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GENERALIZATION OF SOFT SETS IN FUZZY SETTINGS

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ABSTRACT. In 1999, D.Molodtsov defined soft sets and established the fundamental results of the new theory [5], to solve complicated problems and various types of uncertainties. A soft set is an approximate description of an object precisely consisting of two parts, namely predicate and approximate value set. In this paper, the notion of soft set is generalized in fuzzy setting, and introduced several operators for **L**-soft set theory like the complement of an **L**-soft set, **L**-order and **L**-equivalence relation which are continuation of [3].

1. INTRODUCTION

A soft set is introduced by Molodtsov [5] is a collection of approximate function which each of it has two parts: a predicate and an approximate value set that is used for describe an object. In 1982 Pawlak [6] gave first practical application of soft sets in decision making problems. It is based on the notion of knowledge reduction in rough set theory. Research on the soft set theory has been accelerated rapidly. The most important one is researched by Maji et al. [4] and introduced fuzzy soft set theory. The notion of a soft set in L-set theory is studied and introduced several operators of L-soft set theory in [1]. In addition to this, the rough operators on the set of all L-soft sets induced by the rough operators on L^x is investigated in [2]. The aim of this paper is to research the works on generalizations of soft sets in fuzzy settings and several operators for L-soft set theory.

2. BACKGROUND IN SOFT SETS AND FUZZY SOFT SETS

In this section, an overview of L-sets, soft sets, fuzzy soft sets and rough sets is studied which can be named as preliminaries devoted to some main notions for each area, i.e., L-sets [3], soft sets [5], fuzzy soft sets [4] and rough sets [6]. Apart form definitions and theorems are numbered, known concepts are mentioned in the text along with the reference [3].

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Definition 2.1. [5] A pair (f, E) is called a soft set (over U) if and only if f is a mapping or E into the set of all subsets of the set U.

From now on, we will use definitions and operations about soft sets which are more suitable for pure mathematics based on study of [5].

Definition 2.2. [5] A soft set f on the universe U is defined by the set of ordered pairs

$$f = \Big\{ \big(e, f(e)\big) : e \in E \Big\}$$

where $f : E \to \mathcal{P}(U)$ such that $f(e) = \emptyset$ if $e \in E \setminus A$ then $f = f_A$.

Note that the set of all soft sets over U will be denoted by $\mathbb{S}$.

Definition 2.3. [3] (L-sets)
For a universe set X, an L-set in X is a mapping $\widetilde{A} : X \to L$. $\widetilde{A}(x)$ indicates the truth degree of "x belongs to $\widetilde{A}$". We use the symbol L^x to denote the set of all L-sets in X.

Definition 2.4. [4] (Fuzzy Soft Sets)
Let X be a universe and E a set of attributes. Then the pair $\widetilde{(X, E)}$ denotes the collection of all fuzzy soft sets on X with attributes from E and is called a fuzzy soft class.

Definition 2.5. [6] (Rough Sets)
Let X be a universe and E a set of attributes. Then the pair $\widetilde{(X, E)}$ denotes the collection of all fuzzy soft sets on X with attributes from E and is called a fuzzy soft class.

3. Generalization Of Soft Sets In Fuzzy Settings

In this section, the notion of a soft set in fuzzy setting is generalized and defined several operators on the set of all L-soft sets over X. The definitions are accompanies with examples.

First, I define the notion of a soft set in fuzzy setting.

Definition 3.1. [1] A pair (F, A) is called an L-soft set over X, if $A \subseteq E$ and $F \to L^X$, denoted by $\theta = (F, A)$.

Example:

Suppose $X = \{x_1, x_2, x_3\}$, and $L = [0, 1]$ equipped with Gödel Structure. Let $E = \{t_1, t_2, t_3, t_4\}$, $A_1 = \{t_1, t_2, t_3\}$ and let $F_1 : A_1 \to L^X$, where $F_1(t_1) = \{0.7/x_1\}$, $F_1(t_2) = \{1/x_1, 0.5/x_2\}$, $F_1(t_3) = \{0.6/x_1, 0.2/x_2, 0.7/x_3\}$. Then clearly, (F_1, A_1) is an L-soft set.

Let $LS(X)$ be the set of all L-soft sets over X. On which, there exist two kinds of special elements: one is called a absolute soft set $(1_A, A)$, $\forall t \in A, 1_A(t) = \tilde{1}_X$, denoted by $\Gamma_A = (1_A, A)$; the other is called a null soft set $(0_A; A)$, $\forall t \in A, 0_A(t) = \tilde{0}_X$, denoted by $\Phi_A = (0_A, A)$.

Second, I give the relation L-order $\preceq$, and L-equivalence relation $\approx$ are defined in [3] which correspond the relations $\tilde{\subseteq}; =$ in classical case. For two L-soft sets $\theta_1 = (F, A)$, $\theta_2 = (G, B) \in LS(X)$,

$$(\theta_1 \preceq \theta_2) = S(\theta_1, \theta_2) = \bigwedge_{t \in A} S(F(t), G(t)),$$

$$(\theta_1 \approx \theta_2) = S(\theta_1, \theta_2) \wedge S(\theta_2, \theta_1).$$

Example:

Follows above example, (F_1, A_1) is an L-soft set. Let $A_2 = \{t_1, t_2, t_3, t_4\}$ and let $F_2 : A_2 \to L^X$, where $F_2(t_1) = \{0.4/x_1\}$, $F_2(t_2) = \{0.9/x_1, 0.5/x_2, 0.3/x_3\}$, $F_2(t_3) = \{0.4/x_1, 0.2/x_2, 0.5/x_3\}$, $F_2(t_4) = \{1/x_1, 0.7/x_2, 0.6/x_3\}$. Then (F_2, A_2) is also an L-soft set. Thus we obtain

$$S((F_1, A_1), (F_2, A_2)) = \bigwedge_{t \in A_1} S(F_1(t), F_2(t))$$
$$= S(F_1(t_1), F_2(t_1)) \wedge S(F_1(t_2), F_2(t_2)) \wedge S(F_1(t_3), F_2(t_3)) = 0.4,$$
$$S((F_2, A_2), (F_1, A_1)) = \bigwedge_{t \in A_2} S(F_2(t), F_2(t)) = 0.$$

Clearly, we have

$$\theta_1 \tilde{\subseteq} \theta_2 \Leftrightarrow S(\theta_1, \theta_2) = 1 \Leftrightarrow A \subseteq B, \text{ and } \forall t \in A, F(t) \subseteq G(t),$$
$$\theta_1 = \theta_2 \Leftrightarrow S(\theta_1, \theta_2) = 1, S(\theta_2, \theta_1) = 1 \Leftrightarrow A = B, \text{ and } \forall t \in A, F(t) = G(t),$$

So $\langle\langle LS(X), \approx\rangle, \preceq\rangle$ is an L-order set (See[3]).

Example:

Follows the above example, (F_1, A_1) is a L-soft set. Let $A_3 = A_1$ and let $F_3 : A_3 \to L^X$, where $F_3(t_1) = \{0.4/x_1\}$, $F_3(t_2) = \{0.9/x_1, 0.5/x_2\}$, $F_3(t_3) = \{0.4/x_1, 0.2/x_2, 0.5/x_3\}$. Then $(F_3; A_3)$ is also an L-soft set and $(F_3, A_3) \tilde{\subseteq} (F_1, A_1)$.

Third, I study the union and the extended intersection of two L-soft sets is intoduced in [3] Also Maji et al. [4] defined the union of two fuzzy soft sets as follows:

Definition 3.2. [3] Suppose $(F, A), (G, B) \in LS(X)$ are two L-soft sets. Then the union of (F, A) and (G, B) is an L-soft set (H, C), where $C = A \cup B$, and for $t \in C$,

$$H(t) = \begin{cases} F(t) & \text{if } t \in A - B \\ G(t) & \text{if } t \in B - A \\ F(t) \vee G(t) & \text{if } t \in A \cap B \end{cases}$$

and written as $(F, A) \tilde{\bigcup} (G, B) = (H, C)$.

Definition 3.3. [3] The complement of an L-soft set (F, A) is denoted by $(F, A)^c$, and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to L^X$, for every $-t \in \rceil A$,

$$F^c(-t) = F^*(t) = F(t) \to 0.$$

Example:

Suppose $X = \{x_1, x_2, x_3\}$ and $L = [0,1]$ with a $b = \max(a + b - 1, 0)$, $a \to b = \min(1 - a + b, 1)$ (Lukasiewicz Structure). $a^* = a \to 0$.

Consider (F_1, A_1) defined in Example 3.2. Then $(F_1, A_1)^c = (F_1^c, \rceil A_1)$, $\rceil A_1 = \{\neg t_1, \neg t_2, \neg t_3\}$, $F_1^c : \rceil A_1 \to L^X$, where

$$\begin{aligned}
F_1^c(\neg t_1) &= \{0.7/x_1\}^* = \{0.3/x_1, 1/x_2, 1/x_3\},\\
F_1^c(\neg t_2) &= \{1/x_1, 0.5/x_2\}^* = \{0/x_1, 0.5/x_2, 1/x_3\},\\
F_1^c(\neg t_3) &= \{0.6/x_1, 0.2/x_2, 0.7/x_3\}^* = \{0.4/x_1, 0.8/x_2, 0.3/x_3\}.
\end{aligned}$$

Thus Clearly, $\Gamma_A^c = \Phi_A$ and $\Phi_A^c = \Gamma_A$ hold.

Furthermore, L satisfies the law of double negation, if it satisfies $a = (a \to 0) \to 0$, for every $a \in L$, that is $a^{**} = a$ (See [3]).

Finally, I can give definitions of the operators OR,AND on $LS(X)$.

Definition 3.4. [3] Suppose $(F, A), (G, B) \in LS(X)$, $(F; A)$ AND $(G; B)$ is an L-soft set, denoted by $(F, A)\tilde{\wedge}(G, B) = (H, C)$, where $C = A \times B$, for every $t_1 \in A, t_2 \in B, H(t_1, t_2) = F(t_1) \wedge G(t_2)$.

(F, A) OR (G, B) is an L-soft set, denoted by $(F, A)\tilde{\vee}(G, B) = (K, D)$, where $D = A \times B$, for every $t_1 \in A; t_2 \in B, K(t_1, t_2) = F(t_1) \vee G(t_2)$.

Example:

Let (F_1, A_1) and (F_2, A_2) are two L-soft sets. Then we have, $(F_1, A_1)\tilde{\wedge}(F_2, A_2) = (H, C)$, where $C = A \times B$,

C	H
(t_1, t_1)	$\{0.4/x_1\}$
(t_1, t_2)	$\{0.7/x_1\}$
(t_1, t_3)	$\{0.4/x_1\}$
(t_2, t_1)	$\{0.4/x_1\}$
(t_2, t_2)	$\{0.9/x_1, 0.5/x_2\}$
(t_2, t_3)	$\{0.4/x_1, 0.2/x_2\}$
(t_3, t_1)	$\{0.4/x_1\}$
(t_3, t_2)	$\{0.6/x_1, 0.2/x_2, 0.3/x_3\}$
(t_3, t_3)	$\{0.4/x_1, 0.2/x_2, 0.5/x_3\}$

$(F_1, A_1)\tilde{\vee}(F_2, A_2) = (K, D)$, where $D = A \times B$,

D	K
(t_1, t_1)	$\{0.7/x_1\}$
(t_1, t_2)	$\{0.9/x_1, 0.5/x_2, 0.3/x_3\}$
(t_1, t_3)	$\{0.7/x_1, 0.2/x_2, 0.5/x_3\}$
(t_2, t_1)	$\{1/x_1, 0.5/x_2\}$
(t_2, t_2)	$\{1/x_1, 0.5/x_2, 0.3/x_3\}$
(t_2, t_3)	$\{1/x_1, 0.5/x_2, 0.5/x_3\}$
(t_3, t_1)	$\{0.6/x_1, 0.2/x_2, 0.7/x_3\}$
(t_3, t_2)	$\{0.9/x_1, 0.5/x_2, 0.7/x_3\}$
(t_3, t_3)	$\{0.6/x_1, 0.2/x_2, 0.7/x_3\}$

About the operators OR, AND on $LS(X)$, the following De Morgan's types of results hold. For more details, see [3].

Conclusion

This paper contributes generalization of the notion of soft set in fuzzy setting and several operators for L-soft set theory as defined and studied in [3] and supports them with examples and counterexamples.

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