

REDUCTION OF NAVIER-STOKES EQUATION TO A LINEAR EQUATION

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ABSTRACT

In this article, we provide two theorems on pointwise coincidence between solutions of Navier-Stokes equation and solutions of standard linear second order parabolic equations with the same data. We show that the convection, the pressure, and the external forces (if applied) are governed by some sort of balance independent of the equation that governs the solution itself. In light of the well establishment of the theory of existence, regularity and uniqueness of linear second order parabolic equations, this result represents an important step to fully understand the qualitative properties of the solutions to Navier-Stokes equation.

Details: The model equation in hand is:

$$(1) \quad \begin{cases} \mathbf{v}_t + (\mathbf{v} \cdot \nabla)\mathbf{v} - \mu\Delta\mathbf{v} = -\nabla p + \mathbf{f}, & \nabla \cdot \mathbf{v} = 0 \quad \text{in } \Omega_t, \\ \mathbf{v}(\mathbf{x}, 0) = \mathbf{v}_0(\mathbf{x}) & \text{in } \Omega_0, \\ \mathbf{v}(\mathbf{x}, t) = \mathbf{v}^*(\mathbf{x}_{n-1}, t) & \text{on } \partial\Omega_t, \end{cases}$$

The weak solution is defined as introduced first by Leray in [1, 2, 3] to generalize the results to the widest classes of possible solutions. The investigation in this article establishes rigorously the previously investigated results in [4, 5].

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