetilde{x_l}) \widetilde{<} \widetilde{\epsilon}$ for any $n, m \geq k$,

(3) $(\widetilde{x_n})$ *is a Cauchy sequence in the soft metric space* $(\widetilde{X}, d_{\widetilde{G}}, E)$.

Corollary 1.1. *([7]) Every soft G-convergent sequence in any soft G-metric space* $(\widetilde{X}, \widetilde{G}, E)$ *is soft G-Cauchy.*

Proposition 1.5. *([7]) A soft G-metric space* $(\widetilde{X}, \widetilde{G}, E)$ *is soft G-complete if and only if* $(\widetilde{X}, d_{\widetilde{G}}, E)$ *is complete soft metric space.*

It was noticed that in the symmetric case ($(\widetilde{X}, \widetilde{G}, E)$ is symmetric), many fixed point theorems on soft G-metric spaces are particular cases of existing fixed point theorems in soft metric spaces. But for treating the non-symmetric case. For this reason, Bilgili Gungor [[2]] introduced soft quasi-metric space and showed that non-symmetric soft G-metric space have a soft quasi-metric form and then many results on non-symmetric soft G-metric spaces can be reproduced from fixed point on soft quasi-metric spaces.

Definition 1.8. *([2]) Let* X *be a nonempty set and* E *be the nonempty set of parameters. A mapping* $\widetilde{q} : SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ *is said to be soft quasi-metric on* $\widetilde{X}$, *if* $\widetilde{q}$ *satisfies the following conditions:*
$(\widetilde{q}_1)$ $\widetilde{q}(\widetilde{x}, \widetilde{y}) = \bar{0}$ *if and only if* $\widetilde{x} = \widetilde{y}$,
$(\widetilde{q}_2)$ $\widetilde{q}(\widetilde{x}, \widetilde{y}) \widetilde{\leq} \widetilde{q}(\widetilde{x}, \widetilde{z}) + \widetilde{q}(\widetilde{z}, \widetilde{y})$, *for all* $\widetilde{x}, \widetilde{y}, \widetilde{z} \in SE(\widetilde{X})$.
The soft set $\widetilde{X}$ *with a soft quasi-metric* $\widetilde{q}$ *on* $\widetilde{X}$ *is said to be a soft quasi-metric space and is denoted by* $(\widetilde{X}, \widetilde{q}, E)$.

Note that any soft metric space is a soft quasi-metric space, but the converse is not true in general.

Now we touch on that, soft G-metric spaces have soft quasi-metric type structure. Indeed, we have the following results.

Theorem 1.1. *([2]) Let* $(\widetilde{X}, \widetilde{G}, E)$ *be a soft G-metric space. The mapping* $\widetilde{q} : SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ *defined by* $\widetilde{q}(x, y) = \widetilde{G}(x, y, y)$ *satisfies the following properties:*
$(\widetilde{q}_1)$ $\widetilde{q}(\widetilde{x}, \widetilde{y}) = \bar{0}$ *if and only if* $\widetilde{x} = \widetilde{y}$,
$(\widetilde{q}_2)$ $\widetilde{q}(\widetilde{x}, \widetilde{y}) \widetilde{\leq} \widetilde{q}(\widetilde{x}, \widetilde{z}) + \widetilde{q}(\widetilde{z}, \widetilde{y})$, *for all* $\widetilde{x}, \widetilde{y}, \widetilde{z} \in SE(\widetilde{X})$.

Definition 1.9. *([2]) Let* $(\widetilde{X}, \widetilde{q}, E)$ *be a soft quasi-metric space and* $(\widetilde{x_n})$ *be a sequence of soft elements in* $\widetilde{X}$. *The sequence* $(\widetilde{x_n})$ *is said to be soft quasi-converges to* $\widetilde{x}$ *in* $\widetilde{X}$ *if and only if*

$$\lim_{n\to\infty} \widetilde{q}(\widetilde{x_n}, \widetilde{x}) = \lim_{n\to\infty} \widetilde{q}(\widetilde{x}, \widetilde{x_n}) = \bar{0}. \tag{2}$$

Definition 1.10. *([2]) Let* $(\widetilde{X}, \widetilde{q}, E)$ *be a soft quasi-metric space and* $(\widetilde{x_n})$ *be a sequence of soft elements in* $\widetilde{X}$.
The sequence $(\widetilde{x_n})$ *is said to be soft left-Cauchy, if and only if for every* $\widetilde{\epsilon} \widetilde{\geq} \bar{0}$, *chosen arbitrarily, there exists a natural number* k *such that* $\widetilde{q}(\widetilde{x_n}, \widetilde{x_m}) \widetilde{<} \widetilde{\epsilon}$, *whenever* $n \geq m > k$.

Definition 1.11. *([2]) Let* $(\widetilde{X}, \widetilde{q}, E)$ *be a soft quasi-metric space and* $(\widetilde{x_n})$ *be a sequence of soft elements in* $\widetilde{X}$.
The sequence $(\widetilde{x_n})$ *is said to be soft right-Cauchy, if and only if for every* $\widetilde{\epsilon} \widetilde{\geq} \bar{0}$, *chosen arbitrarily, there exists a natural number* k *such that* $\widetilde{q}(\widetilde{x_n}, \widetilde{x_m}) \widetilde{<} \widetilde{\epsilon}$, *whenever* $m \geq n > k$.

Definition 1.12. *([2]) Let $(\widetilde{X}, \widetilde{q}, E)$ be a soft quasi-metric space and $(\widetilde{x_n})$ be a sequence of soft elements in $\widetilde{X}$.*
The sequence $(\widetilde{x_n})$ is said to be soft Cauchy, if and only if for every $\widetilde{\epsilon}\widetilde{\geq}\overline{0}$, chosen arbitrarily, there exists a natural number k such that $\widetilde{q}(\widetilde{x_n}, \widetilde{x_m})\widetilde{<}\widetilde{\epsilon}$, whenever $n, m > k$.

Apparently, a sequence $(\widetilde{x_n})$ in a soft quasi-metric space is soft Cauchy if and only if it is soft left-Cauchy and soft right-Cauchy.

Definition 1.13. *([2]) Let $(\widetilde{X}, \widetilde{q}, E)$ be a soft quasi-metric space. We say that*
(1) $(\widetilde{X}, \widetilde{q}, E)$ is soft left-complete if and only if each soft left-Cauchy sequence in $\widetilde{X}$ is convergent;
(2) $(\widetilde{X}, \widetilde{q}, E)$ is soft right-complete if and only if each soft right-Cauchy sequence in $\widetilde{X}$ is convergent;
(3) $(\widetilde{X}, \widetilde{q}, E)$ is soft complete if and only if each soft Cauchy sequence in $\widetilde{X}$ is convergent.

Theorem 1.2. *([2]) Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space, $\widetilde{d} : SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ be the function defined by $\widetilde{d}(x, y) = \widetilde{G}(x, y, y)$ and $(\widetilde{x_n})$ be a sequence of soft elements in $\widetilde{X}$. Then*

(1) $(\widetilde{X}, \widetilde{d}, E)$ is a quasi-metric space,
(2) $(\widetilde{x_n}) \subset \widetilde{X}$ is soft G-convergent $\widetilde{x} \in \widetilde{X}$ if and only if $(\widetilde{x_n}) \subset \widetilde{X}$ is soft quasi-convergent to $\widetilde{x}$ in $(\widetilde{X}, \widetilde{d}, E)$,
(3) $(\widetilde{x_n}) \subset \widetilde{X}$ is soft G-Cauchy if and only if $(\widetilde{x_n}) \subset \widetilde{X}$ is soft Cauchy in $(\widetilde{X}, \widetilde{d}, E)$,
(4) $(\widetilde{X}, \widetilde{G}, E)$ is soft G-complete if and only if $(\widetilde{X}, \widetilde{d}, E)$ is soft complete.

Every soft quasi-metric induces a soft metric, that is, if $(\widetilde{X}, \widetilde{d}, E)$ is soft quasi-metric space, then the function $\widetilde{\delta} : SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ defined by

$$\widetilde{\delta}(x, y) = \max\{\widetilde{d}(x, y), \widetilde{d}(y, x)\} \tag{3}$$

is a soft metric on $\widetilde{X}$.

Theorem 1.3. *([2]) Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space, $\widetilde{\delta} : SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ be the function defined by $\widetilde{\delta}(x, y) = \max\{\widetilde{G}(x, y, y), \widetilde{G}(y, x, x)\}$. Then*

(1) $(\widetilde{X}, \delta)$ is a soft metric space,
(2) $(\widetilde{x_n}) \subset \widetilde{X}$ is soft G-convergent $\widetilde{x} \in \widetilde{X}$ if and only if $(\widetilde{x_n}) \subset \widetilde{X}$ is soft convergent to $\widetilde{x}$ in $(\widetilde{X}, \widetilde{\delta})$,
(3) $(\widetilde{x_n}) \subset \widetilde{X}$ is soft G-Cauchy if and only if $(\widetilde{x_n}) \subset \widetilde{X}$ is soft Cauchy in $(\widetilde{X}, \widetilde{\delta})$,
(4) $(\widetilde{X}, \widetilde{G}, E)$ is soft G-complete if and only if $(\widetilde{X}, \widetilde{\delta}, E)$ is soft complete.

Definition 1.14. *([1]) Let f and g be self maps of a nonempty set X. If $w = fx = gx$ for some $x \in X$, then x is called a coincidence point of f and g and w is called a point of coincidence of f and g.*

Definition 1.15. *([1]) Let f and g be self maps of a nonempty set X. If f and g commute at their coincidence points, then they called weakly compatible mappings.*

Lemma 1.1. *([1]) Let f and g be weakly compatible self mappings of nonempty set X. If f and g have a unique point of coincidence $w = fx = gx$, then w is the unique common fixed point of f and g.*

Otherwise, Popa ([[10]],[[11]]) started and worked the concept of fixed point for mappings satisfying an implicit relation. And many researchers proved so many fixed point theorems for mappings satisfying several types implicit relations. In [3] Bilgili Gungor defined an implicit contraction mapping via soft real numbers inspired from the article of Popa and Patriciu[[12]].

Definition 1.16. *([3]) Let Γ be the set of all continuous functions $F(\widetilde{t}_1,...,\widetilde{t}_6):\mathbb{R}^6(E)^*\to\mathbb{R}(E)^*$ such that*
$(F1)$: F is nonincreasing in variable $\widetilde{t}_5$,
$(F2)$: There exists $h_1\in\Psi$ such that for all $\widetilde{u},\widetilde{v}\widetilde{\geq}\bar{0}, F(\widetilde{u},\widetilde{v},\widetilde{v},\widetilde{u},\widetilde{u+v},\bar{0})\widetilde{\leq}\bar{0}$ implies $\widetilde{u}\leq h_1(\widetilde{v})$,
$(F3)$: There exists $h_2\in\Psi$ such that for all $\widetilde{u},\widetilde{v}\widetilde{>}\bar{0}, F(\widetilde{u},\widetilde{u},\bar{0},\bar{0},\widetilde{u},\widetilde{v})\widetilde{<}\bar{0}$ implies $\widetilde{u}\widetilde{\leq}h_2(\widetilde{v})$,
where Ψ is the set of functions $\psi:\mathbb{R}(E)^*\to\mathbb{R}(E)^*$, ψ is nondecreasing, $\lim_{n\to\infty}\psi^n(\widetilde{t})\widetilde{=}\bar{0}$ for each $\widetilde{t}\widetilde{\geq}\bar{0}$.

Remark 1.1. *([3]) It is easy to see that if $\psi\in\Psi$, then $\psi(\widetilde{t})\widetilde{<}\widetilde{t}$ for any $\widetilde{t}\widetilde{>}\bar{0}$ and $\psi(\bar{0})\widetilde{=}\bar{0}$.*

Example 1.1. *([3]) $F(\widetilde{t}_1,...,\widetilde{t}_6)\widetilde{=}\widetilde{t}_1-\bar{a}\widetilde{t}_2-\bar{b}\widetilde{t}_3-\bar{c}\widetilde{t}_4-\bar{d}\widetilde{t}_5-\bar{e}\widetilde{t}_6$, where $\bar{a},\bar{b},\bar{c},\bar{d},\bar{e}\widetilde{\geq}\bar{0},\bar{a}+\bar{b}+\bar{c}+2\bar{d}+\bar{e}\widetilde{<}\bar{1}$.*
$(F1)$: Obviously.
$(F2)$: Let $\widetilde{u},\widetilde{v}\widetilde{\geq}\bar{0}$ be and $F(\widetilde{u},\widetilde{v},\widetilde{v},\widetilde{u+v},\bar{0})\widetilde{=}\widetilde{u}-\bar{a}\widetilde{v}-\bar{b}\widetilde{v}-\bar{c}\widetilde{u}-\bar{d}\widetilde{(u+v)}\widetilde{\leq}\bar{0}$ which implies $\widetilde{u}\widetilde{\leq}\frac{\bar{a}+\bar{b}+\bar{d}}{\bar{1}-\bar{c}-\bar{d}}\widetilde{v}$ and $(F2)$ is satisfied for $h_1(\widetilde{t})\widetilde{=}\frac{\bar{a}+\bar{b}+\bar{d}}{\bar{1}-(\bar{c}+\bar{d})}\widetilde{t}$.
$(F3)$: Let $\widetilde{t},\widetilde{t'}\widetilde{>}\bar{0}$ be and $F(\widetilde{t},\widetilde{t},\bar{0},\bar{0},\widetilde{t},\widetilde{t'})\widetilde{=}\widetilde{t}-\bar{a}\widetilde{t}-\bar{d}\widetilde{t}-\bar{e}\widetilde{t'}\widetilde{\leq}\bar{0}$ which implies $\widetilde{t}\widetilde{\leq}\frac{\bar{e}}{\bar{1}-(\bar{a}+\bar{d})}\widetilde{t'}$ and $(F3)$ is satisfied for $h_2(\widetilde{t})\widetilde{=}\frac{\bar{e}}{\bar{1}-(\bar{a}+\bar{d})}\widetilde{t}$.

Example 1.2. *([3]) $F(\widetilde{t}_1,...,\widetilde{t}_6)\widetilde{=}\widetilde{t}_1-\bar{k}\max\{\widetilde{t}_2,...,\widetilde{t}_6\}$, where $\bar{k}\in[\bar{0},\frac{\bar{1}}{2})$.*
$(F1)$: Obviously.
$(F2)$: Let $\widetilde{u},\widetilde{v}\widetilde{\geq}\bar{0}$ be and $F(\widetilde{u},\widetilde{v},\widetilde{v},\widetilde{u},\widetilde{u+v},\bar{0})\widetilde{=}\widetilde{u}-\bar{k}\max\{\widetilde{u},\widetilde{v},\widetilde{u+v}\}\widetilde{\leq}\bar{0}$. Thus, $\widetilde{u}\widetilde{\leq}\frac{\bar{k}}{\bar{1}-\bar{k}}$ and $(F2)$ is satisfied for $h_1(\widetilde{t})\widetilde{=}\frac{\bar{k}}{\bar{1}-\bar{k}}\widetilde{t}$.
$(F3)$: Let $\widetilde{t},\widetilde{t'}\widetilde{>}\bar{0}$ be and $F(\widetilde{t},\widetilde{t},\bar{0},\bar{0},\widetilde{t},\widetilde{t'})\widetilde{=}\widetilde{t}-\bar{k}\max\{\widetilde{t},\widetilde{t'}\}\widetilde{\leq}\bar{0}$. If $\widetilde{t}\widetilde{>}\widetilde{t'}$, then $\widetilde{t}(\bar{1}-\bar{k})\widetilde{\leq}\bar{0}$, a contradiction. Hence $\widetilde{t}\widetilde{\leq}\widetilde{t'}$ which implies $\widetilde{t}\widetilde{\leq}\bar{k}\widetilde{t'}$ and $(F3)$ is satisfied for $h_2(\widetilde{t})\widetilde{=}\bar{k}\widetilde{t}$.

Now, some new examples to implicit contraction mappings via soft real numbers can be given.

Example 1.3. $F(\widetilde{t}_1,...,\widetilde{t}_6)\widetilde{=}\widetilde{t}_1-\bar{k}\max\{\widetilde{t}_2,\widetilde{t}_3,\widetilde{t}_4,\frac{\widetilde{t}_5+\widetilde{t}_6}{2}\}$, *where $\bar{k}\in[\bar{0},1)$.*
$(F1)$: Obviously.
$(F2)$: Let $\widetilde{u},\widetilde{v}\widetilde{\geq}\bar{0}$ be and $F(\widetilde{u},\widetilde{v},\widetilde{v},\widetilde{u},\widetilde{u+v},\bar{0})\widetilde{=}\widetilde{u}-\bar{k}\max\{\widetilde{u},\widetilde{v},\widetilde{\frac{u+v}{2}}\}\widetilde{\leq}\bar{0}$. If $\widetilde{u}\widetilde{>}\widetilde{v}$, then $\widetilde{u}(\bar{1}-\bar{k})\widetilde{\leq}\bar{0}$, is a contradiction. Hence $\widetilde{u}\widetilde{\leq}\widetilde{v}$ which implies $\widetilde{u}\widetilde{\leq}\bar{k}\widetilde{v}$ and $(F2)$ is satisfied for $h_1(\widetilde{t})\widetilde{=}\bar{k}\widetilde{t}$.

$(F3)$: Let $\widetilde{t},\widetilde{t'}\widetilde{>}\bar{0}$ be and $F(\widetilde{t},\widetilde{t},\bar{0},\bar{0},\widetilde{t},\widetilde{t'})\widetilde{=}\widetilde{t}-\bar{k}\max\{\widetilde{t},\widetilde{\frac{t+t'}{2}}\}\widetilde{\leq}\bar{0}$. If $\widetilde{t}\widetilde{>}\widetilde{t'}$, then $\widetilde{t}(\bar{1}-\bar{k})\widetilde{\leq}\bar{0}$, a contradiction. Hence $\widetilde{t}\widetilde{\leq}\widetilde{t'}$ which implies $\widetilde{t}\widetilde{\leq}\bar{k}\widetilde{t'}$ and $(F3)$ is satisfied for $h_2(\widetilde{t})\widetilde{=}\bar{k}\widetilde{t}$.

Example 1.4. $F(\widetilde{t}_1,...,\widetilde{t}_6)\widetilde{=}\widetilde{t}_1-\bar{k}\max\{\widetilde{t}_2,\frac{\widetilde{t}_3+\widetilde{t}_4}{2},\frac{\widetilde{t}_5+\widetilde{t}_6}{2}\}$, *where $\bar{k}\in[\bar{0},1)$.*
$(F1)$: Obviously.

$(F2)$: *Let* $\widetilde{u},\widetilde{v}\widetilde{\geq}\bar{0}$ *be and* $F(\widetilde{u},\widetilde{v},\widetilde{v},\widetilde{u},\widetilde{u+v},\bar{0})\widetilde{=}\widetilde{u}-\bar{k}\max\{\widetilde{v},\widetilde{\frac{u+v}{2}}\}\widetilde{\leq}\bar{0}$. *If* $\widetilde{u}\widetilde{>}\widetilde{v}$, *then* $\widetilde{u}(\bar{1}-\bar{k})\widetilde{\leq}\bar{0}$, *is a contradiction. Hence* $\widetilde{u}\widetilde{\leq}\widetilde{v}$ *which implies* $\widetilde{u}\widetilde{\leq}\bar{k}\widetilde{v}$ *and* $(F2)$ *is satisfied for* $h_1(\widetilde{t})\widetilde{=}\bar{k}\widetilde{t}$.

$(F3)$: *Let* $\widetilde{t},\widetilde{t'}\widetilde{>}\bar{0}$ *be and* $F(\widetilde{t},\widetilde{t},\bar{0},\bar{0},\widetilde{t},\widetilde{t'})\widetilde{=}\widetilde{t}-\bar{k}\max\{\widetilde{t},\widetilde{\frac{t+t'}{2}}\}\widetilde{\leq}\bar{0}$. *If* $\widetilde{t}\widetilde{>}\widetilde{t'}$, *then* $\widetilde{t}(\bar{1}-\bar{k})\widetilde{\leq}\bar{0}$, *a contradiction. Hence* $\widetilde{t}\widetilde{\leq}\widetilde{t'}$ *which implies* $\widetilde{t}\widetilde{\leq}\bar{k}\widetilde{t'}$ *and* $(F3)$ *is satisfied for* $h_2(\widetilde{t})\widetilde{=}\bar{k}\widetilde{t}$.

Example 1.5. $F(\widetilde{t}_1,...,\widetilde{t}_6)\widetilde{=}\widetilde{t}_1-\bar{a}\widetilde{t}_2-\bar{b}\widetilde{t}_3-\bar{c}\max\{\bar{2}\widetilde{t}_4,\widetilde{t}_5+\widetilde{t}_6\}$, *where* $\bar{a},\bar{b},\bar{c}\widetilde{\geq}\bar{0},\bar{a}+\bar{b}+\bar{2}\bar{c}\widetilde{<}\bar{1}$.
$(F1)$: *Obviously.*
$(F2)$: *Let* $\widetilde{u},\widetilde{v}\widetilde{\geq}\bar{0}$ *be and* $F(\widetilde{u},\widetilde{v},\widetilde{v},\widetilde{u},\widetilde{u+v},\bar{0})\widetilde{=}\widetilde{u}-\bar{a}\widetilde{v}-\bar{b}\widetilde{v}-\bar{c}\max\{\bar{2}\widetilde{u},\widetilde{u+v}\}\widetilde{\leq}\bar{0}$. *If* $\widetilde{u}\widetilde{>}\widetilde{v}$, *then* $\widetilde{u}(\bar{1}-(\bar{a}+\bar{b}+\bar{2}\bar{c}))\widetilde{\leq}\bar{0}$, *is a contradiction. Hence* $\widetilde{u}\widetilde{\leq}\widetilde{v}$ *which implies* $\widetilde{u}\widetilde{\leq}\frac{\bar{a}+\bar{b}+\bar{c}}{\bar{1}-\bar{c}}\widetilde{v}$ *and* $(F2)$ *is satisfied for* $h_1(\widetilde{t})\widetilde{=}\frac{\bar{a}+\bar{b}+\bar{c}}{\bar{1}-\bar{c}}\widetilde{t}$.

$(F3)$: *Let* $\widetilde{t},\widetilde{t'}\widetilde{>}\bar{0}$ *be and* $F(\widetilde{t},\widetilde{t},\bar{0},\bar{0},\widetilde{t},\widetilde{t'})\widetilde{=}\widetilde{t}-\bar{a}\widetilde{t}-\bar{c}\widetilde{(t+t')}\widetilde{\leq}\bar{0}$, *which implies* $\widetilde{t}\widetilde{\leq}\frac{\bar{c}}{\bar{1}-(\bar{a}+\bar{c})}\widetilde{t'}$ *and* $(F3)$ *is satisfied for* $h_2(\widetilde{t})\widetilde{=}\frac{\bar{c}}{\bar{1}-(\bar{a}+\bar{c})}\widetilde{t}$.

Example 1.6. $F(\widetilde{t}_1,...,\widetilde{t}_6)\widetilde{=}\widetilde{t}_1-\bar{a}\widetilde{t}_2-\bar{b}\widetilde{t}_3-\bar{c}\max\{\widetilde{t}_4+\widetilde{t}_5,\bar{2}\widetilde{t}_6,\}$, *where* $\bar{a},\bar{b},\bar{c}\widetilde{\geq}\bar{0},\bar{a}+\bar{b}+\bar{3}\bar{c}\widetilde{<}\bar{1}$.
$(F1)$: *Obviously.*
$(F2)$: *Let* $\widetilde{u},\widetilde{v}\widetilde{\geq}\bar{0}$ *be and* $F(\widetilde{u},\widetilde{v},\widetilde{v},\widetilde{u},\widetilde{u+v},\bar{0})\widetilde{=}\widetilde{u}-\bar{a}\widetilde{v}-\bar{b}\widetilde{v}-\bar{c}(\bar{2}\widetilde{u}+\widetilde{v})\widetilde{\leq}\bar{0}$ *which implies* $\widetilde{u}\widetilde{\leq}\frac{\bar{a}+\bar{b}+\bar{c}}{\bar{1}-\bar{2}\bar{c}}\widetilde{v}$ *and* $(F2)$ *is satisfied for* $h_1(\widetilde{t})\widetilde{=}\frac{\bar{a}+\bar{b}+\bar{c}}{\bar{1}-\bar{2}\bar{c}}\widetilde{t}$.

$(F3)$: *Let* $\widetilde{t},\widetilde{t'}\widetilde{>}\bar{0}$ *be and* $F(\widetilde{t},\widetilde{t},\bar{0},\bar{0},\widetilde{t},\widetilde{t'})\widetilde{=}\widetilde{t}-\bar{a}\widetilde{t}-\bar{c}\max\{\widetilde{t},\bar{2}\widetilde{t'}\}\widetilde{\leq}\bar{0}$. *If* $\widetilde{t}\widetilde{>}\bar{2}\widetilde{t'}$, *then* $\widetilde{t}(\bar{1}-(\bar{a}+\bar{c}))\widetilde{\leq}\bar{0}$, *a contradiction. Hence* $\widetilde{t}\widetilde{<}\bar{2}\widetilde{t'}$ *which implies* $\widetilde{t}\widetilde{\leq}\frac{\bar{2}\bar{c}}{\bar{1}-\bar{a}}\widetilde{t'}$ *and* $(F3)$ *is satisfied for* $h_2(\widetilde{t})\widetilde{=}\frac{\bar{2}\bar{c}}{\bar{1}-\bar{a}}\widetilde{t}$.

2. Main Results

In this section, firstly if it exists, the uniqueness of a fixed point of operator which provides a specific inequality is proven. And then, the existence of a fixed point of operator which provides specific inequalities related to implicit contractions via soft real numbers is proven.

Lemma 2.1. *Let* $(\widetilde{X},\widetilde{q},E)$ *be a soft quasi-metric space and* $f:(\widetilde{X},\widetilde{q},E)\rightarrow(\widetilde{X},\widetilde{q},E)$ *satisfying*

$$F(\widetilde{q}(f\widetilde{x},f\widetilde{y}),\widetilde{q}(\widetilde{x},\widetilde{y}),\widetilde{q}(\widetilde{x},f\widetilde{x}),\widetilde{q}(\widetilde{y},f\widetilde{y}),\widetilde{q}(\widetilde{x},f\widetilde{y}),\widetilde{q}(\widetilde{y},f\widetilde{x}))\widetilde{\leq}\bar{0},\forall\widetilde{x},\widetilde{y}\in\widetilde{X} \quad (4)$$

and F *satisfying property* $(F3)$. *Then,* f *has at most one fixed point.*

Proof. We assume that f has two distinct fixed point $\widetilde{p}$ and $\widetilde{r}$ which satisfy $\widetilde{p}\widetilde{=}f\widetilde{p}$ and $\widetilde{r}\widetilde{=}f\widetilde{r}$. Then by using (4) we get

$$F(\widetilde{q}(f\widetilde{p},f\widetilde{r}),\widetilde{q}(\widetilde{p},\widetilde{r}),\widetilde{q}(\widetilde{p},f\widetilde{p}),\widetilde{q}(\widetilde{r},f\widetilde{r}),\widetilde{q}(\widetilde{p},f\widetilde{r}),\widetilde{q}(\widetilde{r},f\widetilde{p}))\widetilde{\leq}\bar{0}, \quad (5)$$

so

$$F(\widetilde{q}(\widetilde{p},\widetilde{r}),\widetilde{q}(\widetilde{p},\widetilde{r}),0,0,\widetilde{q}(\widetilde{p},\widetilde{r}),\widetilde{q}(\widetilde{r},\widetilde{p}))\widetilde{\leq}\bar{0}. \quad (6)$$

By the property (F_3) of F, we get

$$\widetilde{q}(\widetilde{p},\widetilde{r})\widetilde{\leq}h_2(\widetilde{q}(\widetilde{r},\widetilde{p})). \quad (7)$$

Similarly, we get

$$\widetilde{q}(\widetilde{r},\widetilde{p})\widetilde{\leq}h_2(\widetilde{q}(\widetilde{p},\widetilde{r})). \tag{8}$$

With using (7),(8), h is nondecreasing and $h(t) < t$ for $t > 0$,

$$0\widetilde{<}\widetilde{q}(\widetilde{p},\widetilde{r})\widetilde{\leq}h_2(\widetilde{q}(\widetilde{r},\widetilde{p}))\widetilde{\leq}h_2^2(\widetilde{q}(\widetilde{p},\widetilde{r}))\widetilde{<}\widetilde{q}(\widetilde{p},\widetilde{r}). \tag{9}$$

This is a contradiction and $\widetilde{p}\widetilde{=}\widetilde{r}$. Thus, $\widetilde{p}\widetilde{=}f\widetilde{p}\widetilde{=}\widetilde{=}f\widetilde{r}\widetilde{=}\widetilde{r}$.

□

Theorem 2.1. *Let $(\widetilde{X},\widetilde{q},E)$ be a complete soft quasi-metric space and $f:(\widetilde{X},\widetilde{q},E)\to(\widetilde{X},\widetilde{q},E)$ satisfying*

$$F(\widetilde{q}(f\widetilde{x},f\widetilde{y}),\widetilde{q}(\widetilde{x},\widetilde{y}),\widetilde{q}(\widetilde{x},f\widetilde{x}),\widetilde{q}(\widetilde{y},f\widetilde{y}),\widetilde{q}(\widetilde{x},f\widetilde{y}),\widetilde{q}(\widetilde{y},f\widetilde{x}))\widetilde{\leq}\bar{0},\forall\widetilde{x},\widetilde{y}\in\widetilde{X} \tag{10}$$

and

$$F(\widetilde{q}(f\widetilde{x},f\widetilde{y}),\widetilde{q}(\widetilde{x},\widetilde{y}),\widetilde{q}(\widetilde{x},\widetilde{y}),\widetilde{q}(f\widetilde{x},\widetilde{x}),\widetilde{q}(f\widetilde{x},\widetilde{y}),\widetilde{q}(\widetilde{x},f\widetilde{y}))\widetilde{\leq}\bar{0},\forall\widetilde{x},\widetilde{y}\in\widetilde{X} \tag{11}$$

where $F\in\Gamma$. Then f has a unique fixed point.

Proof. If we take $\widetilde{y}\widetilde{=}f\widetilde{x}$ in the inequality (24) we obtain

$$F(\widetilde{q}(f\widetilde{x},f^2\widetilde{x}),\widetilde{q}(\widetilde{x},f\widetilde{x}),\widetilde{q}(\widetilde{x},f\widetilde{x}),\widetilde{q}(f\widetilde{x},f^2\widetilde{x}),\widetilde{q}(\widetilde{x},f^2\widetilde{x}),\bar{0}))\widetilde{\leq}\bar{0}, \tag{12}$$

From $(F1)$ and $(\widetilde{q}_2)$, we obtain

$$F(\widetilde{q}(f\widetilde{x},f^2\widetilde{x}),\widetilde{q}(\widetilde{x},f\widetilde{x}),\widetilde{q}(\widetilde{x},f\widetilde{x}),\widetilde{q}(f\widetilde{x},f^2\widetilde{x}),\widetilde{q}(\widetilde{x},f\widetilde{x})+\widetilde{q}(f\widetilde{x},f^2\widetilde{x}),\bar{0})\widetilde{\leq}\bar{0}. \tag{13}$$

By $(F2)$, we have

$$\widetilde{q}(f\widetilde{x},f^2\widetilde{x})\leq h_1(\widetilde{q}(\widetilde{x},f\widetilde{x})). \tag{14}$$

Let $\widetilde{x_0}$ is an arbitrary and $\widetilde{x}_n\widetilde{=}f\widetilde{x_{n-1}}, n=1,2,....$ Thus,

$$\begin{aligned}\widetilde{q}(\widetilde{x}_n,\widetilde{x}_{n+1}) &\widetilde{\leq}h_1(\widetilde{q}(\widetilde{x}_{n-1},\widetilde{x}_n))\\ &\widetilde{\leq}h_1^2(\widetilde{q}(\widetilde{x}_{n-2},\widetilde{x}_{n-1}))\\ &\widetilde{\leq}....\\ &\widetilde{\leq}h_1^n(\widetilde{q}(\widetilde{x}_0,\widetilde{x}_1))\end{aligned} \tag{15}$$

So with using $(\widetilde{q}_2)$, for $m > n$,

$$\begin{aligned}\widetilde{q}(\widetilde{x}_n,\widetilde{x}_m) &\widetilde{\leq}\widetilde{q}(\widetilde{x}_n,\widetilde{x}_{n+1})+\widetilde{q}(\widetilde{x}_{n+1},\widetilde{x}_{n+2})+...+\widetilde{q}(\widetilde{x}_{m-1},\widetilde{x}_m)\\ &\widetilde{\leq}(h_1^n+h_1^{n+1}+...+h_1^{m-1})(\widetilde{q}(\widetilde{x}_0,\widetilde{x}_1))\\ &\widetilde{\leq}\frac{h_1^n}{1-h_1}(\widetilde{q}(\widetilde{x}_0,\widetilde{x}_1)),\end{aligned} \tag{16}$$

which means that $\widetilde{q}(\widetilde{x}_n,\widetilde{x}_m)\to\bar{0}$. So $\{\widetilde{x}_n\}$ is a soft right-Cauchy sequence. Similarly, if we take $\widetilde{x}\widetilde{=}f\widetilde{y}$ in the inequality (11) we get

$$F(\widetilde{q}(f^2\widetilde{y},f\widetilde{y}),\widetilde{q}(f\widetilde{y},\widetilde{y}),\widetilde{q}(f\widetilde{y},\widetilde{y}),\widetilde{q}(f^2\widetilde{y},f\widetilde{y}),\widetilde{q}(f^2\widetilde{y},\widetilde{y}),\bar{0})\widetilde{\leq}\bar{0}, \tag{17}$$

From $(F1)$ and $(\widetilde{q}_2)$, we obtain

$$F(\widetilde{q}(f^2\widetilde{y},f\widetilde{y}),\widetilde{q}(f\widetilde{y},\widetilde{y}),\widetilde{q}(f\widetilde{y},\widetilde{y}),\widetilde{q}(f^2\widetilde{y},f\widetilde{y}),\widetilde{q}(f^2\widetilde{y},f\widetilde{y})+\widetilde{q}(f\widetilde{y},\widetilde{y}),\bar{0})\widetilde{\leq}\bar{0}, \tag{18}$$

By $(F2)$, we have

$$\widetilde{q}(g\widetilde{x}_{n+1},g\widetilde{x}_n)\leq h_1(\widetilde{q}(g\widetilde{x}_n,g\widetilde{x}_{n-1})). \tag{19}$$

If we continue similar way, we get

$$\widetilde{q}(f^2\widetilde{y}, f\widetilde{y})\widetilde{\leq}h_1(\widetilde{q}(f\widetilde{y}, \widetilde{y})). \tag{20}$$

So with using $(\widetilde{q}_2)$, for $n > m$,

$$\begin{aligned} \widetilde{q}(\widetilde{x}_n, \widetilde{x}_m) \quad & \widetilde{\leq}\widetilde{q}(\widetilde{x}_n, \widetilde{x}_{n-1}) + \widetilde{q}(\widetilde{x}_{n-1}, \widetilde{x}_{n-2}) + ... + \widetilde{q}(\widetilde{x}_{m+1}, \widetilde{x}_m) \\ & \widetilde{\leq}(h_1^m + h_1^{m+1} + ... + h_1^{n-1})(\widetilde{q}(\widetilde{x}_0, \widetilde{x}_1)) \\ & \widetilde{\leq}\tfrac{h_1^m}{1-h_1}(\widetilde{q}(\widetilde{x}_0, \widetilde{x}_1)), \end{aligned} \tag{21}$$

which means that $\widetilde{q}(\widetilde{x}_n, \widetilde{x}_m) \to \bar{0}$. So $\{\widetilde{x}_n\}$ is a soft left-Cauchy sequence.

Thus, $\widetilde{x_n}$ is a soft Cauchy sequence in soft complete quasi metric space and so has a limit $\widetilde{u}$. Now by using the inequality (24) we obtain

$$F(\widetilde{q}(f\widetilde{x}_n, f\widetilde{u}), \widetilde{q}(\widetilde{x}_n, \widetilde{u}), \widetilde{q}(\widetilde{x}_n, f\widetilde{x}_n), \widetilde{q}(\widetilde{u}, f\widetilde{u}), \widetilde{q}(\widetilde{x}_n, f\widetilde{u}), \widetilde{q}(\widetilde{u}, f\widetilde{x}_n))\widetilde{\leq}\bar{0}, \tag{22}$$

so,

$$F(\widetilde{q}(\widetilde{x}_{n+1}, f\widetilde{u}), \widetilde{q}(\widetilde{x}_n, \widetilde{u}), \widetilde{q}(\widetilde{x}_n, \widetilde{x}_{n+1}), \widetilde{q}(\widetilde{u}, f\widetilde{u}), \widetilde{q}(\widetilde{x}_n, f\widetilde{u}), \widetilde{q}(\widetilde{u}, \widetilde{x}_{n+1}))\widetilde{\leq}\bar{0}. \tag{23}$$

Taking limit n tend to infinity we get

$$F(\widetilde{q}(\widetilde{u}, f\widetilde{u}), \bar{0}, \bar{0}, \widetilde{q}(\widetilde{u}, f\widetilde{u}), \widetilde{q}(\widetilde{u}, f\widetilde{u}), \bar{0})\widetilde{\leq}\bar{0}. \tag{24}$$

By $(F2)$ we obtain $\widetilde{q}(\widetilde{u}, f\widetilde{u})\widetilde{\leq}h_1(\bar{0})\widetilde{=}\bar{0}$. And so $\widetilde{u}\widetilde{=}f\widetilde{u}$ that is $\widetilde{u}$ is a fixed point of$f$. From 2.1, $\widetilde{u}$ is the unique fixed point of f.

□

Now, the applications of implicit relations to the fixed point theorems in complete soft quasi metric space can be given as follows:

Corollary 2.1. *Let $(\widetilde{X}, \widetilde{q}, E)$ be a complete soft quasi-metric space and $f : (\widetilde{X}, \widetilde{q}, E) \to (\widetilde{X}, \widetilde{q}, E)$ be a mapping which satisfy $\forall \widetilde{x}, \widetilde{y} \in \widetilde{X}$ the inequalities*

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y})\widetilde{\leq}\bar{a}\widetilde{q}(\widetilde{x}, \widetilde{y}) + \bar{b}\widetilde{q}(\widetilde{x}, f\widetilde{x}) + \bar{c}\widetilde{q}(\widetilde{y}, f\widetilde{y}) + \bar{d}\widetilde{q}(\widetilde{x}, f\widetilde{y}) + \bar{e}\widetilde{q}(\widetilde{y}, f\widetilde{x}), \tag{25}$$

and

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y})\widetilde{\leq}(\bar{a} + \bar{b})\widetilde{q}(\widetilde{x}, \widetilde{y}) + \bar{c}\widetilde{q}(f\widetilde{x}, \widetilde{x}) + \bar{d}\widetilde{q}(f\widetilde{x}, \widetilde{y}) + \bar{e}\widetilde{q}(\widetilde{x}, f\widetilde{y}) \tag{26}$$

where $\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{e}\widetilde{\geq}\bar{0}, \bar{a} + \bar{b} + \bar{c} + \bar{2}d + \bar{e}\widetilde{<}\bar{1}$. Then f has a unique fixed point.

Proof. By Theorem 2.1 and Example 1.1, f has a unique fixed point. □

Corollary 2.2. *Let $(\widetilde{X}, \widetilde{q}, E)$ be a complete soft quasi-metric space and $f : (\widetilde{X}, \widetilde{q}, E) \to (\widetilde{X}, \widetilde{q}, E)$ be a mapping which satisfy $\forall \widetilde{x}, \widetilde{y} \in \widetilde{X}$ the inequalities*

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y})\widetilde{\leq}\bar{k}\max\{\widetilde{q}(\widetilde{x}, \widetilde{y}), \widetilde{q}(\widetilde{x}, f\widetilde{x}), \widetilde{q}(\widetilde{y}, f\widetilde{y}), \widetilde{q}(\widetilde{x}, f\widetilde{y}), \widetilde{q}(\widetilde{y}, f\widetilde{x})\}, \tag{27}$$

and

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y})\widetilde{\leq}\bar{k}\max\{\widetilde{q}(\widetilde{x}, \widetilde{y}), \widetilde{q}(f\widetilde{x}, \widetilde{x}), \widetilde{q}(f\widetilde{x}, \widetilde{y}), \widetilde{q}(\widetilde{x}, f\widetilde{y})\} \tag{28}$$

where $\bar{k} \in [\bar{0}, \frac{\bar{1}}{2})$. Then f has a unique fixed point.

Proof. By Theorem 2.1 and Example 1.2, f has a unique fixed point. □

Corollary 2.3. *Let $(\widetilde{X}, \widetilde{q}, E)$ be a complete soft quasi-metric space and $f : (\widetilde{X}, \widetilde{q}, E) \to (\widetilde{X}, \widetilde{q}, E)$ be a mapping which satisfy $\forall \widetilde{x}, \widetilde{y} \in \widetilde{X}$ the inequalities*

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y}) \widetilde{\leq} \bar{k} \max\{\widetilde{q}(\widetilde{x}, \widetilde{y}), \widetilde{q}(\widetilde{x}, f\widetilde{x}), \widetilde{q}(\widetilde{y}, f\widetilde{y}), \frac{\widetilde{q}(\widetilde{x}, f\widetilde{y}) + \widetilde{q}(\widetilde{y}, f\widetilde{x})}{2}\}, \tag{29}$$

and

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y}) \widetilde{\leq} \bar{k} \max\{\widetilde{q}(\widetilde{x}, \widetilde{y}), \widetilde{q}(f\widetilde{x}, \widetilde{x}), \frac{\widetilde{q}(f\widetilde{x}, \widetilde{y}) + \widetilde{q}(\widetilde{x}, f\widetilde{y})}{2}\} \tag{30}$$

where $\bar{k} \in [\bar{0}, \bar{1})$. Then f has a unique fixed point.

Proof. By Theorem 2.1 and Example 1.3, f has a unique fixed point. □

Corollary 2.4. *Let $(\widetilde{X}, \widetilde{q}, E)$ be a complete soft quasi-metric space and $f : (\widetilde{X}, \widetilde{q}, E) \to (\widetilde{X}, \widetilde{q}, E)$ be a mapping which satisfy $\forall \widetilde{x}, \widetilde{y} \in \widetilde{X}$ the inequalities*

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y}) \widetilde{\leq} \bar{k} \max\{\widetilde{q}(\widetilde{x}, \widetilde{y}), \frac{\widetilde{q}(\widetilde{x}, f\widetilde{x}) + \widetilde{q}(\widetilde{y}, f\widetilde{y})}{2}, \frac{\widetilde{q}(\widetilde{x}, f\widetilde{y}) + \widetilde{q}(\widetilde{y}, f\widetilde{x})}{2}\}, \tag{31}$$

and

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y}) \widetilde{\leq} \bar{k} \max\{\widetilde{q}(\widetilde{x}, \widetilde{y}), \frac{\widetilde{q}(\widetilde{x}, \widetilde{y}) + \widetilde{q}(f\widetilde{x}, \widetilde{x})}{2}, \frac{\widetilde{q}(f\widetilde{x}, \widetilde{y}) + \widetilde{q}(\widetilde{x}, f\widetilde{y})}{2}\} \tag{32}$$

where $\bar{k} \in [\bar{0}, \bar{1})$. Then f has a unique fixed point.

Proof. By Theorem 2.1 and Example 1.4, f has a unique fixed point. □

Corollary 2.5. *Let $(\widetilde{X}, \widetilde{q}, E)$ be a complete soft quasi-metric space and $f : (\widetilde{X}, \widetilde{q}, E) \to (\widetilde{X}, \widetilde{q}, E)$ be a mapping which satisfy $\forall \widetilde{x}, \widetilde{y} \in \widetilde{X}$ the inequalities*

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y}) \widetilde{\leq} \bar{k} \max\{\bar{2}\widetilde{q}(\widetilde{y}, f\widetilde{y}), \widetilde{q}(\widetilde{x}, f\widetilde{y}) + \widetilde{q}(\widetilde{y}, f\widetilde{x})\}, \tag{33}$$

and

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y}) \widetilde{\leq} \bar{k} \max\{\bar{2}\widetilde{q}(f\widetilde{x}, \widetilde{x}), \widetilde{q}(f\widetilde{x}, \widetilde{y}) + \widetilde{q}(\widetilde{x}, f\widetilde{y})\} \tag{34}$$

where $\bar{k} \in [\bar{0}, \frac{\bar{1}}{2})$. Then f has a unique fixed point.

Proof. By Theorem 2.1 and Example 1.5 with $\bar{a} \widetilde{=} \bar{b} \widetilde{=} \bar{0}$ and $\bar{c} \widetilde{=} \bar{k}$, f has a unique fixed point. □

Corollary 2.6. *Let $(\widetilde{X}, \widetilde{q}, E)$ be a complete soft quasi-metric space and $f : (\widetilde{X}, \widetilde{q}, E) \to (\widetilde{X}, \widetilde{q}, E)$ be a mapping which satisfy $\forall \widetilde{x}, \widetilde{y} \in \widetilde{X}$ the inequalities*

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y}) \widetilde{\leq} \bar{k} \max\{\widetilde{q}(\widetilde{y}, f\widetilde{y}) + \widetilde{q}(\widetilde{x}, f\widetilde{y}), \bar{2}\widetilde{q}(\widetilde{y}, f\widetilde{x})\}, \tag{35}$$

and

$$\widetilde{q}(f\widetilde{x}, f\widetilde{y}) \widetilde{\leq} \bar{k} \max\{\widetilde{q}(f\widetilde{x}, \widetilde{x}) + \widetilde{q}(f\widetilde{x}, \widetilde{y}), \bar{2}\widetilde{q}(\widetilde{x}, f\widetilde{y})\} \tag{36}$$

where $\bar{k} \in [\bar{0}, \frac{\bar{1}}{3})$. Then f has a unique fixed point.

Proof. By Theorem 2.1 and Example 1.6 with $\bar{a} \widetilde{=} \bar{b} \widetilde{=} \bar{0}$ and $\bar{c} \widetilde{=} \bar{k}$, f has a unique fixed point. □

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