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INTUITIONISTIC FUZZY MODAL OPERATORS AND RELATIONSHIPS WITH INTUITIONISTIC FUZZY H_ν-IDEALS

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ABSTRACT. In this paper, we wanted to compile the concept of an intuitionistic fuzzy modal operator to intuitionistic fuzzy H_ν-ideal of an H_ν-ring.

1. INTRODUCTION

Davvaz [?] defined the concept of fuzzy H_ν-ideal of an H_ν-ring. Intuitionistic fuzzy modal operator was defined in 1999 by Atanassov [?]. In 2001,Atanassov [?] introduced the generalization of these modal operators.In 2004, Dencheva [?] defined second extension of these operators. Gökhan Çuvalcıoğlu [?] introduced operator $E_{\alpha,\beta}$.He [?] defined a new modal operator $Z^{\omega,\theta}_{\alpha,\beta}$ over intuitionistic fuzzy sets. In 2006 Davvaz, Dudek [?] introduced the notion of intuitionistic fuzzy H_ν-ideal of an H_ν-ring.

Definition 1.1. [?]Let X be a nonempty set.A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X. The complement of μ, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

Definition 1.2. [?]An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$$

where $\mu_A(x), (\mu_A : X \to [0,1])$ is called the "degree of membership of x in A ", $\nu_A(x), (\nu_A : X \to [0,1])$is called the " degree of non- membership of x in A ",and where μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \text{ for all } x \in X.$$

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) >: x\}$.

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Definition 1.3. [?]Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$.

(1) $\Box A = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}$
(2) $\Diamond A = \{< x, 1 - \nu_A(x), \nu_A(x) >: x \in X\}$
(3) $G_{\alpha,\beta}(A) = \{< x, \alpha\mu_A(x), \beta\nu_A(x) >: x \in X\}$ where $\alpha, \beta \in [0, 1]$.

Definition 1.4. [?]Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$,
$\alpha, \beta \in [0, 1]$

(1) ${}_{\alpha}A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha >: x \in X\}$
(2) ${}_{\alpha}A = \{< x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x) >: x \in X\}$

Definition 1.5. [?]Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X), \alpha, \beta \in [0, 1]$.

(1) ${}_{\alpha,\beta}A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + \beta >: x \in X\}$ where $\alpha + \beta \in [0, 1]$.
(2) ${}_{\alpha,\beta}A = \{< x, \alpha\mu_A(x) + \beta, \alpha\nu_A(x) >: x \in X\}$ where $\alpha + \beta \in [0, 1]$.

Definition 1.6. [?]Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$, $\alpha, \beta \in [0, 1]$.

$$E_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + 1 - \alpha), \alpha(\beta\nu_A(x) + 1 - \beta) >: x \in X\}$$

Definition 1.7. [?]Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$,
$\alpha, \beta, \omega \in [0, 1]$

$$Z^{\omega}_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + \omega - \omega.\alpha), \alpha(\beta\nu_A(x) + \omega - \omega.\beta) >: x \in X\}$$

Definition 1.8. [?]Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$, $\alpha, \beta, \omega, \theta \in [0, 1]$.

$$Z^{\omega,\theta}_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + \omega - \omega.\alpha), \alpha(\beta\nu_A(x) + \theta - \theta.\beta) >: x \in X\}$$

Definition 1.9. [?]An H_ν-ring is a system $(R, +, \cdot)$ with two hyperoperations satisfying the following ring-like axioms:

(i) $(R, +, \cdot)$ is an H_ν-group, that is,

$$\forall a \in R,\ a + R = R + a = R$$
$$\forall x, y, z \in R,\ ((x + y) + z) \cap (x + (y + z)) \neq$$

(ii) $(R, \cdot)$ is an H_ν-semigroup, that is,

$$\forall x, y, z \in R,\ ((x \cdot y) \cdot z) \cap (x \cdot (y \cdot z)) \neq$$

(iii) $(\cdot)$ is weak distributive with respect to $(+)$, that is, for all $x, y, z \in R$,

$$((x + y) \cdot z) \cap (x \cdot z + y \cdot z) \neq$$
$$(x \cdot (y + z)) \cap (x \cdot y + x \cdot z) \neq$$

Definition 1.10. [?]A hyperstructure is a non-empty set H together with a map $*: H \times H \to P^*(H)$ which is called hyperoperation, where $P^*(H)$ denotes the set of all non-empty subsets of H. The image of pair (x, y) is denoted by $x * y$. If $x \in H$ and $A, B \subseteq H$, then by $A * B$, $A * x$ and $x * B$ we mean $A * B = \underset{a \in A, b \in B}{\cup} a * b$, $A * x = A * \{x\}$ and $x * B = \{x\} * B$, respectively.

2. Intuitionistic fuzzy H_ν-ideals

Definition 2.1. [?]An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in R is called a left (resp., right) intuitionistic fuzzy H_ν-ideal of R if
(1) $\min\{\mu_A(x), \mu_A(y)\} \leq \inf\{\mu_A(z) : z \in x + y\}$, for all $x, y \in R$
(2) for all $x, a \in R$ there exists $y, z \in R$ such that $x \in (a + y) \cap (z + a)$ and

$$\min\{\mu_A(a), \mu_A(x)\} \leq \min\{\mu_A(y), \mu_A(z)\}$$

(3) $\mu_A(y) \leq \inf\{\mu_A(z) : z \in x \cdot y\}$(resp.,$\mu_A(x) \leq \inf\{\mu_A(z) : z \in x \cdot y\}$) for all $x, y \in R$
(4) $\sup\{\nu_A(z) : z \in x + y\} \leq \max\{\nu_A(x), \nu_A(y)\}$, for all $x, y \in R$
(5) for all $x, a \in R$ there exists $y, z \in R$ such that $x \in (a + y) \cap (z + a)$ and

$$\max\{\nu_A(y), \nu_A(z)\} \leq \max\{\nu_A(a), \nu_A(x)\}$$

(6) $\sup\{\nu_A(z) : z \in x \cdot y\} \leq \nu_A(y)$ (resp.,$\sup\{\nu_A(z) : z \in x \cdot y\} \leq \nu_A(x)$) for all $x, y \in R$

Theorem 2.2. [?]*If $A = (\mu_A, \nu_A)$ is a left intuitionistic fuzzy H_ν-ideal of R if and only if $\Box A$ and $\Diamond A$ are left intuitionistic fuzzy H_ν-ideals of R.*

Theorem 2.3. *If $A = (\mu_A, \nu_A)$ is a left intuitionistic fuzzy H_ν-ideal of R if and only if $\alpha \neq 0$, ${}_\alpha A = (\alpha\mu_A, \alpha\nu_A + 1 - \alpha)$ and ${}_\alpha A = (\alpha\mu_A + 1 - \alpha, \alpha\nu_A)$ are left intuitionistic fuzzy H_ν-ideals of R.*

Theorem 2.4. *If $A = (\mu_A, \nu_A)$ is a left intuitionistic fuzzy H_ν-ideal of R if and only if $\alpha, \beta \in (0, 1)$, $E_{\alpha,\beta}(A) = (\beta(\alpha\mu_A + 1 - \alpha), \alpha(\beta\nu_A + 1 - \beta))$ is left intuitionistic fuzzy H_ν-ideals of R.*

Proof. Suppose that $A = (\mu_A, \nu_A)$ is a left intuitionistic fuzzy H_ν-ideal of R. We show that $E_{\alpha,\beta}(A) = (\beta(\alpha\mu_A + 1 - \alpha), \alpha(\beta\nu_A + 1 - \beta))$ is a left intuitionistic fuzzy H_ν-ideals of R. For $x, y \in R$, we have

$$\min\{\mu_A(x), \mu_A(y)\} \leq \inf\{\mu_A(z) : z \in x + y\}$$

and

$$\begin{aligned}
\inf\{\mu_A(z) : z \in x + y\} &= t_0 \Rightarrow \forall\ z \in x + y,\ t_0 \leq \mu_A(z) \\
&\Rightarrow \forall\ z \in x + y,\ \alpha.t_0 \leq \alpha.\mu_A(z) \\
&\Rightarrow \forall\ z \in x + y,\ \beta.(\alpha.t_0 + 1 - \alpha) \leq \beta.(\alpha.\mu_A(z) + 1 - \alpha) \\
&\Rightarrow \forall\ z \in x + y,\ \beta.(\alpha.t_0 + 1 - \alpha) \leq \inf\{\beta(\alpha\mu_A(z) + 1 - \alpha) : z \in x + y\}
\end{aligned}$$

Hence

$$\begin{aligned}\beta.\left[\alpha.\min\left\{\mu_A(x),\mu_A(y)\right\}+1-\alpha\right] &\leq \beta.\left[\alpha.\inf\left\{\mu_A(z):z\in x+y\right\}+1-\alpha\right]\\ &\leq \inf\left\{\beta(\alpha\mu_A(z)+1-\alpha):z\in x+y\right\}\end{aligned}$$

which implies that

$$\beta.\left[\alpha.\min\left\{\mu_A(x),\mu_A(y)\right\}+1-\alpha\right]\leq \inf\left\{\beta(\alpha\mu_A(z)+1-\alpha):z\in x+y\right\}$$

thus the condition (1) of Definition **??** is valid.
Let $x,a\in R$, then there exists $y,z\in R$ such that $x\in(a+y)\cap(z+a)$ and

$$\min\left\{\mu_A(a),\mu_A(x)\right\}\leq\min\left\{\mu_A(y),\mu_A(z)\right\}$$

so

$$\beta.\left[\alpha.\min\left\{\mu_A(a),\mu_A(x)\right\}+1-\alpha\right]\leq\beta.\left[\alpha.\min\left\{\mu_A(y),\mu_A(z)\right\}+1-\alpha\right]$$

Hence

$$\min\left\{\beta(\alpha\mu_A(a)+1-\alpha),\beta(\alpha\mu_A(x)+1-\alpha)\right\}\leq\min\left\{\beta(\alpha\mu_A(y)+1-\alpha),\beta(\alpha\mu_A(z)+1-\alpha)\right\}$$

in this way, Definition **??** (2) is verified
For $x,y\in R$, we have

$$\mu_A(y)\leq\inf\left\{\mu_A(z):z\in x\cdot y\right\}$$

Hence

$$\begin{aligned}\beta(\alpha\mu_A(y)+1-\alpha) &\leq \beta.\left[\alpha.\inf\left\{\mu_A(z):z\in x\cdot y\right\}+1-\alpha\right]\\ &\leq \inf\left\{\beta(\alpha\mu_A(z)+1-\alpha):z\in x\cdot y\right\}\end{aligned}$$

and Definition **??** (3) is satisfied.
For $x,y\in R$, we have

$$\sup\left\{\nu_A(z):z\in x+y\right\}\leq\max\left\{\nu_A(x),\nu_A(y)\right\}$$

and

$$\begin{aligned}\sup\left\{\nu_A(z):z\in x+y\right\} &= t_0\Rightarrow\forall\ z\in x+y,\ \nu_A(z)\leq t_0\\ &\Rightarrow \forall\ z\in x+y,\ \beta.\nu_A(z)\leq\beta.t_0\\ &\Rightarrow \forall\ z\in x+y,\ \alpha.\left(\beta.\nu_A(z)+1-\beta\right)\leq\alpha.\left(\beta.t_0+1-\beta\right)\end{aligned}$$

Hence

$$\begin{aligned}\sup\left\{\alpha(\beta\nu_A(z)+1-\beta):z\in x+y\right\} &\leq \alpha.\left[\beta.\sup\left\{\nu_A(z):z\in x+y\right\}+1-\beta\right]\\ &\leq \alpha.\left[\beta.\max\left\{\nu_A(x),\nu_A(y)\right\}+1-\beta\right]\end{aligned}$$

which implies that

$$\sup\{\alpha(\beta\nu_A(z)+1-\beta):z\in x+y\}\leq\max\{\alpha(\beta\nu_A(x)+1-\beta),\alpha(\beta\nu_A(y)+1-\beta)\}$$

and Definition ?? (4) is satisfied.
Let $x,a\in R$, then there exists $y,z\in R$ such that $x\in(a+y)\cap(z+a)$ and

$$\max\{\nu_A(y),\nu_A(z)\}\leq\max\{\nu_A(a),\nu_A(x)\}$$

Hence

$$\max\{\alpha(\beta\nu_A(y)+1-\beta),\alpha(\beta\nu_A(z)+1-\beta)\}\leq\max\{\alpha(\beta\nu_A(a)+1-\beta),\alpha(\beta\nu_A(x)+1-\beta)\}$$

and Definition ?? (5) is satisfied.
For $x,y\in R$, we have

$$\sup\{\nu_A(z):z\in x\cdot y\}\leq\nu_A(y)$$

Hence

$$\begin{aligned}\sup\{\alpha(\beta\nu_A(z)+1-\beta):z\in x\cdot y\}&\leq\alpha.[\beta.\sup\{\nu_A(z):z\in x\cdot y\}+1-\beta]\\&\leq\alpha(\beta\nu_A(y)+1-\beta)\end{aligned}$$

which implies that

$$\sup\{\alpha(\beta\nu_A(z)+1-\beta):z\in x\cdot y\}\leq\alpha(\beta\nu_A(y)+1-\beta)$$

and Definition ?? (6) is satisfied.
Coversely suppose that $E_{\alpha,\beta}(A)=(\beta(\alpha\mu_A+1-\alpha),\alpha(\beta\nu_A+1-\beta))$ is a left intuitionistic fuzzy H_ν-ideals of R.
For $x,y\in R$, we have

$$\beta.[\alpha.\min\{\mu_A(x),\mu_A(y)\}+1-\alpha]\leq\inf\{\beta(\alpha\mu_A(z)+1-\alpha):z\in x+y\}$$

Hence

$$\begin{aligned}\frac{1}{\alpha.\beta}.[\beta.(\alpha.\min\{\mu_A(x),\mu_A(y)\}+1-\alpha)-\beta.(1-\alpha)]&\leq\frac{1}{\alpha.\beta}.[\inf\{\beta(\alpha\mu_A(z)+1-\alpha):z\in x+y\}-\beta.(1-\alpha)]\\&\leq\inf\{\mu_A(z):z\in x+y\}\end{aligned}$$

which implies that

$$\min\{\mu_A(x),\mu_A(y)\}\leq\inf\{\mu_A(z):z\in x+y\}$$

thus the condition (1) of Definition ?? is valid.
Let $x,a\in R$, then there exists $y,z\in R$ such that $x\in(a+y)\cap(z+a)$ and

$$\min\{\beta(\alpha\mu_A(a)+1-\alpha),\beta(\alpha\mu_A(x)+1-\alpha)\}\leq\min\{\beta(\alpha\mu_A(y)+1-\alpha),\beta(\alpha\mu_A(z)+1-\alpha)\}$$

so

$$\min\{\mu_A(a),\mu_A(x)\}\leq\min\{\mu_A(y),\mu_A(z)\}$$

in this way, Definition **??** (2) is verified
For $x, y \in R$, we have

$$\frac{1}{\alpha.\beta}.\left[\beta(\alpha\mu_A(y)+1-\alpha)-\beta.(1-\alpha)\right] \leq \frac{1}{\alpha.\beta}.\left[\inf\left\{\beta(\alpha\mu_A(z)+1-\alpha) : z \in x \cdot y\right\}-\beta.(1-\alpha)\right]$$

and so

$$\mu_A(y) \leq \inf\left\{\mu_A(z) : z \in x \cdot y\right\}$$

and Definition **??** (3) is satisfied.
For $x, y \in R$, we have

$$\sup\left\{\alpha(\beta\nu_A(z)+1-\beta) : z \in x+y\right\} \leq \max\left\{\alpha(\beta\nu_A(x)+1-\beta), \alpha(\beta\nu_A(y)+1-\beta)\right\}$$

Hence

$$\begin{aligned} \sup\left\{\nu_A(z) : z \in x+y\right\} &\leq \frac{1}{\alpha.\beta}.\left[\sup\left\{\alpha(\beta\nu_A(z)+1-\beta) : z \in x+y\right\}-\alpha(1-\beta)\right] \\ &\leq \frac{1}{\alpha.\beta}.\left[\max\left\{\alpha(\beta\nu_A(x)+1-\beta), \alpha(\beta\nu_A(y)+1-\beta)\right\}-\alpha(1-\beta)\right] \end{aligned}$$

which implies that

$$\sup\left\{\nu_A(z) : z \in x+y\right\} \leq \max\left\{\nu_A(x), \nu_A(y)\right\}$$

and Definition **??** (4) is satisfied.
Let $x, a \in R$, then there exists $y, z \in R$ such that $x \in (a+y) \cap (z+a)$ and

$$\max\left\{\alpha(\beta\nu_A(y)+1-\beta), \alpha(\beta\nu_A(z)+1-\beta)\right\} \leq \max\left\{\alpha(\beta\nu_A(a)+1-\beta), \alpha(\beta\nu_A(x)+1-\beta)\right\}$$

Hence

$$\max\left\{\nu_A(y), \nu_A(z)\right\} \leq \max\left\{\nu_A(a), \nu_A(x)\right\}$$

and Definition **??** (5) is satisfied.
For $x, y \in R$, we have

$$\sup\left\{\alpha(\beta\nu_A(z)+1-\beta) : z \in x \cdot y\right\} \leq \alpha(\beta\nu_A(y)+1-\beta)$$

Hence

$$\sup\left\{\nu_A(z) : z \in x+y\right\} \leq \frac{1}{\alpha.\beta}.\left[\sup\left\{\alpha(\beta\nu_A(z)+1-\beta) : z \in x \cdot y\right\}-\alpha(1-\beta)\right]$$

which implies that

$$\sup\left\{\nu_A(z) : z \in x \cdot y\right\} \leq \nu_A(y)$$

thus the condition (6) of Definition **??** is valid.

□

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