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THE DYNAMICS OF THE SCALAR PARTICLES IN ROBERTSON-WALKER SPACETIME VIA TELEPARALLEL THEORY

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Abstract. Relativistic particle equations in cosmological backgrounds are studied to analyze the quantum effects in curved space-times. The Klein-Gordon (KG) equation is an eligible relativistic wave equation that describes spin-0 scalar particles. In the present study we investigate the dynamics of scalar particles in the torsion gravity. The torsion gravity (or teleparallel gravity) is an alternative approach to the gravitation theory. This formalism corresponds to a gauge theory for the translation group based on Weitzenbock geometry. In this theory, gravitation is attributed to torsion . In the present study, we investigate the behavior of the massless spin-0 particles by examining KG equation in a Robertson-Walker universe in teleparallel gravity.

1. Introduction

The General Relativity (GR) theory of Einstein depends on the interaction of matter with the spacetime geometry. This theory was then tried to be based on a more general ground with Teleparallel (TP), Einstein-Cartan and Poincare theories [1]-[3].

According to GR Theory, the existance of a gravitational field is assumed to create a curvature in spacetime and no other kind of spacetime corruption is exist. Torsion could also be a reason of the deformation in spacetime, but GR argues that the torsion disappears from the early stages of the universe [4].

The teleparallel counterpart of GR, namely TP theory, can be interpreted as a gauge theory for the translation group. According to TP theory, instead of torsion, curvature is supposed to vanish. Teleparallel theory decribes the gravitational interaction by defining a force field instead of geometrizing the spacetime. The corresponding underlying spacetime of TP theory is called Weitzenböck spacetime. Despite this fundamental diversity, the two theories are found to give same explanations of the gravitational interactions. This implies that both curvature and torsion defines the gravitational interaction equivalent [5].

Key words and phrases. Klein-Gordon equation, teleparallel theory, exact solutions.

Combining the quantum theory and gravitation is one of the most important aim of the contemporary physics. At the atomic scale the weakness of gravitational effects makes the role of general relativistic wave equations insignificant, but for many astrophysical situations the solutions of these equations gain importance due to dominant role of gravitational effects such as in particle creation by black holes [6]. So that, relativistic particle equations are considered in cosmological backgrounds to study quantum effects of curved space-times. In this direction, Klein-Gordon and Dirac equations are the most studied equations. Most of such studies are performed in GR theory. These equations are very rarely studied in the contex of TP theory.

In the present study, we aim to investigate exact solutions of the Klein-Gordon equation in Robertson-Walker space-time following the TP formalism. The space-time model we use represents the early universe and is given by [7]:

$$ds^2 = \frac{1}{a^2(t)}dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{1.1}$$

where the scale factor is $a(t) = \sqrt{\gamma + \Lambda t}$.

By defining the conformal time, $d\eta = \dfrac{dt}{a(t)}$, the metric becomes

$$ds^2 = d\eta^2 - a^2(\eta)(dx^2 + dy^2 + dz^2) \tag{1.2}$$

2. Notations and Definitions in Teleparallel Theory

The curvature and torsion are basicly alternative aspects of describing the gravitational field. They are therefore related to the same degrees of freedom of gravity.

The notion of certain teleparallelism was first presented by Einstein in 1920s, in his unfortunate attempt to unify gravitation and electromagnetism. In the following years, the teleparallel gravity become the gauge theory for the translation group [8].

For the nontrivial tetrad $h^a\mu$, the spacetime is related to tangentspace metrics by

$$g_{\mu\nu} = h^a_\mu h^b_\nu \eta_{ab} \tag{2.1}$$

where η_{ab} is the metric tensor of Minkowski spacetime. The tedrads for the metric given in Eq.(1.2) are $h^\mu_0 = \delta^\mu_0$ and $h^\mu_i = \frac{1}{a(\eta)}\delta^\mu_i$, where $i = 1, 2, 3$.

In the Weitzenböck spacetime, so-called Weitzenböck connections are given by [8]

$$\dot{\Gamma}^\rho_{\nu\mu} = h^\rho_a \partial_\mu h^a\nu \tag{2.2}$$

As we mentioned before, since the teleparallelism attributes gravitation to torsion, the torsion explains gravitation by treating as a force. The torsion of the Weitzenböck connection is [8]

$$\dot{T}^\rho{}_{\mu\nu} = \dot{\Gamma}^\rho{}_{\nu\mu} - \dot{\Gamma}^\rho{}_{\mu\nu} \tag{2.3}$$

This expression shows that the gravitational field strength is nothing but torsion written in the tetrad basis:

$$\dot{T}^a{}_{\mu\nu} = h^a_\rho \dot{\Gamma}^\rho{}_{\mu\nu} \tag{2.4}$$

The contortion of the Weitzenböck torsion is given by [8]

$$\dot{K}^{\rho}{}_{\mu\nu} = \frac{1}{2}\left(\dot{T}_{\mu}{}^{\rho}{}_{\nu} + \dot{T}_{\nu}{}^{\rho}{}_{\mu} - \dot{T}^{\rho}{}_{\mu\nu}\right) \tag{2.5}$$

The teleparallel covariant derivative acting on a B vector in Weitzenböck geometry is given as [8]

$$\dot{D}_{\rho}B^{\mu} \equiv \partial_{\rho}B^{\mu} + \left(\dot{\Gamma}^{\mu}{}_{\lambda\rho} - \dot{K}^{\mu}{}_{\lambda\rho}\right)B^{\lambda} \tag{2.6}$$

3. Klein-Gordon Equation in Teleparallel Gravity

According to TP theory, the interaction of gravitation with any field can be described in terms of torsion. Since the scalar field represented by the Klein-Gordon equation is known to interact with curvature, it may also couple to torsion.

The KleinGordon equation in teleparallel gravity reads [8]

$$\left\{\partial_{\mu}\partial^{\mu} + (\dot{\Gamma}^{\mu}{}_{\lambda\mu} - \dot{K}^{\mu}{}_{\lambda\mu})\partial^{\lambda} + m^2\right\}\phi = 0 \tag{3.1}$$

Making use of metric given in Eq. (1.2) for calculation of Weitzenböck connections and contortion given in Eqs. (2.2) and (2.5), one obtains the below non-vanishing components, respectively:

$$\dot{\Gamma}^{i}{}_{i0} = \frac{\dot{a}}{a}, i = 1, 2, 3 \tag{3.2}$$

where the dot represents the derivative with respect conformal time η and,

$$\dot{K}^{0}{}_{ij} = -\delta_{ij}a\dot{a} \tag{3.3}$$

$$\dot{K}^{i}{}_{0j} = -\delta_{ij}\frac{\dot{a}}{a} \tag{3.4}$$

After insertion of these expressions into Eq.(3.1), we obtain for Klein-Gordon equation

$$\left\{\partial_{\eta}^2 - \frac{1}{a^2(\eta)}(\partial x^2 + \partial y^2 + \partial z^2) + \frac{3\dot{a}}{a}\partial_{\eta} + m^2\right\}\phi = 0 \tag{3.5}$$

By defining the wave-function as $\phi(\eta, \vec{x}) = e^{i\vec{k}\cdot\vec{x}}\eta^{-\frac{3}{2}}\varphi(\eta)$, the Klein-Gordon equation becomes to the following form:

$$\left\{\partial_{\eta}^2 - \left(\frac{3}{4} + \frac{4\vec{k}^2}{\lambda^2}\right)\frac{1}{\eta^2} + m^2\right\}\varphi = 0 \tag{3.6}$$

Definition 3.1. The Bessel differential equation in general form is given as follow [9]

$$x^2\partial_x^2 y + ax\partial_x y + (bx^q + c)y = 0, q \neq 0; \tag{3.7}$$

The solution is

$$b \neq 0 : y = x^{\frac{1-a}{2}} Z_{\nu}\left(\frac{2}{q}\sqrt{b}x^{\frac{q}{2}}\right) \tag{3.8}$$

where Z_{ν} are Bessel functions and $\nu = \frac{1}{q}\sqrt{(1-a)^2 - 4c}$

From this definition, we will find the solution of Eq.(3.6) as

$$\varphi(\eta) = \sqrt{\eta} Z_\nu(m\sqrt{\eta}) \tag{3.9}$$

where $\nu = \sqrt{\frac{13}{4} + \frac{16k^2}{\lambda}}$.

Hence, exact solution for our problem is obtained as follow:

$$\phi(\eta, \vec{x}) = e^{i\vec{k}\cdot\vec{x}} \eta^{-1} Z_\nu(m\sqrt{\eta}) \tag{3.10}$$

4. Conclusion

In this study we obtained exact solutions of the Klein-Gordon equation via teleparallel gravity formalism. The wave function of the relativistic scalar particle is found in terms of the Bessel functions.

Compared to the General Relativity theory, there are fewer studies in literature in which the complete solutions of scalar particles have been obtained via Teleparallel theory. So, this study might be useful for further studies in this direction.

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