

## A Common Fixed Points Theorems for Ordered F-Contractions on Partial Metric Space

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**Abstract:** Matthews introduced the concept of partial metric spaces and proved an analogue of Banach fixed point theorem in partial metric spaces. After this remarkable contribution, many authors took interest in partial metric spaces and its topological properties and presented several well known fixed point results in the framework of partial metric spaces. Banach presented a landmark fixed point result (Banach Contraction Principle). This result proved a gateway for the fixed point researchers and opened a new door in metric fixed point theory. A number of efforts have been made to enrich and generalize Banach Contraction Principle. Following Banach, in 2012, Wardowski presented a new contraction (known as F -contraction). Since 2012, a number of fixed point results have been established by using F-contraction or ordered F-contraction. In his recent paper, Wardowski presented the concept of F-contraction. Then, some generalizations of F-contractions including multivalued case are obtained. In this talk, we prove a common fixed point theorem for a pair of generalized rational type ordered F-contractions in complete partial metric spaces. An example is constructed to illustrate this result and to show that our result is more general than the result in literature. We apply the mentioned theorem to show the existence of solution of implicit type integral equations.

**Keywords and phrases:** fixed point, partial metric, F-contraction, integral type

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## Some Properties of $\mathbb{B}(-1)$ -convex Sets

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Abstract:  $\mathbb{B}^{-1}$ -convexity which is an abstract convexity type is studied in various articles ([1, 1, 3, 4]).  $\mathbb{B}^{-1}$ -convex sets was introduced in [1].  $\mathbb{B}^{-1}$ -convex functions was examined in [3]. In this work, some important properties of  $\mathbb{B}^{-1}$ -convex sets are given.

**Keywords and phrases:** Abstarct convexity,  $\mathbb{B}^{-1}$ -convexity,  $\mathbb{B}^{-1}$ -convex sets.

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## On the Basis Properties for a Discontinuous Sturm-Liouville Operator

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Abstract: We consider discontinuous Sturm-Liouville equation with boundary conditions depending eigenparameter and transmission conditions

$$\begin{aligned}ly &:= -y''(x) + q(x)y(x) = \lambda y(x), x \in [-1, 0) \cup (0, 1], \\ \alpha_{11}y(-1) - \alpha_{12}y'(-1) &= \lambda(\alpha_{21}y(-1) - \alpha_{22}y'(-1)), \\ \beta_{11}y(1) - \beta_{12}y'(1) &= \lambda(\beta_{21}y(1) - \beta_{22}y'(1)), \\ \gamma_3y(+0) - \gamma_4y(-0) &= 0, \\ \gamma_2y'(+0) - \gamma_1y'(-0) + (\lambda\delta_1 + \delta_2)y(0) &= 0,\end{aligned}$$

where  $\lambda$  is a complex parameter,  $q(x)$  is real-valued continuous function on the intervals  $[-1, 0)$  and  $(0, 1]$  has a finite limits  $q(\pm 0) := \lim_{x \rightarrow \pm 0} q(x)$  and  $\gamma_i, \delta_j, (i = \overline{1, 4}, j = 1, 2)$  are real numbers.

In this study, we research the completeness, the minimality and the basis properties of the system eigenfunctions of the boundary value problem. For another different boundary value problem, the same results are investigated in [1], [2].

**Keywords and phrases:** Discontinuous Sturm-Liouville operator, basic property, spectral parameter in boundary conditions, transmission conditions,

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## New bounds for the zeros of polynomials

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**ABSTRACT** Let  $p(z) = z^n + a_{n-1}z^{n-1} + \dots + a_2z + a_1$  be a monic polynomial of degree  $n \geq 3$ , where  $a_1, a_2, \dots, a_n$  are complex numbers with  $a_1 \neq 0$  and let  $C(p)$  be the Frobenius companion matrix of  $p$ . In this article we apply numerical radius estimates to derive new bounds for the zeros of  $p$ . Also we obtain some known bounds as special cases from our bounds.

**Keywords and phrases:** Frobenius companion matrix, zeros of polynomials, numerical radius, spectral radius

**2010 Mathematic Subject Classification:** 47A12, 15A60, 26C10,30C15.

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### Porosity Convergence in Metric Spaces

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Abstract: Porosity was defined by Denjoy in [1]. A detailed information about this type porosity was given by Thomson in [1]. The notion of porosity can also be used in metric space by replacing intervals with balls (see [3]) Let  $(X, d)$  be a metric space. Let  $M \subset X$ ,  $x \in X$  and  $R > 0$ . Then we define

$$\theta(x, R, M) := \sup\{r > 0 : \text{there exist an open ball } B_d(z, r) \text{ such that} \\ d(x, z) < R \text{ and } B_d(z, r) \cap M = \emptyset\},$$

$$p(x, M) := \limsup_{R \rightarrow 0^+} \frac{\theta(x, R, M)}{R}. \quad (1)$$

In this study, we will define a new type of convergence called  $d_p$ -convergence for metric valued sequences by using (9). Then we will give some properties of this new concept.

**Keywords and phrases:** metric space, sequence, porosity.

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**On Fredholm Integral Equations with Separable Kernels**

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**Abstract:**

The Fredholm integral equations of the first kind are usually considered as incorrectly posed problems. The conventional way of solving them is to first convert them into the Fredholm integral equations of the second kind by means of the regularization method. Applying the method of regularization combined with any other standard methods to obtain solutions sometimes can require a tedious work. One of the aims of this article is to obtain a simple formula which provides not only the solution obtained by the regularization method followed by some existing techniques such as direct computation, Adomian decomposition, successive approximations, etc., but also other solutions.

**Keywords and phrases:** Fredholm integral equation, ill-posed problems, regularization.

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**An Application of the Weighted Mean Value Method to Multidimensional Fredholm  
Integral Equations**

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**Abstract:**

This study introduces a new algorithm for solving multidimensional Fredholm integral equations of the first kind. The regularization and weighted mean-value methods constitute the algorithm. The former is used to transform the Fredholm integral equations of the first kind to the Fredholm integral equations of the second kind and the latter is employed to solve the resulting Fredholm integral equations. Examples will be provided to demonstrate the reliability and applicability of the proposed approach.

**Keywords and phrases:** Fredholm integral equation, weighted-mean value method, regularization.

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## On graded semi-primary submodules

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**ABSTRACT** Let  $G$  be a group with identity  $e$  and let  $R$  be a  $G$ -graded ring. In this work, we introduce and study graded semi-primary submodules of graded modules over graded commutative rings. A number of results concerning of these classes of submodules are given.

**Keywords and phrases:** graded primary submodule, graded semi-primary submodules, graded submodules.

**2010 Mathematic Subject Classification:** 13A02, 16W50

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### A note on Soft Normed Spaces

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In this paper we discuss some properties of soft normed spaces and soft inner product spaces. We also investigate some properties of soft continuous operators in these spaces.

**Keywords and phrases:** soft set, soft normed space, soft inner product space, soft continuous operators.

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## Some Ideal Convergent Sequence Spaces Defined by $M_\lambda$ -summability

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Abstract: In the present work, we introduce some ideal convergent sequence spaces by using  $M_\lambda$ -summability method which is defined by P. N. Natarajan [On the  $(M, \lambda_n)$  method of summability, Analysis] as a typically generalization of Nörlund method. Further, we examine some of their topological properties.

**Keywords and phrases:**  $I$ -convergence, Orlicz function,  $M_\lambda$ -summability, sequence space.

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## ON INTUITIONISTIC FUZZY SIMILARITIES OF THE AUDIO SIGNALS BASED ON SPECTRAL ANALYSIS

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**Abstract:** In this paper, we compute similarity degrees between audio signals which is obtained after transfer over the communication channel and original audio signal. Fuzzification of audio signals which frequency spectrum analysis performed on the frequency-time domain are made. Fuzzy and intuitionistic fuzzy similarity measures are used to calculate the degree of similarity between intuitionistic fuzzified audio signals. In the simulation performed with different noise levels may occur during transfer, the performance of similarity algorithms used in the system are tested .

**Keywords and phrases:** intuitionistic fuzzy set, distance measure, similarity measure, audio signal,

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**Actions of some simple compact Lie groups on themselves**

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Let  $G$  be a compact connected simple Lie group and let it act non-transitively, non-trivially on itself. Hsiang conjectured that the principal isotropy subgroup type must be the maximal torus and the action must be cohomologically similar that of the adjoint action. But Bredon found a simple counterexample where  $G = SU(3)$  and the action is defined by  $(A, B) \mapsto ABA^t$ . In this work, we define adjoint-like action and we prove that if  $SO(n)$ , ( $n \geq 6$ ) or  $SU(3)$  acts smoothly (and nontrivially) on itself with non-empty fixed point set, then the action is adjoint-like.

**Keywords and phrases:** Lie groups, Differentiable transformation groups, Characteristic classes.

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## Split Semi-Quaternions and Unit Tangent Bundle of Minkowski 3-Space

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**Abstract:** In this study, firstly the basic algebraic structures of split semi-quaternions are given. Afterwards, we have defined the unit tangent bundle of Minkowski 3-space in terms of unit split semi-quaternions.

**Keywords and phrases:** Minkowski 3-space, Split Semi-Quaternion, Unit Tangent Bundle.

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## Choquet and Sugeno Integrals as Aggregation Operators for Pattern Recognition

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**Abstract:** In this paper, a comparison of the Choquet and Sugeno integrals is presented. The proposed methods enable the calculation of the Choquet and Sugeno integrals for combining multiple source of information with a degree of uncertainty. The methods are used to combine the output of the modules of a modular neural network for face recognition. In this paper, the focus is on aggregation operators that use measures as inputs, in particular the Choquet and Sugeno integrals. Recognition results with the Choquet integral are better or comparable to results produced by the Sugeno integral.

**Keywords and phrases:** Aggregation operators, Choquet Integral, Sugeno integral, Modular Neural networks, fuzzy measures, fuzzy densities.

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ABSTRACT NO.14 (2017)

## Isoclinic Rotations and Semi-Quaternions

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Abstract: In this study, the basic algebraic structures of semi-quaternions are given. Furthermore, isoclinic rotation is defined by using semi-quaternions.

**Keywords and phrases:** Semi-Quaternion, Isoclinic Rotation.

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ABSTRACT NO.15 (2017)

**Solving fuzzy wave fractional differential equation by mean Fourier transform**

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Abstract:

In this work, we studied the integral solution of wave fractional differential equation with fuzzy initial data under generalized fuzzy Caputo derivative by mean the Fourier transform, the exact solution is given in the case of . Some examples are presented to illustrate the results.

**Keywords and phrases:** Intuitionistic fuzzy sets, Distance between intuitionistic fuzzy sets, intuitionistic fractional derivative.

## References

[1] ...

## Semi-Quaternions and Unit Tangent Bundle of Euclidean 3-Space

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Abstract: In this study, the basic algebraic structures of semi-quaternions are given. Moreover, the unit tangent bundle of Euclidean 3-space is stated in terms of unit semi-quaternions.

**Keywords and phrases:** Euclidean 3-space, Semi-Quaternion, Unit Tangent Bundle.

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***k*-Pell, *k*-Pell Lucas and Modified *k*-Pell Matrix Sequences**

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**Abstract:** In this study, we define *k*-Pell, *k*-Pell Lucas and Modified *k*-Pell matrix sequences. We obtain the matrices given *n*th general term of these sequences. Also, we present the relationships and some important identities involving terms of *k*-Pell matrix sequence, *k*-Pell Lucas matrix sequence and Modified *k*-Pell matrix sequence.

**Keywords and phrases:** *k*-Pell sequence, *k*-Pell Lucas sequence, Modified *k*-Pell sequences.

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**On Determination a Saturation Class for  $(M, \lambda_n^\pi)$  Methods**

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Abstract: One of the main problem in approximation theory is determination a saturation class for given method. In this paper, motivating Natarajan, we defined  $(M, \lambda_n^\pi)$  method of summability. The object of the present paper is to give some of its nice properties and determine a saturation class for  $(M, \lambda_n^\pi)$  method.

**Keywords and phrases:** Regularity,  $(M, \lambda_n^\pi)$  method, Abel-summability, Saturation class

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### Basis properties of eigenfunctions of a discontinuous Sturm-Liouville problem

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Abstract: In this work, we studied the boundary value problem

$$-y'' + q(x)y = \lambda^2 \rho(x)y, \quad 0 \leq x \leq \pi, \quad (2)$$

$$y(0) = 0, \quad (3)$$

$$\alpha_1 y(\pi) + \alpha_2 y'(\pi) + \lambda^2 (\beta_1 y'(\pi) + \beta_2 y(\pi)) = 0. \quad (4)$$

where  $q(x) \in L_2(0, \pi)$  is a real valued function,  $\lambda$  is a complex parameter,  $\alpha_i, \beta_j$  ( $i, j = 1, 2$ ) are arbitrary real numbers and

$$\rho(x) = \begin{cases} 1, & 0 \leq x < a, \\ \alpha^2, & a < x \leq \pi, \end{cases}$$

where  $0 < \alpha \neq 1$ .

We shall encounter the boundary value problem (9)-(11) in the mathematical physics problems which consist a boundary condition with a time-derivative. For example, in [1] the relationship between diffusion processes and Sturm-Liouville problem with eigen-parameter in the boundary conditions has been shown. The operator theoretic formulations for these kinds of problems are investigated in [2] which also has examples of physical applications of boundary value problems including spectral parameters in boundary conditions. The discontinuous coefficient  $\rho(x)$  describes the processes with different densities.

In this work, it has been shown that one can associate a self-adjoint operator in an adequate Hilbert space with the boundary value problem (9)-(11) and basis properties are considered.

**Keywords and phrases:** Sturm-Liouville operator, minimality, completeness.

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### Basis properties of eigenfunctions of a discontinuous Sturm-Liouville problem

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Abstract: In this work, we studied the boundary value problem

$$-y'' + q(x)y = \lambda^2 \rho(x)y, \quad 0 \leq x \leq \pi, \quad (5)$$

$$y(0) = 0, \quad (6)$$

$$\alpha_1 y(\pi) + \alpha_2 y'(\pi) + \lambda^2 (\beta_1 y'(\pi) + \beta_2 y(\pi)) = 0. \quad (7)$$

where  $q(x) \in L_2(0, \pi)$  is a real valued function,  $\lambda$  is a complex parameter,  $\alpha_i, \beta_j$  ( $i, j = 1, 2$ ) are arbitrary real numbers and

$$\rho(x) = \begin{cases} 1, & 0 \leq x < a, \\ \alpha^2, & a < x \leq \pi, \end{cases}$$

where  $0 < a \neq 1$ .

We shall encounter the boundary value problem (9)-(11) in the mathematical physics problems which consist a boundary condition with a time-derivative. For example, in [1] the relationship between diffusion processes and Sturm-Liouville problem with eigen-parameter in the boundary conditions has been shown. The operator theoretic formulations for these kinds of problems are investigated in [2] which also has examples of physical applications of boundary value problems including spectral parameters in boundary conditions. The discontinuous coefficient  $\rho(x)$  describes the processes with different densities.

In this work, it has been shown that one can associate a self-adjoint operator in an adequate Hilbert space with the boundary value problem (9)-(11) and basis properties are considered.

**Keywords and phrases:** Sturm-Liouville operator, minimality, completeness.

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## Connectedness in Fuzzy Soft Topological Spaces

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**Abstract:** In this talk, we aim to investigate the connectedness of an L-fuzzy soft set with respect to parameters in fuzzy soft topological spaces. We first define separatedness of two L-fuzzy soft sets with respect to parameters. Then we introduce the notion of connectedness in fuzzy soft topological spaces and study some of its fundamental properties.

**Keywords and phrases:** fuzzy soft set, fuzzy soft topology, continuity, connectedness.

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## **A survey for spectrum of discrete Klein-Gordon s-wave equation**

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Abstract: Spectral theory of differential and discrete operators plays a crucial role for many branches of science from mathematical physics to engineering. Naimark investigated the spectral analysis of one dimensional Schrödinger operator in his study [1]. He gave necessary and sufficient conditions for the finiteness of the eigenvalues and spectral singularities. Besides continuous case of the operators, discrete versions of the differential operators have become the subject matter of various studies [2,3]. In this study, we shall present the spectral properties of the discrete Klein-Gordon equation.

**Keywords and phrases:** Spectral analysis, Klein-Gordon equation, spectral singularities.

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**Application of Naimark and Pavlov conditions for discrete Sturm-Liouville equation with hyperbolic eigenparameter**

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Abstract: Spectral analysis of differential and discrete operators have many application areas in science like quantum mechanics, mathematical physics, economics and engineering. Spectral theory of Sturm-Liouville equation was begun by Naimark [1]. He showed that spectrum of this operator consists of eigenvalues, spectral singularities and continuous spectrum. Also, these spectral singularities and eigenvalues are of finite number under certain conditions. In this study, we shall present necessary and sufficient conditions for the finiteness of the eigenvalues and spectral singularities of the discrete analogue of Sturm-Liouville equation consisting of eigenparameter dependent boundary condition and hyperbolic eigenparameter [2,3].

**Keywords and phrases:** Spectral analysis, Sturm-Liouville equation, spectral singularities.

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**Some inequalities on a Kaehler manifold endowed with complex semi-symmetric metric connection**

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Abstract: In this paper, we establish some inequalities for a Kaehler manifold endowed with complex semi-symmetric metric connection. Using these inequalities, we obtain the relation between Ricci curvature, scalar curvature and the mean curvature for a Kaehler manifold endowed with complex semi-symmetric metric connection.

**Keywords and phrases:** Kaehler manifold, Chen inequalities, Ricci quarter-symmetric metric connection, Levi-Civita connection.

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## Geometry of Lightlike Einstein Hypersurfaces

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**Abstract:** In this paper, firstly, we present lightlike Einstein hypersurfaces. Then, we give some theorems and results for Einstein screen homothetic lightlike hypersurfaces and Einstein screen conformal lightlike hypersurfaces of a Lorentz manifold  $(\bar{M}(c), \bar{g})$  of constant curvature  $c$ .

**Keywords and phrases:** Einstein hypersurfaces, lightlike hypersurfaces, Lorentzian manifold.

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**A characterization theorem of screen conformal half-lightlike submanifolds of an indefinite complex space form admitting a quarter symmetric metric connection**

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Abstract: There are two cases of codimension 2 lightlike submanifolds  $M$  since for this type the dimension of their radical distribution  $Rad(TM)$  is either one or two. A codimension 2 lightlike submanifold is called half-lightlike if  $\dim(RadTM) = 1$  [1]. The objective of this study is to present the geometry of screen conformal half-lightlike submanifolds of an indefinite complex space form equipped with a quarter symmetric metric connection and give an important characterization theorem for them.

**Keywords and phrases:** Half-lightlike submanifold, screen conformal submanifold, quarter-symmetric metric connection, complex space form.

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## Geometry of Minimal STCR Lightlike Submanifolds of an Indefinite Kaehler Manifold

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**Abstract:** In this study, we present a new class of lightlike submanifolds of an indefinite Kaehler manifold which called as Screen Transversal Cauchy Riemann (STCR)-lightlike submanifolds. Firstly, we investigate geometry of minimal STCR-lightlike submanifolds of an indefinite Kaehler manifold. Then, we give some theorems and results.

**Keywords and phrases:** Lightlike submanifold, minimal submanifold, indefinite Kaehler manifold.

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## A Note on Ricci Solitons in CR-Submanifolds with Concurrent Vector Fields

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**Abstract** In this paper, we focus on Ricci soliton with concurrent potential field. We survey conditions for a CR-submanifold to be a Ricci soliton in a Kaehlerian manifold with equipped with a concurrent vector field.

**Keywords and phrases:** Kaehlerian manifold, Ricci soliton, concurrent vector field.

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**Local Campanato estimates for multilinear commutator operators with rough kernel on generalized vanishing local Morrey spaces**

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Abstract: In this paper, we consider the behavior of multilinear commutator operators with rough kernel both on generalized local Morrey spaces and generalized vanishing local Morrey spaces, respectively.

**Keywords and phrases:** Calderón–Zygmund operator, rough kernel, generalized vanishing local Morrey space, multilinear commutator, local Campanato space.

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**Vanishing generalized Morrey spaces and commutators of Marcinkiewicz integrals with rough kernel associated with schrödinger operator**

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Abstract: Let  $L = -\Delta + V(x)$  be a Schrödinger operator, where  $\Delta$  is the Laplacian on  $\mathbb{R}^n$ , while nonnegative potential  $V(x)$  belonging to the reverse Hölder class. We establish the boundedness of the commutators of Marcinkiewicz integrals with rough kernel associated with schrödinger operator on vanishing generalized Morrey spaces.

**Keywords and phrases:** Marcinkiewicz operator, rough kernel, Schrödinger operator, vanishing generalized Morrey space, commutator, *BMO*.

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## On The Spectral Properties of a Boundary Value Problem

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Abstract: We consider the spectral problem

$$\begin{aligned} -y'' + q(x)y &= \lambda y, \quad 0 < x < 1, \\ y(0) &= 0, \quad y'(0) = \lambda (ay(1) + by'(1)), \end{aligned}$$

where  $\lambda$  is a spectral parameter,  $q(x) \in L_1(0, 1)$  is a complex-valued function,  $a$  and  $b$  are arbitrary nonzero complex numbers which satisfy the condition  $|a| + |b| \neq 0$ .

We research the spectral properties (existence of eigenvalues, asymptotic formulae for eigenvalues and eigenfunctions, minimality and basicity of the system of eigenfunctions) of the considered boundary value problem.

There are many articles which investigate the various aspects of boundary value problems for ordinary differential operators with a spectral parameter in the boundary condition. This study is related to the articles [1]-[3].

**Keywords and phrases:** Eigenvalues, Eigenfunctions, Minimal System and Basis.

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**Conformable fractional order Dirac system on time scales**Tuba GULSEN<sup>1</sup>, Emrah YILMAZ<sup>2</sup> and Sertac GOKTAS<sup>3</sup><sup>1,2</sup>Department of Mathematics, Faculty of Science, Firat University, Elazig, TURKEY<sup>3</sup>Department of Mathematics, Mersin University, Mersin, TURKEY

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Abstract: Fractional calculus means differentiation and integration of a noninteger order. This topic occurred by the ideas of the mathematicians Leibniz, Liouville, Riemann, Letnikov and Grunwald [1]. It is one of the most intensively developing area of mathematical physics. Fractional calculus has many applications in science and engineering such as memory of a variety of materials, signal identification, temperature field problems in oil strata and diffusion problems. Recently, many researchers have started to deal with the discrete versions of fractional calculus benefitting from the theory of time scales. For example, Benkhettou and his coworkers [1, 3] introduced the concept of conformable fractional derivative on time scales. They explained all properties of conformable fractional derivative on that time scale. Conformable fractional derivative of a function defined on time scale reduces to Hilger derivative [4, 5] when  $\alpha = 1$ .

In this study, we consider conformable fractional order Dirac system [6, 7, 8] with boundary conditions on time scales. Some results about spectral properties of this problem are obtained. Then, we obtain asymptotic estimates for the eigenfunction of this problem. Obtained results are more general than classical Dirac boundary value problem.

**Keywords and phrases:** Time scale calculus, Conformable fractional derivative.

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## Rotational Hypersurfaces in $\mathbb{E}^6$

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**Abstract:** We consider some rotational hypersurfaces in the six dimensional Euclidean space  $\mathbb{E}^6$ . We compute the mean curvature and the Gaussian curvature, and obtain some results of rotational hypersurfaces.

**Keywords and phrases:** 6-space, rotational hypersurface, Gaussian curvature, mean curvature.

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## Differential Equations on the Rotational Hypersurfaces in $\mathbb{E}^5$

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Abstract: In this talk, we study some rotational hypersurfaces in the five dimensional Euclidean space  $\mathbb{E}^5$ . We compute the mean curvature and the Gaussian curvature, and obtain some differential equations on the rotational hypersurfaces.

**Keywords and phrases:** 5-space, rotational hypersurface, Gaussian curvature, mean curvature.

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## On The Properties of New Families of s-Pell and s-Pell-Lucas Numbers

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**Abstract:** In this work, new families of s-Pell and s-Pell-Lucas numbers are introduced. We also present the recurrence relations and the generating functions of the new families for  $k=2$ .

**Keywords and phrases:** Pell numbers, Pell-Lucas numbers, recurrence relations.

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## Application of Asymptotic Iteration Method to Cornell Potential

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Abstract: Energy eigenvalues of Schrödinger particles are acquired for the case of Cornell potential. For calculations, Asymptotic Iteration Method (AIM) is used to get the numeric results. Furthermore, a semi-analytical formula for eigenvalues is achieved by using the method in scope of the perturbation theory. The formula obtained brings benefit to practical usage.

**Keywords and phrases:** Asymptotic iteration method, Cornell potential, Schrödinger equation

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## APPROXIMATION BY DEFERRED-NÖRLUND PRODUCT MEANS IN $L_p$

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Abstract: Let  $a = (a_n)$  and  $b = (b_n)$  be sequences of nonnegative integers with conditions

$$a_n < b_n \quad n = 1, 2, 3, \dots$$

and

$$\lim_{n \rightarrow \infty} b_n = +\infty.$$

The deferred Cesáro means,  $D_a^b$ , determined by  $a$  and  $b$

$$D_a^b = \frac{s_{a_n+1} + s_{a_n+2} + \dots + s_{b_n}}{b_n - a_n} = \frac{1}{b_n - a_n} \sum_{k=a_n+1}^{b_n} s_k,$$

where  $(s_k)$  is a sequence of real or complex numbers [1]. Taking into deferred Cesáro means, deferred-Nörlund product means  $(D_a^b.N_p)$  which the product of  $D_a^b$  mean with a  $N_p$  mean are defined by transformation

$$t_n^{D_a^b.N_p}(f; \cdot) := t_n^{D_a^b.N_p} = \frac{1}{b-a} \sum_{k=a+1}^b \left( P_k^{-1} \sum_{v=0}^k p_{k-v} s_v(f; \cdot) \right)$$

where  $s_n(f; \cdot)$  denotes the partial sum of the first  $(n+1)$  terms of the Fourier series of a  $f \in L$ . In this study, we investigate the degree of approximation by the deferred-Nörlund product means of their Fourier series to functions belonging to generalized Lipschitz class under the some conditions in  $L_p$  space for  $p \geq 1$ .

**Keywords and phrases:** Lipchitz class, deferred-Nörlund product means, trigonometric approximation.

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### Intuitionistic fuzzy fractional differential equation

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Abstract: In this paper we investigate the existence and uniqueness of intuitionistic fuzzy solution for three-point boundary value problem for fractional differential equation:

$$\begin{cases} D^\alpha X(t) = F(t, X_t, D^\beta X(t)) & t \in J := [0, 1] \\ X(t) = \phi(t) & t \in [-r, 0] \\ X(1) = X(\xi) \end{cases}$$

where  $D^\alpha$ ,  $D^\beta$  are standard Riemann-Liouville fractional derivatives. ( $\alpha - \beta > 0$ ) and ( $1 < \alpha < 2$ ), ( $\xi \in [0, 1]$ ),  $F : J \times C_0 \times IF^1 \rightarrow IF^1$  is an intuitionistic fuzzy function.  $\phi \in C_0$ ,  $\phi(0) = 0_{IF}$  and  $C_0 = C([-r, 0], IF^1)$  we denote by  $X_t$  the element of  $C_0$  defined by  $X_t(\theta) = X(t + \theta)$ ,  $\theta \in [-r, 0]$ .

**Keywords and phrases:** Intuitionistic fuzzy sets, Distance between intuitionistic fuzzy sets, Intuitionistic fractional derivative..

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### On the Volkenborn Integral of Some $p$ -adic Trigonometric Function

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**Abstract:** Let  $p$  be a fixed odd prime number. By  $\mathbb{Z}_p, \mathbb{Q}_p$  and  $\mathbb{C}_p$  we denote the ring of  $p$ -adic integers, the field of  $p$ -adic numbers and the completion of the algebraic closure of  $\mathbb{Q}_p$ , respectively.

The Volkenborn integral of  $f \in C^1(\mathbb{Z}_p \rightarrow \mathbb{C}_p)$  on  $\mathbb{Z}_p$  is defined by the formula

$$\int_{\mathbb{Z}_p} f(x) dx := \lim_{n \rightarrow \infty} p^{-n} \sum_{j=0}^{p^n-1} f(j).$$

If  $f(x) = c$  is a constant function, then

$$\int_{\mathbb{Z}_p} f(x) dx = c$$

and for any  $f \in C^1(\mathbb{Z}_p \rightarrow K)$  the relation

$$\int_{\mathbb{Z}_p} f(x+1) dx - \int_{\mathbb{Z}_p} f(x) dx = f'(0)$$

holds (for detail see [3], [1], [2]). We note that in  $p$ -adic analysis the elementary functions are defined by power series. For example, the exponential function is defined by

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

and it converges for  $|x|_p < p^{-\frac{1}{p-1}}$ .

In the present work we consider some  $p$ -adic trigonometric and inverse trigonometric functions. We give some results on the Volkenborn integral of  $p$ -adic trigonometric and inverse trigonometric functions.

**Keywords and phrases:**  $p$ -adic numbers;  $p$ -adic trigonometric functions; Volkenborn integral.

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## The Eigenvalues of $r$ -Periodic Tridiagonal Matrices

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**Abstract:** We introduce  $r$ -periodic tridiagonal matrices for given integer  $r \geq 2$ . We give a new algorithm to find the eigenvalues of certain  $r$ -periodic tridiagonal matrices and implement it in Maple to give some results. Our algorithm also finds the zeros of some families of polynomials with integer coefficients. The degree of these polynomials can be chosen very high. Finally, we generalize existing results by giving the explicit formulas for the eigenvalues of some  $r$ -periodic tridiagonal matrices for  $r = 2, 3$  and  $4$ .

**Keywords and phrases:** Tridiagonal matrices; eigenvalues; characteristic polynomials, linear recurrences; conditional recurrences; continued fractions; Fibonacci sequence; continuants.

This talk is about a joint work with Semih YILMAZ

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## General Conditional Recurrences

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Abstract: A general conditional recurrence sequence  $\{q_n\}$  is one in which the recurrence satisfied by  $q_n$  depends on the residue of  $n$  modulo some integer  $r \geq 2$ . The properties of such sequences are studied, and in particular it is shown that any such sequence  $\{q_n\}$  satisfies a single recurrence equation not dependent on the modulus  $r$ . We also obtain the closed formulas for generating functions and Binet-like formulas for some special cases of such sequences by using continuants and integer partitions. Obtaining the closed formulas for other cases remains as an open problem.

**Keywords and phrases:** Linear recurrences; characteristic polynomials; conditional recurrences; continuants; Fibonacci sequences; integer partitions.

This talk is about a joint work with Daniel Panario and Qiang Wang

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**Exact Solutions of the DKP Equation for an Isotropic Robertson-Walker Metric**

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Abstract: Exact solutions of the Duffin-Kemmer-Petiau (DKP) equation are studied for an Isotropic Robertson-Walker (RW) Metric in the presence of a time-dependent electric field. By considering the solutions we discuss the current density of vector bosons with usage of the Gordon decomposition of the current. Harmonic oscillator behaviour of the vector bosons is also found.

**Keywords and phrases:** DKP equation, RW metric, Gordon Decomposition, Harmonic Oscillator.  
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**On the zeros of generating functions of additive arithmetical semigroups**

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**Abstract:**

Let  $(G, \partial)$  be an additive arithmetical semigroup. By definition,  $G$  is a free commutative semigroup with identity element 1, generated by a countable set  $P$  of primes and admitting an integer valued degree mapping

$\partial : G \rightarrow \mathbb{N} \cup \{0\}$  with the properties

- (i)  $\partial(1) = 0$  and  $\partial(p) > 0$  for all  $p \in P$ ,
- (ii)  $\partial(ab) = \partial(a) + \partial(b)$  for all  $a, b \in G$ ,
- (iii) the total number  $G(n)$  of elements  $a \in G$  of degree  $\partial(a) = n$  is finite for each  $n \geq 0$ .

Let

$$\pi(n) := \# \{p \in P : \partial(p) = n\}$$

denote the total number of primes of degree  $n$  in  $G$ .

The generating function is defined in the formal sense by

$$F(y) := \sum_{n=0}^{\infty} G(n)y^n.$$

In this talk, we prove the zeros of generating functions of additive arithmetical semigroups and then we give as application the abstract prime number theorems for additive arithmetical semigroups.

This is a joint work with Karl-Heinz Indlekofer.

**Keywords and phrases:** Zeros of generating functions, additive arithmetical semigroups, abstract prime number theorems.



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## On the statistical convergence of arithmetical functions

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**Abstract:** Let  $K$  be a subset of  $\mathbb{N}$  and

$$K(n) := \{k : k \leq n, k \in K\}.$$

Then, the asymptotic density of  $K$ , denoted by  $\delta(K)$ , is defined by

$$\delta(K) := \lim_{n \rightarrow \infty} \frac{1}{n} |K(n)| \quad (8)$$

if the limit exists. In (1), the vertical bars indicate the cardinality of the enclosed set.

A real or complex valued arithmetical function  $f$  is said to be statistical convergent to the number  $L$ , if for every  $\varepsilon > 0$ , the set

$$K(n, \varepsilon) := \{k : k \leq n, |f(k) - L| \geq \varepsilon\},$$

has asymptotic density zero.

In this talk, we give some properties of the statistical convergent of arithmetical functions.

This is a joint work with Mehmet Küçükbaşlan and Robert Wagner.

**Keywords and phrases:** Statistical convergence, arithmetical functions.

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## TWISTED SURFACES IN $G_3^1$

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**Abstract:** In this paper, we construct the twisted surfaces according to the supporting plane and type of rotations in pseudo-Galilean space  $G_3^1$ . Also, we find the Gaussian curvatures and mean curvatures of the different types of these twisted surfaces and draw some figures for these twisted surfaces.

**Keywords and phrases:** Twisted surface, Pseudo-Galilean space, Pseudo-Euclidean rotation, Isotropic rotation.

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**Intuitionistic Fuzzy Modal Operators on Intuitionistic Fuzzy  $H_\nu$ -Ideals**Sinem Tarsuslu(Yılmaz)<sup>1</sup>, Yelda Yorulmaz<sup>1</sup>, Arif Bal<sup>1</sup>, Gökhan Çuvalcıoğlu<sup>1</sup><sup>1</sup>Department of Mathematics, Mersin University, Mersin, Turkey

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Abstract: Intuitionistic fuzzy modal operator firstly was defined in 1999 by Atanassov[2]. In following years, the generalization of these modal operators were defined by several authors[3, 4, 5, 7]. The concept of intuitionistic fuzzy  $H_\nu$ -ideal of an  $H_\nu$ -ring was introduced by Davvaz, Dudek [6] in 2006 as a generalization of the fuzzy  $H_\nu$ -ideal. In this paper, we studied the effect of intuitionistic fuzzy modal operator on intuitionistic fuzzy  $H_\nu$ -ideal of an  $H_\nu$ -ring.

**Keywords and phrases:** Intuitionistic fuzzy set, Intuitionistic fuzzy modal operator, Intuitionistic fuzzy  $H_\nu$ -ideal of an  $H_\nu$ -ring.

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## On Homomorphism between Intuitionistic Fuzzy Universal Algebras

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Abstract: The concept of universal algebra on intuitionistic fuzzy sets were introduced by authors and some basic theorems were proved. In this study, homomorphism between intuitionistic fuzzy universal algebras was defined and effect of homomorphism on intuitionistic fuzzy universal algebras was examined.

**Keywords and phrases:** Intuitionistic fuzzy sets, intuitionistic fuzzy universal algebra, intuitionistic fuzzy subalgebra, intuitionistic fuzzy homomorphism.

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## 2-MAGNETIC CURVES IN $E^3$

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Abstract: In this paper, we define the notion of 2- $T$ -magnetic (respectively, 2- $N$ -magnetic and 2- $B$ -magnetic) curve according to Frenet frame in Euclidean 3-space. Also we obtain the 2-magnetic vector field  $V$  when the curve is a 2- $T$ -magnetic (respectively, 2- $N$ -magnetic and 2- $B$ -magnetic) trajectory of  $V$  according to Frenet frame and give some results and examples for 2-magnetic curves according to Frenet frame.

**Keywords and phrases:** Magnetic curves, 2-Magnetic curves, Lorentz force, Frenet frame.

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## Pseudosymmetric Null Hypersurfaces in Indefinite Kenmotsu Space Forms

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**Abstract:** In this study, firstly we investigate pseudosymmetric null hypersurfaces of an indefinite Kenmotsu space form, tangent to the structure vector field. We obtain sufficient conditions for null hypersurface to be pseudosymmetric in an indefinite Kenmotsu space form. Also, we give sufficient conditions for a null hypersurface to be totally geodesic in indefinite Kenmotsu space form:

**Keywords and phrases:** Pseudosymmetric lightlike hypersurface, Indefinite Kenmotsu space form, Lightlike hypersurface.

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**Special Weakly Ricci Symmetric Lightlike Hypersurfaces  
in Indefinite Kenmotsu Space Forms**

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**Abstract:** In this study, we investigate special weakly Ricci symmetric lightlike hypersurfaces of indefinite Kenmotsu space form, tangent to the structure vector field. We obtain sufficient conditions for a lightlike hypersurface to be a special weakly Ricci symmetric (SWRS) lightlike hypersurface in indefinite Kenmotsu space form and we show that a special weakly Ricci symmetric (SWRS) lightlike hypersurface is totally geodesic under certain a condition.

**Keywords and phrases:** Special Weakly Ricci symmetric, Indefinite Kenmotsu space form, Lightlike hypersurface.

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New Convergence Definitions for a Sequence of Order  $\alpha$  of Random Variables in ProbabilityÖmer Kişi<sup>1</sup> and <sup>2</sup>Erhan Güler<sup>1,2</sup>Department of Mathematics, Bartın University, Bartın, Turkey

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Abstract: In recent years, lots of notions have begun to be studied in probability. In this study, following a very recent and new approach, we further generalize recently introduced summability methods, namely,  $\mathcal{I}$ -statistical convergence of order  $\alpha$  and  $\mathcal{I}$ -lacunary statistical convergence of order  $\alpha$ , where  $0 < \alpha < 1$  (which extend the important summability methods, statistical convergence and lacunary statistical convergence using ideals of  $\mathbb{N}$ ). We will study this important concepts in probability and we will discuss some relations between our definitions. We will also examine some results when  $0 < \alpha < \beta \leq 1$ .

**Keywords and phrases:** Statistical convergence,  $\mathcal{I}$ -convergence, probability theory, lacunary sequence.

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$\sigma$ -Asymptotically Lacunary Statistical Equivalent Functions on Amenable SemigroupsÖmer Kişî<sup>1</sup> and <sup>2</sup>Erhan Güler<sup>1,2</sup>Department of Mathematics, Bartın University, Bartın, Turkey

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Abstract: This study presents the following definitions which is a natural combination of the definition for asymptotically equivalent,  $\lambda$ -statistical convergence, lacunary convergence and  $\sigma$ -convergence. In this study we introduce the concepts of  $S_\sigma$ -asymptotically equivalent,  $S_{\sigma,\lambda}$ -asymptotically equivalent,  $\sigma$ -asymptotically lacunary statistical equivalent and strong  $(\sigma, \theta)$ -asymptotically equivalent functions defined on discrete countable amenable semigroups. In addition to these definitions, we give some inclusion theorems.

**Keywords and phrases:** Lacunary statistical convergence,  $\sigma$ -convergence, Amenable Semigroups.

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**Extension of Meir-Keeler fixed-point theorem in fuzzy metric space**

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Abstract:

Mier and Keeler formulated their fixed-point theorem for contractive mapping with purely metric condition. Numerous mathematicians extended this idea. In this paper, we present a simple method of proving such theorem in the fuzzy metric space and give a new result.

**Keywords and phrases:** xxx.

## References

1 ...

[9] **Motion of Klein Gordon Particles In External Fields via Asymptotic Iteration Method**

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Abstract: We study the motion of the Klein-Gordon particles in the existence of the hyperbolic external electromagnetic fields. Exact solutions are obtained in terms of the hypergeometric functions with the usage of the asymptotic iteration method (AIM). The external fields used for the problem is defined as follow:

$$A_\mu = \left\{ 0, 0, B_0x, -\frac{E_0}{\xi} [1 - e^{-\xi t}] \right\}$$

**Keywords and phrases:** Asymptotic iteration method, hyperbolic external fields, Klein-Gordon equation IFSCOM2017

ABSTRACT NO. ? PP. 000-000 (2017)

**On Separation Axioms in Temporal Intuitionistic Fuzzy Šostak's Topology**

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Abstract: In this study, we define temporal intuitionistic fuzzy point and the separation axioms  $T_0, T_1$  and  $T_2$  in temporal intuitionistic fuzzy Šostak's topology and investigate some properties of them.

**Keywords and phrases:** temporal intuitionistic fuzzy set, temporal intuitionistic fuzzy topology, separation axioms, Hausdorffness

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## Derivation of the Ladder Operators for spin-1/2 Particles Moving in a Magnetic Field

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**Abstract:** In the present study we investigate the motion of charged spin-1/2 particles moving in a varying magnetic field given as  $B_z = B_0/x^2$ . We obtain the creation and annihilation operators for the wavefunction of Dirac particles. The helicity raising and lowering canonical momentum operators will be also constructed.

**Acknowledgement:** Supported by the Research Fund of Mersin University in TURKEY with project number: 2016-1-AP4-1425.

**Keywords and phrases:** Ladder operator, Dirac equation, creation and annihilation operators. IFSCOM2017

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## Landsberg Finsler Structures

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**Abstract:** The slit tangent bundle manifold  $F^m = (M, F)$  admits the Riemann metric  $G$  with Finslerian coefficients. Then the totally geodesic vertical foliation  $F_V$  on  $(TM_0, G)$  can be found which means that  $F^m$  is the Landsberg manifold. In this study, some structure theorems for these kind of spaces are discussed.

**Keywords and phrases:** Finsler structure, Landsberg manifold, Riemannian metric, tangent bundle

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### Thermodynamics properties of a Schrödinger particle interacting with general molecular potential

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**Abstract:** We explore the thermodynamics properties such as, mean energy  $U$ , specific heat  $C$ , free energy  $F$  and entropy  $S$ , of the non-relativistic spinless particles interacting with general molecular potential (GMP) by using the vibrational partition function  $Z$  which depends on the energy eigenvalues obtained for GMP potential. We give also some numerical analysis for each thermodynamics properties graphically.

**Keywords and phrases:** Thermodynamics properties, Schrödinger particles, General molecular potential.

**Acknowledgement:** This study was supported by the Research Fund of Mersin University in Turkey with the project number: 2015-AP4-1244. IFSCOM2017

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### Twenty years of the $q$ -Bernstein polynomials: achievements and perspectives

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**Abstract:** The Bernstein polynomials were introduced in 1912 by S. N. Bernstein, who used them to provide an elegant proof of the Weierstrass Approximation Theorem. Subsequently, many remarkable properties and important applications of these polynomials were discovered, while a great number of generalizations and analogues were introduced.

Generalized Bernstein polynomials based on the  $q$ -integers, otherwise known as  $q$ -Bernstein polynomials, were defined by G. M. Phillips in 1997 and studied by a number of researchers from different angles during the last decades. When  $q = 1$ , these polynomials coincide with the Bernstein ones, while, for  $q \neq 1$ , we obtain new polynomials. Conventionally, the name ' $q$ -Bernstein polynomials' is reserved for the case  $q \neq 1$ .

It has been known that the  $q$ -Bernstein polynomials inherit some properties of the classical Bernstein polynomials. For example, they possess the end-point interpolation property, leave linear functions invariant, and admit representation via divided differences. However, the theory of the  $q$ -Bernstein polynomials is not reduced to drawing analogies between the classical case and the  $q$ -one. Even for the results that can be viewed as direct generalizations of those known, their study either requires different tools or leads to problems which cannot even arise in the classical setting, such as, for example, the dependence of outcomes on parameter  $q$  or the analytical and geometric properties of the limit operators. Their investigation reveals some new phenomena and also establishes new connections between the relatively narrow class of operators and other areas, not only inside approximation theory, but also within functional analysis, algorithms, complex analysis, and others.

In this talk, a review of the main achievements in the theory of the  $q$ -Bernstein polynomials is presented. The focus is on the results which have no counterparts in the classical case.

**Keywords and phrases:**  $q$ -integers,  $q$ -Bernstein polynomials, limit  $q$ -Bernstein operator.

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## A Nonstandard Numerical Scheme for Predator-Prey Model with Ratio-Dependent Functional Response

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**Abstract:** Mathematical models have become useful tools to analyze the dynamical behaviour of predator-prey systems. In this talk, we investigate the dynamical properties of predator-prey theory that depends on the prey and predator density with ratio-dependent functional response. We transform a continuous-time predator-prey model with ratio-dependent functional response into a discrete-time predator-prey model by Nonstandard Finite Difference schemes (NSFD). The proposed new numerical scheme preserves the positivity of the solutions with positive initial conditions. Numerical simulations have been done to compare the dynamics of the discrete-time predator-prey system and continuous-time predator-prey system.

**Keywords and phrases:** Ratio-dependent, stability analysis, nonstandard finite difference scheme.

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## More on The Intuitionistic Fuzzy Homotopy

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**Abstract:** The homotopy theory is used several areas in science. The fuzzy homotopy theory was introduced by authors [4] in 2006. After this paper, some topological other structures were studied by several authors [2, 3, 5, 6]. Intuitionistic fuzzy homotopy defined firstly by authors [5] in 2014. There were some properties of intuitionistic fuzzy homotopic functions and concept of intuitionistic fuzzy homotopy theory. In this study we have been made additional features and arrangements.

**Keywords and phrases:** fuzzy soft set, fuzzy soft topology, continuity, connectedness.

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## Discretization Method for The Predator-Prey System with Allee Effect

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Abstract: The predator-prey dynamics, which have been studied extensively in the last few decades, play an important role in mathematical biology. In this talk, stabilizer power of Allee effect is mentioned and the Allee effect on prey population for a discrete-time predator-prey systems are analyzed. A new Nonstandard Finite Difference schemes (NSFD) is constructed to transform the continuous time predator-prey model into the discrete time predator-prey model. NSFD method preserve the physical and biological properties. We use the Schur-Cohn criteria which deal with coefficient matrix of the linearized system for the local asymptotic stability.

**Keywords and phrases:** Allee effect, stability analysis, nonstandard finite difference scheme.

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## Holographic Quintessence in Fractal Framework

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Abstract: Making use of the non-flat Friedmann-Robertson-Walker type universe dominated by dark energy interacting with dark matter and radiation in the fractal gravity[1, 1], we investigate the power-law corrected description of holographic dark energy[3, 4] in order to reconstruct quintessence scalar field[5].

**Keywords and phrases:** Fractal geometry, Dark energy, Cosmology.

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## Strong and Weak Convergence of Soft Linear Functionals

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In this paper we define strong and weak convergence of soft linear functionals and their properties are studied. Hahn Banach Theorem on soft normed spaces was given in [5] and in this study the consequences of Hahn Banach Theorem are discussed. Furthermore, Uniform Boundedness Principle Theorem is given in soft normed spaces.

**Keywords and phrases:** soft set, soft normed space, soft linear functionals.

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## APPLICATION OF INTUITIONISTIC FUZZY SET IN HIGH SCHOOL DETERMINATION VIA NORMALIZED EUCLIDEAN DISTANCE METHOD

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Abstract: Intuitionistic fuzzy set is very profitable model to elaborate uncertainty and vagueness involved in decision making. In this paper, we proposed an application of intuitionistic fuzzy set in school determination using normalized euclidean distance method. Normalized euclidean distance method was utilized to measure the distance between each student and each school. School which settled each student determined using normalized euclidean distance method according to examination transition to high school education. Solution is determined by measuring the smallest distance between each student and each school.

**Keywords and phrases:** intuitionistic fuzzy sets, high school determination, distance measures.

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## Combinatorial Results for Semigroups of Order-preserving and $A$ -Decreasing ( $A$ -Increasing) Finite Transformations

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Abstract: For  $n \in \mathbb{N}$ , let  $O_n$  be the semigroup of all order-preserving transformations on the finite chain  $X_n = \{1, \dots, n\}$ , under its natural order. Moreover, for any non-empty subset  $A$  of  $X_n$ , let

$$\begin{aligned} O_n(A) &= \{\alpha \in O_n : (\forall x \in A) \quad x\alpha \leq x\} \text{ and} \\ O_n^+(A) &= \{\alpha \in O_n : (\forall x \in A) \quad x\alpha \geq x\}. \end{aligned}$$

In this talk we obtain formulae for the number of idempotents and (when  $1 \in A$ ) nilpotents in  $O_n(A)$  and  $O_n^+(A)$ .

The results presented in my talk have been obtained in collaboration with Hayrullah Ayık and Leyla Bugay.

**Keywords:** (Full) Transformation, order-preserving,  $A$ -decreasing ( $A$ -increasing), idempotent, nilpotent.

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**APPROXIMATION BY RIESZ MEANS IN THE CLASS  $C_{\infty}^{\bar{\psi}}$**

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Abstract: Let  $L$  denote the space of integrable  $2\pi$ -periodic functions, and let

$$S[f] = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx) \equiv \sum_{k=0}^{\infty} A_k(f; x)$$

be the Fourier series of a function  $f \in L$ , i.e., for any  $k = 0, 1, 2, \dots$

$$a_k = a_k(f) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos ktdt \quad , \quad b_k = b_k(f) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin ktdt.$$

In [1, see Chpt. IV], we know that  $C_{\infty}^{\bar{\psi}}$  is class of  $2\pi$  - periodic continuous functions which represented in the form of convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(x-t)\Psi(t)dt = \frac{a_0}{2} + (f^{\bar{\psi}} * \Psi)(x) ,$$

where  $\Psi(x)$  is a certain function that has the Fourier series

$$\sum_{k=1}^{\infty} (\psi_1(k) \cos kx + \psi_2(k) \sin kx),$$

$\bar{\psi} = (\psi_1, \psi_2)$  is a pair of arbitrary fixed systems of numbers  $\psi_1(k)$  and  $\psi_2(k)$ ,  $k = 1, 2, \dots$ . Here, the function  $\varphi$  is called  $\bar{\psi}$  - derivative of function  $f$ , and is denoted by  $f^{\bar{\psi}}(\cdot)$ ,  $\text{ess sup}_t |f^{\bar{\psi}}(t)| \leq$

$1, \int_{-\pi}^{\pi} f^{\bar{\psi}}(t)dt = 0$ . In this study, the value

$$\mathcal{E}_n(C_{\infty}^{\bar{\psi}}, R_n)_C = \sup_{f \in C_{\infty}^{\bar{\psi}}} \|f(x) - R_n(f; \cdot)\|_C$$

is the main subject of our studying where  $R_n(f; \cdot)$  denotes Riesz means of the sequence of partial sums of Fourier series of a  $f \in C_{\infty}^{\bar{\psi}}$  and  $\|\varphi\|_C = \max_x |\varphi(x)|$ . In other words, the aim of this study is to obtain asymptotic equalities for the value  $\mathcal{E}_n(C_{\infty}^{\bar{\psi}}, R_n)_C$  under various conditions on functions  $\psi_1(\cdot)$  and  $\psi_2(\cdot)$ .

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## Generating Sets Of Partial Transformation Semigroups $PT_{n,r}$

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**Abstract:** Let  $PT_{n,r} = S_n \cup PK_{n,r}$  where  $S_n$  is the symmetric group on  $X_n$  and  $PK_{n,r}$  is the semigroup of partial maps  $\alpha : X_n \rightarrow X_n$  such that represent the generating sets of semigroups  $PT_{n,r}$  by considering a set which contains only one element for each partition of  $n - s$  with  $r$  terms in any fixed  $\mathcal{L}$ -class of  $D_r^P$ . Moreover we show that the relative rank of  $PT_{n,r}$  modulo  $S_n$ .

The results presented in my talk have been obtained in collaboration with Gonca Ayık and Hayrullah Ayık.

**Keywords and phrases:** (Partial) transformation semigroup; (Minimal) generating set.

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### Some Remarks on 3-Dimensional Trans-Sasakian Manifolds Admitting Ricci Soliton

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**Abstract** In this paper, we study 3-dimensional trans-Sasakian manifolds admitting Ricci solitons. According to constants  $\alpha$ ,  $\beta$ , we classify trans-Sasakian manifolds admits a Ricci soliton. Then, we obtain such a Ricci soliton is shrinking, steady or expanding according to  $\lambda$ .

**Keywords and phrases:** Sasakian manifold, Ricci soliton.

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**Approximation by Double Deferred Nörlund Means of Double Fourier Series for Lipschitz Functions**

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**Abstract:** In this study, we investigate the rate of uniform approximation by double Deferred Nörlund means of the rectangular partial sums of the double Fourier series of a function  $f(x, y)$  belonging to the class  $Lip\alpha$ ,  $0 < \alpha \leq 1$ , on the two dimensional torus  $-\pi < x, y \leq \pi$ . In addition, we obtain the rate of uniform approximation by double Cesàro means.

**Keywords and phrases:** double Fourier series, double Deferred means, double Deferred Nörlund means.

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**Cobb-Douglas firm production model with stochastic coefficients in imprecise environment: A geometric programming approach**

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**Abstract:** In this paper, we discuss Cobb-Douglas firm production model with stochastic coefficients and exponents via geometric programming technique in imprecise environment. We may note that the Cobb-Douglas production function is a particular functional form of the production function, widely used to represent the technological relationship between the amounts of two or more inputs, particularly physical capital and labor, and the amount of output that can be produced by those inputs. On the other hand, in last few decades, we may observe increasing interest in geometric programming (GP) technique. Advances in numerical methods as well as fast computers may allow us to solve large problems by GP technique. Further, stochastic GP technique may be used to deal with real life situations. Sensitivity analysis may be performed to find effects of alterations of parameters in the model. Numerical examples may further illustrate the proposed technique. Finally, conclusions are drawn and future research directions are given.

**Keywords and phrases:** xxx,xxx, xxx.

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**On a Basic Problem for One Class Discontinuous Sturm-Liouville Operator**

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Abstract: Let us consider the following boundary value problem depending on spectral parameter both in the equation and in boundary conditions;

$$-y'' + q(x)y = \lambda y, 0 \leq x \leq \pi, \tag{9}$$

$$-(\beta_{11}y(0) - \beta_{12}y'(0)) = \lambda(\alpha_{11}y(0) - \alpha_{12}y'(0)), \tag{10}$$

$$-(\beta_{12}y(\pi) - \beta_{22}y'(\pi)) = \lambda(\alpha_{21}y(\pi) - \alpha_{12}y'(\pi)), \tag{11}$$

here  $\alpha_{ij}, \beta_{ij} (i, j = 1, 2)$  are real numbers,  $q(x)$

$$q(x) = \begin{cases} q_1(x), & 0 \leq x < c, \\ q_2(x), & c < x \leq \pi \end{cases}$$

real valued which has discontinuity at  $x = c, q_1(x) \in C[0, c), q_2(x) \in C[0, c)$  has a finite limits  $q(c \pm 0) := \lim_{x \rightarrow c \pm 0} q(x)$  and

$$\begin{aligned} y(c+0) &= y(c-0), \\ y'(c+0) &= y'(c-0) \end{aligned} \tag{12}$$

conjugating conditions.

Let be  $z(x) \in L_2(0, \pi), a_0, a_1 \in \mathbb{C}$  in  $H = L_2(0, \pi) \otimes \mathbb{C}$  space  $\|\tilde{z}\|_H = \|z\|_{L_2} + k_1|a_0|^2 + k_2|a_1|^2, k_1, k_2 > 0$ .

To investigate the basic property of the spectral problem (9)-(12), it is written an operator equation as the form  $L\tilde{y} = \lambda\tilde{y}$  in  $H$  space.

Let  $G$  be non-singular real symmetric matrix. Defining indefinite metric in  $H$  space, it is shown that selfadjoint operator of  $GA$  in  $H$  space on the domain  $D(GL) = D(L)$ . Using these properties it is proved the following theorems:

Let be  $\gamma = \alpha_{12}\beta_{11} - \alpha_{11}\beta_{12}, \sigma = \alpha_{21}\beta_{22} - \alpha_{22}\beta_{21}$ .

**Theorem 1:** In the case  $\sigma\gamma \neq 0$  the eigenfunctions of operator  $L$  form a Riesz basis in the Hilbert space  $H = L_2(0, \pi) \otimes \mathbb{C}^2$ : In the case  $\sigma = 0, \gamma \neq 0$  or  $\sigma \neq 0, \gamma = 0$  then the eigenfunctions of operator  $L$  form a Riesz basis in the Hilbert space  $H = L_2(0, \pi) \otimes \mathbb{C}$ .

**Theorem 2:** By omitting two (or one) elements from  $\{y_n(x)\}, n = 0, 1, 2, \dots$  eigenfunction sequences then the operator  $L$  form a Riesz basis in the Hilbert space  $L_2(0, 1)$ space.

**Keywords and phrases:** Riesz basis, Spectral parameter, Eigenfunctions.

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