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ON INTUITIONISTIC FUZZY HYPERSTRUCTURE WITH T-NORM

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Abstract. In this paper, for any ring R, the T-intuitionistic fuzzy H_ν-subring of R is defined. Using this definition, some related properties with fuzzy H_ν-subrings were given. In addition to above study, some fundamental relation properties are studied.

1. Introduction

The theory of hyperstructures has been introduced by Marty [16] in 1934.Marty introduced the notion of a hypergroup and then many researchers have been worked on this new field of modern algebra and developed it.H_ν-rings first were introduced by Vougiouklis [20] in 1990.The largest class of algebraic systems satisfying ring-like axioms is the H_ν-ring.So, he defined the fundamental definition of H_ν-rings theory.The concept of fuzzy subhypergroup as well as the fuzzy H_ν-group were introduced by Davvaz [5] in 1999.Davvaz [6] defined the concept of fuzzy H_ν-ideal of an H_ν-ring which is a generalization of the concept of fuzzy ideal.The notion of intuitionistic fuzzy H_ν-ideal of an H_ν-ring were introduced by Davvaz, Dudek [9] in 2006.

Definition 1.1. [21] Let X be a nonempty set.A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X. The complement of μ, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

Definition 1.2. [1] An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$$

where $\mu_A(x), (\mu_A : X \to [0,1])$ is called the "degree of membership of x in A ", $\nu_A(x), (\nu_A : X \to [0,1])$is called the " degree of non- membership of x in A ",and

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where μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \text{ for all } x \in X.$$

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) >: x\}$.

Definition 1.3. [2] Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy sets in X .Then

(1) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(2) $A^c = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$
(3) $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: x \in X\}$
(4) $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: x \in X\}$
(5) $A = B :\Leftrightarrow A \subseteq B \wedge B \subseteq A$

Definition 1.4. [16]A hyperstructure is a non-empty set H together with a map $* : H \times H \to P^*(H)$ which is called hyperoperation, where $P^*(H)$ denotes the set of all non-empty subsets of H. The image of pair (x, y) is denoted by $x * y$. If $x \in H$ and $A, B \subseteq H$, then by $A * B$, $A * x$ and $x * B$ we mean
$A * B = \bigcup_{a \in A, b \in B} a * b$, $A * x = A * \{x\}$ and $x * B = \{x\} * B$, respectively.

Definition 1.5. [5]A hyperstructure $(H, *)$ is called a hypergroup if the following axioms hold:
(i) $(H, *)$ is a semihypergroup, that is,

$$\forall x, y, z \in H, (x * (y * z)) = ((x * y) * z)$$

(ii) $x * H = H * x = H$ for all x in H

Definition 1.6. [19]An H_ν-ring is a system $(R, +, \cdot)$ with two hyperoperations satisfying the following ring-like axioms:

(i) $(R, +, \cdot)$ is an H_ν-group, that is,

$$\forall a \in R,\ a + R = R + a = R$$
$$\forall x, y, z \in R,\ ((x + y) + z) \cap (x + (y + z)) \neq$$

(ii) $(R, \cdot)$ is an H_ν-semigroup, that is,

$$\forall x, y, z \in R,\ ((x \cdot y) \cdot z) \cap (x \cdot (y \cdot z)) \neq$$

(iii) $(\cdot)$ is weak distributive with respect to $(+)$, that is, for all $x, y, z \in R$,

$$((x + y) \cdot z) \cap (x \cdot z + y \cdot z) \quad \neq$$
$$(x \cdot (y + z)) \cap (x \cdot y + x \cdot z) \quad \neq$$

Definition 1.7. [5]Let $(H, \cdot)$ be a hypergroup (or H_ν-group) and let μ be a fuzzy subset of H. Then μ is said to be a fuzzy subhypergroup (or fuzzy H_ν-subgroup) of H if the following axioms hold:
(i) $\min\{\mu(x), \mu(y)\} \leq \inf_{\alpha \in x \cdot y}\{\mu(\alpha)\}$, $\forall x, y \in H$

(ii) for all $x, a \in H$ there exists $y \in H$ such that
$x \in a \cdot y$ and $\min\{\mu(a), \mu(x)\} \leq \mu(y)$.

Definition 1.8. [5]Let $(H, \cdot)$ be an H_ν-group and let μ be a fuzzy subset of H.Then μ is said to be a T-fuzzy H_ν-subgroup of H with repect to T-norm T if the following axioms hold:
(i) $T(\mu(x), \mu(y)) \leq \inf\limits_{\alpha \in x \cdot y}\{\mu(\alpha)\}$, $\forall x, y \in H$
(ii) for all $x, a \in H$ there exists $y \in H$ such that
$x \in a \cdot y$ and $T(\mu(a), \mu(x)) \leq \mu(y)$.

Definition 1.9. [9]Let $(R, +, \cdot)$ be an H_ν-ring and μ a fuzzy subset of R. Then μ is said to be a left (resp., right) fuzzy H_ν-ideal of R if the following axioms hold:
1) $\min\{\mu(x), \mu(y)\} \leq \inf\{\mu(z) : z \in x + y\}$, for all $x, y \in R$
2) for all $x, a \in R$ there exists $y \in R$ such that $x \in a + y$ and

$$\min\{\mu(a), \mu(x)\} \leq \mu(y)$$

3) for all $x, a \in R$ there exists $z \in R$ such that $x \in z + a$ and

$$\min\{\mu(a), \mu(x)\} \leq \mu(z)$$

4) $\mu(y) \leq \inf\{\mu(z) : z \in x \cdot y\}$ (resp.,$\mu(x) \leq \inf\{\mu(z) : z \in x \cdot y\}$) for all $x, y \in R$

Definition 1.10. [9]An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in R is called a left (resp., right) intuitionistic fuzzy H_ν-ideal of R if
1) $\min\{\mu_A(x), \mu_A(y)\} \leq \inf\{\mu_A(z) : z \in x + y\}$, for all $x, y \in R$
2) for all $x, a \in R$ there exists $y, z \in R$ such that $x \in (a + y) \cap (z + a)$ and

$$\min\{\mu_A(a), \mu_A(x)\} \leq \min\{\mu_A(y), \mu_A(z)\}$$

3) $\mu_A(y) \leq \inf\{\mu_A(z) : z \in x \cdot y\}$(resp.,$\mu_A(x) \leq \inf\{\mu_A(z) : z \in x \cdot y\}$) for all $x, y \in R$
4) $\sup\{\nu_A(z) : z \in x + y\} \leq \max\{\nu_A(x), \nu_A(y)\}$, for all $x, y \in R$
5) for all $x, a \in R$ there exists $y, z \in R$ such that $x \in (a + y) \cap (z + a)$ and

$$\max\{\nu_A(y), \nu_A(z)\} \leq \max\{\nu_A(a), \nu_A(x)\}$$

6) $\sup\{\nu_A(z) : z \in x \cdot y\} \leq \nu_A(y)$ (resp.,$\sup\{\nu_A(z) : z \in x \cdot y\} \leq \nu_A(x)$) for all $x, y \in R$

Definition 1.11. [15]By a t-norm T , we mean a function $T : [0, 1] \times [0, 1] \to [0, 1]$ satisfying the following conditions:
1) $T(x, 1) = x$
2) $T(x, y) \leq T(x, z)$ *if* $y \leq z$
3) $T(x, y) = T(y, x)$
4) $T(x, T(y, z)) = T(T(x, y), z)$
for all $x, y, z \in [0, 1]$

For a t-norm T on $[0, 1]$, denote by Δ_T the set of element $\alpha \in [0, 1]$ such that $T(\alpha, \alpha) = \alpha$, i.e., $\Delta_T := \{\alpha \in [0, 1] \ : \ T(\alpha, \alpha) = \alpha\}$

Proposition 1. [15]Every t-norm T has a useful property:

$$T(\alpha,\beta) \leq \min(\alpha,\beta)$$

for all $\alpha, \beta \in [0,1]$

Definition 1.12. [14]A t-norm T is continuous if for all convergent sequences $\{x_n\}$, $\{y_n\}$ we have

$$T\left(\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n\right) = \lim_{n\to\infty} T\left(x_n, y_n\right)$$

2. On intuitionistic fuzzy hyperstructure with T-norm

Definition 2.1. Let $(R,+,\cdot)$ be an H_ν-ring and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of R.Then $A = (\mu_A, \nu_A)$ is said to be a T-intuitionistic fuzzy H_ν-subring of R with respect to t-norm T if the following axioms hold:
1) $T\left(\mu_A\left(x\right), \mu_A\left(y\right)\right) \leq \inf\left\{\mu_A\left(z\right) : z \in x+y\right\}$, for all $x, y \in R$
2) $\sup\left\{\nu_A\left(z\right) : z \in x+y\right\} \leq 1 - T\left(1-\nu_A\left(x\right), 1-\nu_A\left(y\right)\right)$, for all $x, y \in R$
3) for all $x, a \in R$ there exists $y, z \in R$ such that $x \in (a+y) \cap (z+a)$ and

$$T\left(\mu_A\left(a\right), \mu_A\left(x\right)\right) \leq T\left(\mu_A\left(y\right), \mu_A\left(z\right)\right)$$

4) $T\left(\mu_A\left(x\right), \mu_A\left(y\right)\right) \leq \inf\left\{\mu_A\left(z\right) : z \in x\cdot y\right\}$, for all $x, y \in R$
5) $\sup\left\{\nu_A\left(z\right) : z \in x\cdot y\right\} \leq 1 - T\left(1-\nu_A\left(x\right), 1-\nu_A\left(y\right)\right)$, for all $x, y \in R$
6) for all $x, a \in R$ there exists $y, z \in R$ such that $x \in (a+y) \cap (z+a)$ and

$$T\left(1-\nu_A\left(a\right), 1-\nu_A\left(x\right)\right) \leq T\left(1-\nu_A\left(y\right), 1-\nu_A\left(z\right)\right)$$

Proposition 2. Let T be an t- norm and $A = (\mu_A, \nu_A)$ be an T-intuitionistic fuzzy H_ν-subring of R.Let μ_A , $1-\nu_A$ have idempotent property.Then the following sets are H_ν-subring of R

$$R^w = \{x \in R : \mu_A\left(x\right) \geq \mu_A\left(w\right)\} \ , \quad L^w = \{x \in R : \nu_A\left(x\right) \leq \nu_A\left(w\right)\}$$

Proposition 3. Let H be a non-empty subset of a H_ν-ring R and let fuzzy sets μ, ν in R are defined by

where $0 \leq \alpha_1 < \alpha_0$, $0 \leq \beta_0 < \beta_1$ and $\alpha_i + \beta_i \leq 1$ for $i = 0, 1$.
Let μ , $1-\nu$ have idempotent property .Then
$A = (\mu, \nu)$ be an T-intuitionistic fuzzy H_ν-subring of R⇔H is a H_ν-subring of R.

Definition 2.2. [13]Let $(R,+,\cdot)$ be an H_ν-ring.The relation γ_R^* is the smallest equivalence relation on R such that the quotient $R\gamma_R^*$, the set of all equivalence classes, is a ring.γ_R^* is called the fundamental relation on R , and $R\gamma_R^*$ is called the fundamental ring.
If Ω denotes the set of all finite polynomials of elements of R, over $\mathbb{N}$ (the set of all natural numbers), then a relation γ_R can be defined on R whose transitive closure is the fundamental relation γ_R^* .

The relation γ_R is as follow:
For x,y in R we write $x\gamma_R y$ if and only if $\{x,y\} \subseteq \Lambda$ for some $\Lambda \in \Omega$.
Suppose $\gamma_R^*(a)$ is the equivalence class containing $a \in R$.Then both the sum $\oplus$ and the product $\odot$ on $R\gamma_R^*$ are defined as follows:

$$\begin{aligned}
\gamma_R^*(a) \oplus \gamma_R^*(b) &= \gamma_R^*(c)\ , \quad for\ all\ c \in \gamma_R^*(a) + \gamma_R^*(b) \\
\gamma_R^*(a) \odot \gamma_R^*(b) &= \gamma_R^*(d)\ , \quad for\ all\ d \in \gamma_R^*(a) \cdot \gamma_R^*(b)
\end{aligned}$$

Here we also denote w_R the zero element of $R\gamma_R^*$.

Definition 2.3. [9]Let $(R,+,\cdot)$ be an H_ν-ring and $A = (\mu_A, \nu_A)$ be an left intuitionistic fuzzy H_ν-ideal of R.The intuitionistic fuzzy set
$A\gamma_R^* = \left(\mu_{\gamma_R^*}, \nu_{\gamma_R^*}\right)$ is defined as follows:

$$\begin{aligned}
\mu_{\gamma_R^*} &: R\gamma_R^* \to [0,1] \\
\mu_{\gamma_R^*}\left(\gamma_R^*(x)\right) &= \begin{cases} \sup\left\{\mu_A(a) : a \in \gamma_R^*(x)\right\} & ,\gamma_R^*(x) \neq w_R \\ 1 & ,\gamma_R^*(x) = w_R \end{cases}
\end{aligned}$$

and

$$\begin{aligned}
\nu_{\gamma_R^*} &: R\gamma_R^* \to [0,1] \\
\nu_{\gamma_R^*}\left(\gamma_R^*(x)\right) &= \begin{cases} \inf\left\{\nu_A(a) : a \in \gamma_R^*(x)\right\} & ,\gamma_R^*(x) \neq w_R \\ 0 & ,\gamma_R^*(x) = w_R \end{cases}
\end{aligned}$$

Theorem 2.4. *Let T be a continuous t-norm and $A = (\mu_A, \nu_A)$ be an T-intuitionistic fuzzy H_ν-subring of R.Considering $R\gamma_R^*$ as a hyperring, then $A\gamma_R^* = \left(\mu_{\gamma_R^*}, \nu_{\gamma_R^*}\right)$ is a T-intuitionistic H_ν-subring of $R\gamma_R^*$.*

Proof. Let $\gamma_R^*(x)$, $\gamma_R^*(y) \in R\gamma_R^*$.we can write:

$$\begin{aligned}
T\left(\mu_{\gamma_R^*}\left(\gamma_R^*(x)\right), \mu_{\gamma_R^*}\left(\gamma_R^*(y)\right)\right) &= T\left(\sup_{a\in\gamma_R^*(x)}\left\{\mu_A(a)\right\}\ , \sup_{b\in\gamma_R^*(y)}\left\{\mu_A(b)\right\},\right) \\
&= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{T\left(\mu_A(a), \mu_A(b)\right)\right\} \\
&\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{\inf\left\{\mu_A(z) : z \in a+b\right\}\right\} \\
&\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{\sup\left\{\mu_A(z) : z \in a+b\right\}\right\} \\
&\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{\sup\left\{\mu_A(z) : z \in \gamma_R^*(a+b)\right\}\right\} \\
&= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{\mu_{\gamma_R^*}\left(\gamma_R^*(a+b)\right)\right\} \\
&= \mu_{\gamma_R^*}\left(\gamma_R^*(a+b)\right) = \mu_{\gamma_R^*}\left(\gamma_R^*(a) \oplus \gamma_R^*(b)\right)
\end{aligned}$$

Therefore the first condition in definition 2.1 is satisfied.

Let $\gamma_R^*(x)$, $\gamma_R^*(y) \in R\gamma_R^*$.we can write:

$$\begin{aligned}
T\left(1-\nu_{\gamma_R^*}\left(\gamma_R^*(x)\right), 1-\nu_{\gamma_R^*}\left(\gamma_R^*(y)\right)\right) &= T\left(1-\inf_{a\in\gamma_R^*(x)}\{\nu_A(a)\}, 1-\inf_{b\in\gamma_R^*(y)}\{\nu_A(b)\}\right) \\
&= T\left(\sup_{a\in\gamma_R^*(x)}\{1-\nu_A(a)\} \ , \sup_{b\in\gamma_R^*(y)}\{1-\nu_A(b)\},\right) \\
&= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)} \{T(1-\nu_A(a), 1-\nu_A(b))\} \\
&\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)} \{1-\sup\{\nu_A(z) : z\in a+b\}\} \\
&\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)} \{1-\inf\{\nu_A(z) : z\in a+b\}\} \\
&\leq \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)} \{1-\inf\{\nu_A(z) : z\in \gamma_R^*(a+b)\}\} \\
&= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)} \left\{1-\nu_{\gamma_R^*}\left(\gamma_R^*(a+b)\right)\right\} \\
&= 1-\nu_{\gamma_R^*}\left(\gamma_R^*(a+b)\right) \\
&= 1-\nu_{\gamma_R^*}\left(\gamma_R^*(a)\oplus\gamma_R^*(b)\right)
\end{aligned}$$

in this way, Definition 2.1 (2) is verified.
Now suppose $\gamma_R^*(x)$ and $\gamma_R^*(a)$ are two arbitrary elements of $R\gamma_R^*$.Since $A = (\mu_A, \nu_A)$ be an T-intuitionistic fuzzy H_ν-subring of R, it follows that for all $r \in \gamma_R^*(a)$, $s \in \gamma_R^*(x)$ there exists $y_{r,s}$, $z_{r,s} \in R$ such that $r \in (s + y_{r,s}) \cap (z_{r,s} + s)$ and

$$T(\mu_A(r), \mu_A(s)) \leq T(\mu_A(y_{r,s}), \mu_A(z_{r,s}))$$

From $r \in (s + y_{r,s}) \cap (z_{r,s} + s)$ it follows that

$$\gamma_R^*(s) \oplus \gamma_R^*(y_{r,s}) = \gamma_R^*(r) \ \ , \ \gamma_R^*(z_{r,s}) \oplus \gamma_R^*(s) = \gamma_R^*(r)$$

which implies

$$\gamma_R^*(x) \oplus \gamma_R^*(y_{r,s}) = \gamma_R^*(a) \ \ , \ \gamma_R^*(z_{r,s}) \oplus \gamma_R^*(x) = \gamma_R^*(a)$$

Now if $r_1 \in \gamma_R^*(a)$ and $s_1 \in \gamma_R^*(x)$, then there exists there exists y_{r_1,s_1} , $z_{r_1,s_1} \in R$ such that

$$\gamma_R^*(s_1) \oplus \gamma_R^*(y_{r_1,s_1}) = \gamma_R^*(r_1)$$

and since $\gamma_R^*(r_1) = \gamma_R^*(r)$ we get $\gamma_R^*(s_1) \oplus \gamma_R^*(y_{r_1,s_1}) = \gamma_R^*(s) \oplus \gamma_R^*(y_{r,s})$ and therefore $\gamma_R^*(y_{r,s}) = \gamma_R^*(y_{r_1,s_1})$ .Similarly, we have $\gamma_R^*(z_{r,s}) = \gamma_R^*(z_{r_1,s_1})$.
So all the $y_{r,s}$, $z_{r,s}$ satisfying

$$T(\mu_A(r), \mu_A(s)) \leq T(\mu_A(y_{r,s}), \mu_A(z_{r,s}))$$

belong to the same equivalence class.Now we have:

$$\begin{aligned}
T\left(\mu_{\gamma_R^*}\left(\gamma_R^*(x)\right),\mu_{\gamma_R^*}\left(\gamma_R^*(a)\right)\right) &= T\left(\sup_{r\in\gamma_R^*(a)}\{\mu_A(r)\}\ ,\sup_{s\in\gamma_R^*(x)}\{\mu_A(s)\},\right)\\
&= \sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{T\left(\mu_A(r),\mu_A(s)\right)\}\\
&\leq \sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{T\left(\mu_A(y_{r,s}),\mu_A(z_{r,s})\right)\}\\
&= T\left(\sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{\mu_A(y_{r,s})\}\ ,\sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{\mu_A(z_{r,s})\},\right)\\
&\leq T\left(\sup_{y\in\gamma_R^*(y_{r,s})}\{\mu_A(y)\}\ ,\sup_{z\in\gamma_R^*(z_{r,s})}\{\mu_A(z)\},\right)\\
&= T\left(\mu_{\gamma_R^*}\left(\gamma_R^*(y_{r,s})\right)\ ,\mu_{\gamma_R^*}\left(\gamma_R^*(z_{r,s})\right)\right)
\end{aligned}$$

and Definition 2.1 (3) is satisfied.
Similary, we have

$$\begin{aligned}
T\left(1-\nu_{\gamma_R^*}\left(\gamma_R^*(a)\right),1-\nu_{\gamma_R^*}\left(\gamma_R^*(x)\right)\right) &= T\left(1-\inf_{r\in\gamma_R^*(a)}\{\nu_A(r)\},1-\inf_{s\in\gamma_R^*(x)}\{\nu_A(s)\}\right)\\
&= T\left(\sup_{r\in\gamma_R^*(a)}\{1-\nu_A(r)\}\ ,\sup_{s\in\gamma_R^*(x)}\{1-\nu_A(s)\},\right)\\
&= \sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{T\left(1-\nu_A(r),1-\nu_A(s)\right)\}\\
&\leq \sup_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{T\left(1-\nu_A(y_{r,s}),1-\nu_A(z_{r,s})\right)\}\\
&= T\left(\sup_{r\in\gamma_R^*(a)}\{1-\nu_A(y_{r,s})\}\ ,\sup_{s\in\gamma_R^*(x)}\{1-\nu_A(z_{r,s})\},\right)\\
&= T\left(1-\inf_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{\nu_A(y_{r,s})\},1-\inf_{r\in\gamma_R^*(a),s\in\gamma_R^*(x)}\{\nu_A(z_{r,s})\}\right)\\
&\leq T\left(1-\inf_{y\in\gamma_R^*(y_{r,s})}\{\nu_A(y_{r,s})\},1-\inf_{z\in\gamma_R^*(z_{r,s})}\{\nu_A(z_{r,s})\}\right)\\
&= T\left(1-\nu_{\gamma_R^*}\left(\gamma_R^*(y_{r,s})\right),1-\nu_{\gamma_R^*}(\gamma_R^*(z_{r,s}))\right)
\end{aligned}$$

and Definition 2.1 (6) is satisfied.
Let $\gamma_R^*(x)\ ,\ \gamma_R^*(y)\in R\gamma_R^*$.we can write:

$$\begin{aligned}
T\left(\mu_{\gamma_R^*}\left(\gamma_R^*(x)\right), \mu_{\gamma_R^*}\left(\gamma_R^*(y)\right)\right) &= T\left(\sup_{a\in\gamma_R^*(x)}\left\{\mu_A(a)\right\}, \sup_{b\in\gamma_R^*(y)}\left\{\mu_A(b)\right\},\right) \\
&= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{T\left(\mu_A(a), \mu_A(b)\right)\right\} \\
&\le \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{\inf\left\{\mu_A(z) : z\in a\cdot b\right\}\right\} \\
&\le \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{\sup\left\{\mu_A(z) : z\in a\cdot b\right\}\right\} \\
&\le \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{\sup\left\{\mu_A(z) : z\in \gamma_R^*(a\cdot b)\right\}\right\} \\
&= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{\mu_{\gamma_R^*}\left(\gamma_R^*(a\cdot b)\right)\right\} \\
&= \mu_{\gamma_R^*}\left(\gamma_R^*(a\cdot b)\right) = \mu_{\gamma_R^*}\left(\gamma_R^*(a)\odot\gamma_R^*(b)\right)
\end{aligned}$$

and Definition 2.1 (4) is satisfied.
Let $\gamma_R^*(x)$, $\gamma_R^*(y) \in R\gamma_R^*$.we can write:

$$\begin{aligned}
T\left(1-\nu_{\gamma_R^*}\left(\gamma_R^*(x)\right), 1-\nu_{\gamma_R^*}\left(\gamma_R^*(y)\right)\right) &= T\left(1-\inf_{a\in\gamma_R^*(x)}\left\{\nu_A(a)\right\}, 1-\inf_{b\in\gamma_R^*(y)}\left\{\nu_A(b)\right\}\right) \\
&= T\left(\sup_{a\in\gamma_R^*(x)}\left\{1-\nu_A(a)\right\}, \sup_{b\in\gamma_R^*(y)}\left\{1-\nu_A(b)\right\},\right) \\
&= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{T\left(1-\nu_A(a), 1-\nu_A(b)\right)\right\} \\
&\le \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{1-\sup\left\{\nu_A(z) : z\in a\cdot b\right\}\right\} \\
&\le \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{1-\inf\left\{\nu_A(z) : z\in a\cdot b\right\}\right\} \\
&\le \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{1-\inf\left\{\nu_A(z) : z\in \gamma_R^*(a\cdot b)\right\}\right\} \\
&= \sup_{b\in\gamma_R^*(y), a\in\gamma_R^*(x)}\left\{1-\nu_{\gamma_R^*}\left(\gamma_R^*(a\cdot b)\right)\right\} \\
&= 1-\nu_{\gamma_R^*}\left(\gamma_R^*(a\cdot b)\right) \\
&= 1-\nu_{\gamma_R^*}\left(\gamma_R^*(a)\odot\gamma_R^*(b)\right)
\end{aligned}$$

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