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IFSCOM2016

**SECTION II  
ABSTRACTS**

## Fracral Reconstruction of Dilaton Field

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### Abstract

Numerous papers have been presented[1,2,3,4,5] to implement the dynamics of scalar field describing nature of the dark energy by establishing a connection between the pilgrim/new agegraphic/Ricci/ghost/holographic energy density and a scalar field definition. These works showed that the analytical form of potential in terms of the scalar field cannot be obtained due to the complexity of the involved equations. On the other hand, writing a meaningful quantum gravity theory is one of the tough puzzles in modern theoretical physics[6,7]. In the quantum gravity theories, the universe is described as a dimensional flow and one can discuss whether and how these attractive features are connected with the ultraviolet-divergence problem[8]. That's why, such important points motivated us to reconstruct the potential and dynamics of the dilaton scalar field model[9] according to the evolutionary behavior of the extended holographic energy description[10] in fractal geometry.

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## The Cauchy Problem for Complex Intuitionistic Fuzzy Differential Equations

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### Abstract

In this paper, we discuss the existence of a solution to the Cauchy problem for complex intuitionistic fuzzy differential equations. We first propose definitions of complex intuitionistic fuzzy sets and discuss entailed results which parallel those of complex fuzzy sets.

**Keywords :** complex intuitionistic fuzzy sets, complex intuitionistic fuzzy differential equations.

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## Numerical Solution of Intuitionistic Fuzzy Differential Equations by Runge-Kutta Method of Order Four

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### Abstract

This paper presents solution for first order fuzzy differential equation by Runge-Kutta method of order four. This method is discussed in detail and this is followed by a complete error analysis. The accuracy and efficiency of the proposed method is illustrated by solving an intuitionistic fuzzy initial value problem.

**Keywords :** intuitionistic fuzzy Cauchy problem, Runge-Kutta method of order four.

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## Fractional Differential Equations with Intuitionistic Fuzzy Data

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### Abstract

The purpose of this paper is to study the existence and uniqueness of solution for fractional differential equation with intuitionistic fuzzy data where the intuitionistic fuzzy fractional derivatives and integral are considered in the Riemann-Liouville sense. Finally we give an example.

**Keywords :** intuitionistic fuzzy number, Fractional differential equations.

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## Solving Second Order Intuitionistic Fuzzy Initial Value Problems with Heaviside Function

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### Abstract

In this work, we examined the solution of the following second order intuitionistic fuzzy initial value problem through intuitionistic Zadeh's Extension Principle [17]:

$$y''(x) + \bar{a}_1^i y'(x) + \bar{a}_2^i y(x) = \sum_{j=1}^r \bar{b}_j^i g_j(x); \quad (0.0.1)$$

$$y(0) = \bar{\gamma}_0^i; \quad (0.0.2)$$

$$y'(0) = \bar{\gamma}_1^i. \quad (0.0.3)$$

Here  $\bar{a}_1^i$ ,  $\bar{a}_2^i$ ,  $\bar{\gamma}_0^i$ ,  $\bar{\gamma}_1^i$  and  $\bar{b}_j^i$  ( $j=1,2,\dots,r$ ) are intuitionistic fuzzy numbers and  $g_i(x)$  ( $i=1,2,\dots,r$ ) are continuous functions on the interval  $[0, \infty)$ . We reformulated the approach in [2] and [3] for finding an analytical form of alpha and beta cuts for the solution of intuitionistic fuzzy initial value problem for the second order differential equation with the help of Heaviside step function. Firstly we reformulated the general solution of the crisp differential equation corresponding to Eq. 0.1 and applied intuitionistic Zadeh's Extension Principle to intuitionistically fuzzify the solution. Then, we obtained the analytical form

of  $(\alpha, \beta)$ -cuts of the solution of the fuzzy initial value problem by using interval operations and Heaviside step function. Finally, we have illustrated some examples by using this algorithm.

**Keywords :** Intuitionistic Fuzzy Initial Value Problem, Intuitionistic Zadeh's Extension Principle, Heaviside Function.

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## Equivalence Among Three 2-Norms on the Space of $p$ -Summable Sequences

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### Abstract

There are two known 2-norms defined on the space of  $p$ -summable sequences of real numbers. The first 2-norm is a special case of Gähler's formula [Mathematische Nachrichten, 1964], while the second is due to Gunawan [Bulletin of the Australian Mathematical Society, 2001]. The aim of this paper is to define a new 2-norm on  $\ell^p$  and prove the equivalence among these three 2-norms.

**Keywords :** 2-normed spaces; the space of  $p$ -summable sequences; completeness; norm equivalence.

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## Some Properties of Soft Mappings on Soft Metric Spaces

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### Abstract

In this study we define the soft topology generated by the soft metric and show that every soft metric space is a soft normal space. We also investigate some properties of soft continuous mappings on soft metric spaces and finally we give a few examples of soft contraction mapping on soft metric spaces.

**Keywords:** soft metric space, soft normal space, soft continuous mapping, soft contraction mapping.



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## Soft Totally Bounded Spaces in Soft Metric Spaces

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### Abstract

In this study we define the soft topology generated by the soft metric and show that every soft metric space is a soft normal space. We also investigate some properties of soft continuous mappings on soft metric spaces and finally we give a few examples of soft contraction mapping on soft metric spaces.

**Keywords:** Soft set, soft metric space, soft sequential compact, soft totally bounded sets.

### Referances

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## Some Generalized Fixed Point Type Theorems on an $S$ -Metric Space

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### Abstract

In this talk, we give new contractive mappings on an  $S$ -metric space. We investigate some generalizations of the Banach's contraction principle and new fixed point type theorems using the notion of periodic index on an  $S$ -metric space.

**Keywords:**  $S$ -metric, Banach's contraction principle, periodic index.

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## A New Generalization of Soft Metric Spaces

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### Abstract

In this talk, we describe the notion of a soft  $S$ -metric as a generalization of a soft metric. We investigate some basic and topological properties of this new metric. Also we give some existence and uniqueness conditions of fixed-point theorems on a complete soft  $S$ -metric space. We verify our results with some examples.

**Keywords:** Soft  $S$ -metric space, fixed point, topological properties

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## Some Separation Axioms in Fuzzy Soft Topological Spaces

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### Abstract

Molodtsov (1999) proposed a completely new concept called soft set theory to model uncertainty, which associates a set with a set of parameters. Pei and Miao (2005) showed that soft sets are a class of special information systems. Later, Maji et al. (2001) introduced the concept of a fuzzy soft set which combines a fuzzy set and a soft set. From then on, many authors have contributed to (fuzzy) soft set theory in the different fields such as algebra, topology and etc. Soft topology is a relatively new and promising domain which can lead to the development of new mathematical models and innovative approaches that will significantly contribute to the solution of complex problems in natural sciences. Separation is an essential part of topology, on which a lot of work has been done. The aim of this work is to generalize some low-level separation axioms in fuzzifying topology and fuzzy topology to the fuzzifying soft topology and fuzzy soft topology by considering parametrization. So, we obtain some fundamental properties and characterizations of proposed separations.

**Keywords:** Fuzzy soft set, fuzzy soft topology, separation axiom

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## Fuzzy Equilibrium Analysis of a Transportation Network Problem

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### Abstract

In this paper, we focused on the solution process of a fuzzy transportation network equilibrium problem. This problem aims to minimize the total travel time of vehicles on traffic flows between specified origin and destination points. The link travel time for a vehicle is taken as a linear function of link flow (the number of vehicles on that link). Thus, the objective function can be formulated in terms of link flows and link travel times in a quadratic form while satisfying the flow conservation constraints. The parameters of this problem are path lengths, number of lanes, average velocity of a vehicle, vehicle-length, clearance, spacing, link capacity and free flow travel time. Considering a road network, path lengths and number of lanes are taken as crisp numbers. The average velocity of a vehicle and the vehicle-length are imprecise in nature, so these are taken as triangular fuzzy numbers. Since the remaining parameters, that are clearance, spacing, link capacity and free flow travel time, are determined by the average velocity of a vehicle and vehicle-length, all of them will be triangular

fuzzy numbers. Finally, the original fuzzy transportation network problem is converted to a fuzzy quadratic programming problem, and it is solved with an existing approach from the literature. A numerical experiment is illustrated.

**Keywords:** Fuzzy transportation network equilibrium problem, fuzzy quadratic programming, triangular fuzzy numbers.

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## A Special Type of Sasakian Finsler Structures on Vector Bundles

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### Abstract

Sasakian Finsler structures can be obtained on horizontal and vertical distributions of vector bundles. In this paper, Sasakian Finsler structures satisfying  $R(X^H, Y^H)C^* = 0$  on horizontal distribution of vector bundles are examined where  $R$  is Riemann curvature tensor,  $C^*$  is quasi-conformal curvature tensor and  $X^H, Y^H$  are elements of family of vector fields on horizontal distribution. In this regard some structure theorems are examined.

**Keywords:** Quasi-conformal curvature tensor, Sasakian Finsler structure.

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## On Some Properties and Applications of the Quasi-Resolvent Operators of the Infinitesimal Operator of a Strongly Continuous Linear Representation of the Unit Circle Group in a Complex Banach Space

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### Abstract

Let  $A$  be the infinitesimal operator of a strongly continuous linear representation of the unit circle group on a complex Banach space  $H$ . In this talk, the quasi-resolvent operator of  $A$  which is denoted by  $R_\lambda$  is defined by the spectrum of  $A$ . Some properties and inter relations of operators  $R_\lambda$  are introduced, and by using them, some theorems on existence of periodic solutions to the non-linear equations  $\phi(A)x = f(x)$  are stated and proven, where  $\phi(A)$  is a polynomial of  $A$  and  $f$  is a continuous mapping of  $H$  into itself.

## Existence and Nonexistence for Nonlinear Problems with Singular Potential

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### Abstract

Let  $\Omega \subset \mathbb{R}^N$  be a bounded regular domain of  $\mathbb{R}^N$  we consider the following class of elliptic problem

$$\begin{cases} -\Delta u = \frac{u^q}{d^2} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $0 < q \leq 2^* - 1$ . We investigate the question of existence and nonexistence of positive solutions depending on the range of the exponent  $q$ .

## Intuitionistic Fuzzy Soft Generalized Superconnected

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### Abstract

Shabir and Naz [7] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also studied some of basic concepts of soft topological spaces. In the present study, we introduce some new concepts in intuitionistic fuzzy soft topological spaces such



as intuitionistic fuzzy soft generalized superconnected. We also give characterizations and properties of this notion.

**Keywords:** intuitionistic fuzzy soft set, Intuitionistic fuzzy soft topology, intuitionistic fuzzy soft mapping.

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## Vietoris Topology in the Context of Soft Set

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### Abstract

In the present paper, we study the notion of a Vietoris topology by using soft sets. We obtain some properties related to the first countability of soft Vietoris topology. Then, we focus on second countability of it.

**Keywords:** Soft set, Soft Vietoris topology, Soft first countability, Soft second countability.

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## On Totally Umbilical and Minimal Cauchy Riemannian Lightlike Submanifolds of an Indefinite Kaehler Manifold

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### Abstract

In this talk, we survey Cauchy Riemannian lightlike submanifolds of an indefinite Kaehler manifold. Firstly, we mention definition of Cauchy Riemannian (CR, SCR and GCR) lightlike submanifolds of an indefinite Kaehler manifold. Then, we investigate minimal and totally umbilical Cauchy Riemannian lightlike submanifolds and give some examples for these classes.

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## Algebraic Properties of Dual Quasi-Quaternions

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### Abstract

In this work, we consider the algebra of dual quasi-quaternion and give some algebraic properties of this algebra.

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## Fixed Intuitionistic Fuzzy Point Theorem in Hausdorff Intuitionistic Fuzzy Metric Spaces

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### Abstract

Atanassov (see [2,3]) introduced and studied the concept of intuitionistic fuzzy sets (i-fuzzy set, for short) and later there has been much progress in the study of i-fuzzy sets by many authors (see [1,4,5,10]). Using the idea of i-fuzzy set, Park [10] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [7] and proved some known results of metric spaces for intuitionistic fuzzy metric space. In 2001 Estruch and Vidal [6] introduced the concept of intuitionistic fuzzy mapping (i-fuzzy mapping, for short) and gave an intuitionistic version of Heilpern's mentioned theorem (see [9]). After that, Gregori et al [8] defined Hausdorff intuitionistic fuzzy metric on a family of non-empty compact subsets of a given intuitionistic fuzzy metric space.

In this study we modify concept of Hausdorff intuitionistic fuzzy metric using i-fuzzy sets and obtain fixed i-fuzzy point results for i-fuzzy mappings.

**Keywords:** i-fuzzy mapping, i-fuzzy point.

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## Orthonormal Systems in Spaces of Number Theoretical Functions

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### Abstract

In this paper we regard some (for number theory important) examples of set algebras  $\mathcal{A}$  on  $\mathbb{N}$ . In each example we obtain the measure space  $\Omega := (\beta\mathbb{N}, \sigma(\overline{\mathcal{A}}), \bar{\delta})$  by the model of Indlekofer which is based on the Stone-Cech compactification of  $\mathbb{N}$ .

Let  $\mathcal{E}(\mathcal{A})$  be the set of simple functions on  $\mathcal{A}$  and let  $\mathcal{L}^{*\alpha}(\mathcal{A})$  be  $\|\cdot\|_\alpha$  the closure of  $\mathcal{E}(\mathcal{A})$  with

$$\|f\|_\alpha := \left\{ \limsup_{x \rightarrow \infty} \frac{1}{x} \sum_{n \leq x} |f(n)|^\alpha \right\}^{\frac{1}{\alpha}}, \quad 1 \leq \alpha < \infty.$$

Now our aim was to give a description of a complete orthonormal system for  $L^{*2}(\mathcal{A})$  in each regarded case where  $L^{*2}(\mathcal{A})$  is denoted the quotient space  $\mathcal{L}^{*2}(\mathcal{A})$  modulo null-functions.

**Keywords:** Stone-Cech compactification, function spaces, complete orthonormal systems.

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## A Step Size Strategy for Numerical Integration of the Hurwitz Stable Differential Equation Systems

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### Abstract

For the efficiency of the numerical integration of the Cauchy problems, it is not practical to use constant step size. There are some studies in the literature about the variable step size for numerical integration (for example see; [3,4,5]). One of these studies has given in [1,2]. On the region  $D = (t, X) : |t - t_0| \leq T, |x_j - x_{j0}| \leq b_j$ , in [1,2] the step size strategy for the Cauchy Problem

$$X' = AX, X(t_0) = X_0$$

has proposed such that the local error  $\|LE_i\| \leq \delta_L$ , where  $\delta_L$  is the error level that is determined by user. Here  $X(t) = (x_j(t))$ ,  $X_0 = (x_{j0})$ ;  $x_{j0} = x_j(t_0)$ ,  $A \in R^{N \times N}$ ,  $X(t)$ ,  $X_0$  and  $b = (b_j) \in R^N$ .

In this study, we aimed to develop the step size strategy in [1,2] for the system

$$X'^{N \times N} - \text{Hurwitz stable matrix.}$$

A step size strategy and an algorithm for the Hurwitz systems which calculate the step sizes based on the given strategy and numerical solutions are given. The numerical solutions obtained with the new strategy and algorithm are compared with the results in [1,2]. The given strategy and algorithm are applied to some industrial problems.

**Keywords:** Variable step size, Hurwitz stable differential equation systems, numerical integration, step size strategy

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## Schur Stabilitiy in Floating Point Arithmetic: Systems with Constant Coefficients

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### Abstract

The representation of the numbers  $\mathbb{F} = \mathbb{F}(\gamma, p_-, p_+, k) = \{0\} \cup \{z | z = \pm \gamma^{p(z)} m_\gamma(z)\}$ , which are called floating point numbers [1,9,13,14]. Computers use floating point numbers for computing. These numbers are also called computer numbers or machine numbers [3,10,15,17]. If the results of arithmetic operations are elements of set  $\mathbb{F}$ , they are directly stored in the memory. Otherwise they are stored with error [4,7,11,12,16,18].

The linear difference equation in order of  $N$  as  
$$y(n + N) = a_N y(n + N - 1) + \dots + a_1 y(n)$$

can be transform to one order system as

$$x(n+1) = Ax(n), \text{ } n\text{-integer number,}$$

where matrix  $A$  ( $N \times N$  dimension) is the companion matrix. It is well known that the solution of the Cauchy problem

$$x(n+1) = Ax(n), \text{ } x(0) = x_0$$

is  $x(n) = A^n x_0$  (see, [1,8]). In [2], an algorithm has given which have computed power of the companion matrix.

The matrix  $fl(A^n)$  is the computed companion matrix  $A^n$  in floating point arithmetic. The effects of floating point arithmetic in the computation of the companion matrix  $A^n$  were investigated [5,6]. Error bounds were obtained for  $\|A^n - fl(A^n)\|$ , where  $A \in M_N(D)$ . Additionally, Schur stability were investigated according to floating point arithmetic. The obtained results were supported with numerical examples.

**Keywords:** floating point arithmetics, difference equation, error analysis, companion matrix.

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## The Numerical Solution of Some SIR Epidemic Models with Variable Step Size Strategy

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### Abstract

Selection of step size is one of the most important concepts in numerical integration of differential equation systems. Even to use constant step size, it must be investigated how should be selected the step size in the first step of numerical integration. Because, if the selected step size is large in numerical integration, computed solution can diverge from the exact solution. And if the chosen step size is small; calculation time, number of arithmetic operations, the calculation errors start to increase. So, it will be sensible to use small step sizes in the region where the solution changes rapidly and to use bigger step size in the region where the solution changes slowly. So, it is not practical to use constant step size in numerical integration. In literature, step size strategies have been given for the numerical integration. One of these strategies is given for the Cauchy problem

$$X'(t) = AX(t) + \varphi(t, X), X(t_0) = X_0 \quad (0.0.4)$$

where  $A = (a_{ij}) \in R^{N \times N}$ ,  $X \in R^N$  and  $\varphi \in C^1([t_0 - T, t_0 + T] \times R^N)$  in [3,4].

Many dynamical system models are represented by non-linear differential equation systems as in (0.0.4). The epidemic models is one of these systems also attracted attention in recent years (for example see, [1,2,5,6,7]). The classical epidemic model is SIR model.

In this study, we have aimed to investigate the effectiveness of the variable step size strategy for some SIR epidemic models. We have applied the variable step size strategy to the SIR model and its modifications.

**Keywords:** Step size strategy, variable step size, epidemic model, SIR model, system of non-linear differential equations.

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## When is an Archimedean $f$ -Algebra Finite Dimensional?

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Abstract

In this note, we give necessary and sufficient conditions for an Archimedean  $f$ -algebra to be of finite dimensional. As an application, we give a positive answer to a question raised by Bresar in [1].

### Referances

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## By Calculating for Some Linear Positive Operators to Compare of the Errors in the Approximations

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### Abstract

In mathematics, we investigate a Korovkin-type approximation theorem for sequences of positive linear operators on the space of all continuous real valued functions defined on  $[a, b]$  in "Approximation theory". In this paper, we get some approximation properties for sequences of positive linear operators constructed by means of the Bernstein operator and give a Korovkin-type approximations properties for them. We research convergence and approximation properties for type generalized Stancu operators and Bernstein operator to give some examples. We also made a comparison between the approximations obtained by them with calculating the errors in the approximations for different continuous functions. Recently, some authors draw graphics of some modified operators and calculating the errors in approximations[1,2]. Figures of these kind of operator are very difficult because these operators have many of properties such that integrals, summations etc. Consequently figures and numerical results verify the theoretical results in the view of different aspects.

**Keywords:** Approximation, Positive linear operators, Korovkin-type theorem, Comparison, Errors, Figures.

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## Intuitionistic Fuzzy Soft Neighborhoods

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### Abstract

In this paper, we introduce the concept of intuitionistic fuzzy soft point as a generalization of intuitionistic fuzzy point and study some basic properties. We consider the neighborhood structures of an intuitionistic fuzzy point and generate an intuitionistic fuzzy soft topology by using the systems of neighborhood.

**Keywords:** Intuitionistic fuzzy soft set, intuitionistic fuzzy soft point, intuitionistic fuzzy soft topology, neighborhood.

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## On Nonlightlike Offset Curves in Minkowski 3-Space

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### Abstract

In this study, the offset curves of nonlightlike curves are investigated in three different cases. Then the curvature, torsion and arclength of a given offset curve are expressed in terms of the curvature, torsion of the main curve and constants A and B for each case. Moreover, it is proved that the offset curve constitutes another Bertrand curve.

**Keywords:** Bertrand Curve, Offset Curve, Minkowski Space.

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## Weierstrass Representation, Degree and Classes of the Surfaces in the Four Dimensional Euclidean Space

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### Abstract

In this talk, we present on a minimal surface using Weierstrass representation in the four dimensional Euclidean space. We compute implicit equations, degree and class of the surface.

**Keywords:** 4-space, Weierstrass representation, minimal surface, degree, class.

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## A New Type Graph and Their Parameters

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### Abstract

The graph theory has been improved fastly since it has applications in different fields of science. In this paper a special algebraic graph has been defined then some parameters of this graph have been studied.

**Keywords:** Graph, Graph Parameter.

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## The Dot Product Graph of Monogenic Semigroup

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### Abstract

We can identify  $S$  as a cartesian product of finite times a finite semigroup  $S_M^n$  which has some elements like  $\{0, x, x^2, \dots, x^n\}$  Let  $\Gamma(S)$  be a dot product graph whose vertices are the nonzero elements of  $S$ . In this study we are going to analyze some parameters of  $\Gamma(S)$ .

**Keywords:** Dot Product, Monogenic Semigroup, Graph.

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## Some Number Theoretical Results Related to the Suborbital Graphs for the Congruence Subgroup $\Gamma_0\left(\frac{n}{h}\right)$

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### Abstract

In this work, we study the congruence subgroup  $\Gamma_0\left(\frac{n}{h}\right)$  of the Modular group  $\Gamma$  acting transitively on the subset  $\bar{Q}(h)$ . From the suborbital graph  $F(1, n)$  we obtain some interesting number theoretical results, for instance, for all  $n \in \mathbb{N}$ , the numbers  $n(n-4)b^2 - 4$  are not squares.

**Keywords:** Graph Theory, Number Theory.

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## Schur Stabilitiy in Floating Point Arithmetic: Systems with Periodic Coefficients

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### Abstract

The representation of the numbers

$$\mathbb{F} = \mathbb{F}(\gamma, p_-, p_+, k) = \{0\} \cup \left\{ z \mid z = \pm \gamma^{p(z)} m_\gamma(z) \right\}, \quad (0.0.5)$$

which are called floating point numbers [2, 13, 17, 18]. Computers use floating point numbers for computing. These numbers are also called computer numbers or machine numbers [6, 14, 19, 21]. If the results of arithmetic operations are elements of set  $\mathbb{F}$ , they are directly stored in the memory. Otherwise they are stored with error [7, 11, 15, 16, 20, 22].

Let  $A_n$  be an  $N$ -dimensional periodic matrix ( $T$ -period) and difference equation system  $x_{n+1} = A_n x_n$ . The matrix  $X_n$  is called the fundamental matrix of the system, and the matrix  $X_T$  is called the monodromy matrix of the system [1, 3, 4, 5, 12]. The matrix  $Y_n = fl(A_{n-1} Y_{n-1})$  is the computed fundamental matrix  $X_N$  in floating point arithmetic. Cauchy problem of difference equation system can be written

$$fl(A_{n-1} Y_{n-1}) = Y_n = A_{n-1} Y_{n-1} + \phi_N; Y_0 = I, \quad (0.0.6)$$

where  $\phi_N$  is computation error of  $(A_{n-1} Y_{n-1})$ .

The effects of floating point arithmetic in the computation of the fundamental matrix  $X_N$  were investigated. Error bounds were obtained for  $\|X_N - Y_N\|$ , where  $A_N \in M_N(D)$ . The obtained results were investigated for Schur stability of the system [8, 9, 10]. These results were supported with numerical examples.

**Keywords:** floating point arithmetic, difference equation, error analysis, Schur stability.

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## $\mathcal{I}$ -Limit Inferior and $\mathcal{I}$ -Limit Superior of Sequences of Sets

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### Abstract

In this talk, we extend the concepts of  $\mathcal{I}$ -Limit superior and  $\mathcal{I}$ -Limit inferior for real number sequences to  $\mathcal{I}$ -Limit superior and  $\mathcal{I}$ -Limit inferior for sequences of sets, study their certain properties and establish some basic theorems.

**Keywords:** Statistical convergence,  $I$ -convergence and  $I^*$ -convergence, sequence of sets, Wijsman convergence.

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## A Ditopological Fuzzy Structural View of Inverse Systems and Inverse Limits

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### Abstract

One of the theories defined to develop some complement-free concepts is the *texture (fuzzy structure) theory* and it was constructed in [1] as a point-based setting for the study of classical sets and fuzzy sets. Add to that, the notion of *ditopology* described on a texture is essentially a topology for which there is no a priori relation between the open and closed sets, and thus ditopological fuzzy structures [1] were conceived as a unified setting for the study of fuzzy topology. Especially, some useful relationships with fuzzy topology may be found in [3].

In the previous study [2], the foundations of a corresponding theory of *inverse (projective) systems* and their limits, called *inverse limits* were laid in the category **ifPTex** of plain textures which are special types of textures, and point functions satisfying a compatibility condition, named *w*-preserving. Therefore firstly, a detailed analysis of inverse systems and inverse limits was presented in [2] insofar as the category of plain textures is concerned. Evidently, this theory was constituted as an analogue of the inverse system theory in the classical categories **Set**, **Top** and **Rng** in algebra, simultaneously.

As the main theme of this presentation, a suitable theory of inverse systems and their limits is established for some subcategories of the category **ifPDitop** topological over **ifPTex**, whose objects are ditopological fuzzy structures which have plain texturing and morphisms are bicontinuous **ifPTex**-morphisms. In addition, many useful properties of inverse limits in **ifPDitop** are studied via examples in the context of ditopological fuzzy structures, as natural counterparts of the classical cases.

**Keywords:** Inverse Limit, Fuzzy Topology, Category, Texture, Ditopology.

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## On Nullnorms on Bounded Lattices

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### Abstract

t-operators and nullnorms were introduced in [9], [7] respectively, which are also generalizations of the notions of t-norms and t-conorms. And then in [10], it is pointed out that nullnorms and t-operators are equivalent since they have the same block structures in  $[0, 1]^2$ . Namely, if a binary operator  $F$  is a nullnorm then it is also a t-operator and vice versa.

**Definition 0.0.1** *Let  $(L, \leq, 0, 1)$  be a bounded lattice. A commutative, associative, non-decreasing in each variable function  $F : L^2 \rightarrow L$  is called a nullnorm if there is an element  $a \in L$  such that  $F(x, 0) = x$  for all  $x \leq a$ ,  $F(x, 1) = x$  for all  $x \geq a$ .*

In this study, given a bounded lattice  $L$  and a nullnorm on it, taking into account the properties of nullnorms, we investigate an order induced by nullnorms and equivalence relation on bounded lattice. In this way, we obtain that interesting results.

**Keywords:** Nullnorm, Bounded lattice, Partial order.

**Acknowledgement:** The work on this study was supported by the Research Fund of Karadeniz Technical University, project number FBB-2015-5218.

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## Ideal Version of Weighted Lacunary Statistical Convergence of Sequences of Order $\alpha$

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### Abstract

In this work, we are interested in ideal version of weighted lacunary statistical convergence of sequences of order  $\alpha$  and we examine some inclusion relations.

**Keywords:**  $I$ -convergence;  $I$ -statistical convergence; weighted lacunary  $I$ -statistical convergence of order  $\alpha$ ; sequence space

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## $AK(S)$ and $AB(S)$ Properties of a $K$ -Space

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### Abstract

A typical sum  $s$  on a  $K$ -space  $S$  has often the representation,

$$s(z) = \lim_{\gamma \in \Gamma} \sum_k u_{\gamma k} z_k, \quad z = (z_k) \in S \quad (0.0.7)$$

where  $\Gamma$  is a directed index set and  $u_\gamma = (u_{\gamma k}) \in \phi$ , the space of finitely non-zero sequences, for each  $\gamma \in \Gamma$ . Let a  $K$ -space  $S$  be equipped with a sum (0.0.7). Then, for each  $x = (x_k)$  and  $\gamma \in \Gamma$ , the sequence  $P_\gamma(x) = \sum_k u_{\gamma k} x_k \delta^k$ , ( $\gamma \in \Gamma$ ) is called the  $\gamma$ th  $S$ -section of  $x$  [2]. Here  $\delta^k$  is the sequence whose  $k$ th component is 1 all the others are 0.

If  $\lambda \supset \phi$  is a  $K$  space, then Boos and Leiger defined the spaces  $\lambda_{AB(S)}$  and  $\lambda_{AK(S)}$  in [?] as

$$\lambda_{AB(S)} = \{x \in \omega | (P_\gamma(x))_{\gamma \in \Gamma} \text{ is a bounded net in } \lambda\},$$

and

$$\lambda_{AK(S)} = \{x \in (\lambda_{AB(S)} \cap \lambda) | \lim_\gamma P_\gamma(x) \text{ exists in } \lambda\}.$$

In this work, we investigate some properties of these spaces and give some theorems related to the duals.

**Keywords:**  $K$ - spaces,  $n$ -th section of a sequence,  $\beta$ -,  $\gamma$ -,  $f$ -duality.

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# On the Second Homology of the Schützenberger Product of Monoids

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## Abstract

For two finite monoids  $S$  and  $T$ , we prove that the second integral homology of the Schützenberger product  $S\Diamond T$  is equal to

$$H_2(S\Diamond T) = H_2(S) \times H_2(T) \times (H_1(S) \otimes_{\mathbb{Z}} H_1(T))$$

as the second integral homology of the direct product of two monoids.

This is joint work with Hayrullah Ayık and Leyla Bugay.

(Melek Yağcı is supported by Ç.Ü. BAP.)

**Keywords:** Monoid, Schützenberger product, Second integral homology.

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## New Sequence Spaces with Respect to a Sequence of Modulus Functions

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### Abstract

In this talk, we introduce the notion of  $A^{\mathcal{I}}$ -invariant statistical convergence,  $A^{\mathcal{I}}$ -lacunary invariant statistical convergence with respect to a sequence of modulus functions. We establish some inclusion relations between these spaces under some conditions.

**Keywords:** Lacunary invariant statistical convergence; Invariant statistical convergence, ideal convergence, modulus function.

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## TF-Type Hypersurfaces in 4-Space

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### Abstract

We study on translation and factorable hypersurfaces in the four dimensional Euclidean space. We calculate implicit algebraic equations of the hypersurfaces.

**Keywords:** 4-space, translation hypersurface, factorable hypersurface, algebraic equation.

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## A Note on $q$ -Binomial Coefficients

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### Abstract

The  $q$ -calculus has been developing fast. In the present work we study on a  $q$ -extension of binomial coefficients. The infinite sum of  $q$ -extension of binomial coefficients is obtained. Then, by using its infinite sum, we obtain Volkenborn integral value of  $q$ -extension of binomial coefficients.

**Keywords:**  $p$ -adic number, Indefinite sum,  $q$ -analogue of the binomial coefficients, Volkenborn integral.

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## On Statistical Convergence of Sequences of $p$ -Adic Numbers

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### Abstract

Let  $p$  be a fixed prime number. By  $\mathbb{Q}_p$  we denote the field of  $p$ -adic numbers, the completion of the rational numbers field  $\mathbb{Q}$  with respect to the  $p$ -adic norm  $|\cdot|_p$ . The concept of statistical convergence was introduced by H. Fast (1951) [1] and R. C. Buck (1953) [2] independently for real or complex sequences. This concept was studied by T.Salat (1980) [3], J. A. Fridy (1985) [4] and many authors. We note that the field of  $p$ -adic numbers  $\mathbb{Q}_p$  is non-Archimedean, means that the ultrametric inequality is valid

$$|x + y|_p \leq \max \left\{ |x|_p, |y|_p \right\}$$

for all  $x, y \in \mathbb{Q}_p$ . In the present work we define the concept of statistical convergence of sequences for  $p$ -adic numbers and give some its properties.

**Keywords:**  $p$ -adic number, statistical convergence of sequence of  $p$ -adic numbers, statistical Cauchy sequence of  $p$ -adic numbers.

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## Lightlike Hypersurface of an Indefinite Kaehler Manifold with a Complex Semi-Symmetric Metric Connection

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### Abstract

In this paper, we study lightlike hypersurface of an indefinite Kaehler manifold admitting a complex semi-symmetric metric connection. We get the equations of Gauss and Codazzi. Then, we give some characterizations of lightlike hypersurface in an indefinite complex space form with a complex semi-symmetric metric connection. Finally, we show that the Ricci tensor of lightlike hypersurface of an indefinite Kaehler manifold with complex semi-symmetric metric connection is not symmetric.

**Keywords:** Lightlike Hypersurface, Indefinite Complex space form, Complex Semi-Symmetric Metric Connection, Levi-Civita connection, Ricci tensor.

## Intuitionistic Fuzzy Fractional Evolution Problem

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### Abstract

We introduce the generalized intuitionistic fuzzy derivative, this concept used in order to give a generalized intuitionistic fuzzy Caputo fractional derivative. And we discuss the intuitionistic fuzzy fractional evolution problem.

**Keywords:** Generalized intuitionistic fuzzy Hukuhara difference, Generalized intuitionistic fuzzy derivative, generalized intuitionistic fuzzy Caputo-derivative, intuitionistic fuzzy fractional evolution problem.

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## A General Tableaux Method for Contact Logics Interpreted over Intervals

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### Abstract

In this paper, we focus our attention on tableau methods for contact logics interpreted over intervals on the reals. Contact logics provide a natural framework for representing and reasoning about regions in several areas of computer science such as geological information systems, artificial intelligence and etc. In this paper, we focus our attention on tableau methods for contact logics interpreted over intervals on the reals. Contact logics provide a natural framework for representing and reasoning about regions in several areas of computer science such as geological information systems, artificial intelligence and etc.[1,3,4]. However,

while various tableau methods have been developed for classical logic, modal logics and intuitionistic logic, not much work has been done on tableau methods for contact logics [1,2]. We develop a general tableau method for contact logic interpreted over intervals. In this paper we give sound and complete tableaux-based decision procedures for contact logics. Developing such tableaux-based decision procedures, we obtain new decidability/complexity results.

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## Solving Intuitionistic Fuzzy Differential Equations with Linear Differential Operator by Adomain Decomposition Method

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### Abstract

In this paper we presented intuitionistic fuzzy differential equation with linear differential operator which can be of any order and it also involves nonlinear functional. So our solution procedure gives the solutions of a large area of problems involving intuitionistic fuzzy differential equations. Adomain decomposition method (ADM) has been used to find the approximate solution. Note that we used ADM which gives solution even for some nonlinear problems that can't be solved by classical methods. We have given two numerical examples and by comparing the numerical results obtain from ADM with the exact solution, we have studied their accuracy.

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## Involution Matrices of $\frac{1}{4}$ –Quaternions

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**Abstract**

In this work, we consider the  $\frac{1}{4}$ -quaternion algebra and give the matrix representations of the involution and anti-involution maps obtained by this algebra.

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## Topological Full Groups of Cantor Minimal Systems

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**Abstract**

In this work, we study topological full groups of Cantor minimal systems. In recent years, this subject has been very popular since it supplies a connection between dynamical systems and group theory. We will investigate the relationship between conjugation of dynamical systems and isomorphism of their topological full groups. Moreover, topological full groups provide the first examples of finitely generated, simple and amenable groups. We will survey the ideas behind the proofs of these facts.

**Keywords:** Cantor Space, Topological Full Group, Simple Grup, Amenable Group.

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## Global Stability to Nonlinear Neutral Differential Equations of First Order

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### Abstract

In this paper, we study globally asymptotically stability of zero solution to a nonlinear neutral differential of first order. The technique of the proof involves the fixed point method. By this way, we extend and improve some recent works in the literature.

**Keywords:** Fixed point theorem, globally asymptotically stability, neutral differential equation, first order.

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## A Study on the Cartesian Product of a Special Graphs

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### Abstract

Our main aim in this presentation is to extend these studies over  $\Gamma(S_M)$  to the cartesian product. In here,  $\Gamma(S_M)$  is a graph of monogenic semigroup  $S_M = \{x, x^2, x^3, \dots, x^n\}$  with zero. In detail, we will investigate some important graph parameters for the cartesian product of any two (not necessarily different) graphs  $\Gamma(S_{M_1})$  and  $\Gamma(S_{M_2})$ .

**Keywords:** Monogenic semigroup graph, Graph product.

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## Unit Dual Lorentzian Sphere and Tangent Bundle of Lorentzian Unit 2-Sphere

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### Abstract

The purpose of this study is two-fold, firstly to recall some basic concepts and notions of unit dual Lorentzian sphere. Secondly, to define a one-to-one relationship between the unit dual Lorentzian sphere and tangent bundle of Lorentzian unit 2-sphere.

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## On the Inverse Problem for a Sturm-Liouville Equation with Discontinuous Coefficient

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### Abstract

In this study, it is investigated nonhomogenous boundary value problem. During the solution it is encountered Sturm-Liouville problem with piecewise continuous coefficients and that contained eigenvalue parameter. One transmission condition, which given by as relations between the right and left hand limit of the solution at the point of discontinuity are added to the boundary conditions. We examined some spectral properties of the problem. The numeric solutions of eigenvalues are obtained. According to the spectral datas the inverse problem are researched.

**Keywords:** Discontinuous Sturm-Liouville Problem, Inverse Problem, Transmission Condition.

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## Calculation and Analysis of Electronic Parameters of Electroluminescent Device Cells Through I-V Based Modeling

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### Abstract

Light-emitting electrochemical cells (LECs) is one of the simplest kinds of electroluminescent devices. LEC is constituted to an organic single layer structure that was sandwiched between a cathode and an anode. In this study we calculated theoretically of the electronic parameter of LECs device through I-V based modeling. The LEC diode electronic parameters as the ideality factor  $n$  and barrier height  $\phi_b$  were obtained using a method developed by Cheung and confirmed by Werner. The net current of a LEC device is due to the thermionic emission and it can be expressed as

$$I = I_0 \exp\left(\frac{q(V - IR_s)}{nkT}\right)$$

where  $V$  is applied voltage and saturation current  $I_0$  is defined as

$$I_0 = AA^*T^2 \exp\left(-\frac{q\phi_b}{kT}\right).$$

**Keywords:** LECs device, Electronic parameters, I-V modeling.

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## On Some Properties of Sum Spaces

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### Abstract

A sum is a continuous linear functional  $s$  defined on a  $K$ -space  $\lambda \supset \phi$  (space of finitely non-zero sequences) such that,  $s(z) = \sum_k z_k$ ,  $\forall z = (z_k) \in \phi$ . A  $K$ -space

$\lambda$  is called a sum space if and only if  $\lambda \supset \phi$  and  $\lambda^f = \lambda^\lambda$ , where  $\lambda^f = \{(f(\delta^k)) : f \in \lambda'\}$  and  $\lambda^\lambda$  is the set of all sequences  $x$  such that  $xy \in \lambda$ ,  $\forall y \in \lambda$  [4,6]. Here  $\delta^k$  is the sequence whose  $k$ th component is 1 all the others are 0,  $xy = (x_k y_k)$  for  $x = (x_k), y = (y_k)$  and  $\lambda'$  is the space of continuous linear functionals on  $\lambda$ .

An FK space  $\lambda \supset \phi$  is generalized semiconservative FK space if  $\lambda^f \subset \lambda^{\lambda^2}$ , where  $\lambda^{\lambda^2} = \lambda^{\lambda\lambda} = (\lambda^\lambda)^\lambda$ .

In this work, we give some definitions and theorems related with sum spaces and generalized semiconservative FK spaces.

**Keywords:** FK spaces,  $\beta$ - dual,  $f$ - dual, Semiconservative FK spaces.

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## On $q^\lambda$ and $q_0^\lambda$ Invariant Sequence Spaces

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### Abstract

Invariant sequence spaces are very helpful for investigations of the duality of sequence spaces. For instance, if the sequence space  $X$  satisfies the condition  $\ell_\infty.X = X$  then its  $\alpha$ -,  $\beta$ - and  $\gamma$ - duals are same [4]. Garling [1] investigated  $B$ - and  $B_0$ - invariant sequence spaces and Buntinas [2] introduced and investigated  $q$ - and  $q_0$ - invariant sequence spaces and recently, Grosse- Erdmann [3] studied on  $\ell_1$  invariant sequence spaces.

In this work, we define  $q^\lambda$  and  $q_0^\lambda$  invariant sequence spaces,  $X$  with  $q^\lambda.X = X$  and  $q_0^\lambda.X = X$ , respectively. and give some related theorems.



**Keywords:** K- spaces,  $\lambda$ -boundedness and  $\lambda$ -convergence of a sequence,  $\beta$ -,  $\gamma$ -,  $f$ - duality.

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## Some New Results on a Graph of Monogenic Semigroup

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### Abstract

In [2], it has been defined a new graph  $\Gamma(S_M)$  on monogenic semigroups  $S_M$  (with zero) having elements  $\{0, x, x^2, x^3, \dots, x^n\}$ . Many researchers have been working on this area after that work, for example [1,3,4]. As a continues study of these studies, in this paper, it will be investigated define some new graph parameters (such as covering number, accessible number, Zagreb indices, ect.) for monogenic semigroup graph  $\Gamma(S_M)$ .

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## Computational Solution of Katugampola Conformable Fractional Differential Equations Via RBF Collocation Method

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### Abstract

In conjunction with the development of fractional calculus, conformable derivatives and integrals has been widely used a number of scientific areas. In this talk, we provide a numerical scheme to solve Katugampola conformable fractional differential equations via radial basis function (RBF) collocation technique. In order to confirm our numerical scheme, we present some numerical experiments results.

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## Some Integral Inequalities Via Conformable Calculus

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### Abstract

The purpose of this talk is making generalization of Gronwall, Volterra and Pachpatte type inequalities for conformable differential equations. Then we provide some upper or lower bound for fractional derivatives and integrals with the help of Katugampola definition for conformable calculus. These results are extensions of some existing Gronwall, Volterra and Pachpatte type inequalities in the previous studies.

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## Weighted Ostrowski, Chebyshev and Grüss Type Inequalities for Conformable Fractional Integrals

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### Abstract

In this presentation, we have obtained weighted versions of Ostrowski,  $\tilde{C}$ ebysev and Grüss type inequalities for conformable fractional integrals. In accordance with this purpose we have used Katugompola type conformable fractional integrals. The present study confirms previous findings and contributes additional evidence that provide the bounds for more general functions.

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## Bifurcation and Stability Analysis of a Discrete-Time Predator-Prey Model

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### Abstract

We consider discrete-time Leslie Model. We first determine its non-negative fixed point. Later on, we study local stability of the fixed point and determine the conditions on the parameters to show the existence of flip bifurcation by taking the step-size as a bifurcation parameter. Analytical results are also supported by some numerical simulation. Moreover, using Center Manifold Theory, we show the existence of flip bifurcation and its properties.

## Applications of Hermite-Hadamard Inequalities for $\mathbb{B}$ -Convex Functions and $\mathbb{B}^{-1}$ -Convex Functions

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### Abstract

Hermite-Hadamard Inequality that is expressed in the following form

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(t) dt \leq \frac{1}{2}(f(a) + f(b))$$

was proven by Hermite in [1] and then, ten years later, Hadamard rediscovered in [2] (for the historical consideration see also [3]). Next, Hermite-Hadamard Inequalities for different kinds of functions were examined in numerous article [4,5].

$\mathbb{B}$ -convex functions were introduced and studied in [6].  $\mathbb{B}^{-1}$ -convex functions were defined and examined in [7]. Hermite-Hadamard Inequalities for  $\mathbb{B}$ -convex Functions and  $\mathbb{B}^{-1}$ -convex Functions were introduced in [8].

In this work, we give the applications of Hermite-Hadamard Inequalities for  $\mathbb{B}$ -convex functions and  $\mathbb{B}^{-1}$ -convex functions.

**Keywords:** Hermite-Hadamard Inequalities,  $\mathbb{B}$ -convex functions,  $\mathbb{B}^{-1}$ -convex functions.

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## Operations and Extension Principle under T-Intuitionistic Fuzzy Environment

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### Abstract

In this paper, we introduce necessary definitions and related theorems on intuitionistic fuzzy T-set theory. In existing crisp set theory, characteristic functions, defined by two valued logic, can take values: zero and one only. In fuzzy set theory, introduced by Zadeh (1965), membership functions can take any value in closed unit interval . And, in intuitionistic fuzzy set theory, introduced by K. T. Atanassov (1986), both membership functions and non-membership functions can take suitable values in closed unit interval . But, we may observe that in those existing theories, we have to assign same membership value unity to elements, even if belongingness of one element is more certain than other to the subsets. In other words, certainly belongingness and more certainly belongingness of elements to subsets of universal sets are treated at par in all these existing theories, including classical crisp set theory. Similarly, in non-membership functions of intuitionistic fuzzy sets, zero is assigned as non-membership values to elements, both not belonging and not belonging certainly to subsets of universal sets. In order to overcome these limitations of existing theories, in 2015, we proposed intuitionistic fuzzy T-set theory, in which real numbers are suitably assigned to membership and non-membership functions. In this paper, we further introduce necessary definitions and related theorems

on intuitionistic fuzzy  $T$ -set theory. Those may be considered as generalizations of existing definitions and theorems from existing fuzzy and/or intuitionistic fuzzy set theory. In particular, we have generalized the concepts of extension principle and alpha, beta cut under existing intuitionistic fuzzy environment to  $T$ -intuitionistic fuzzy environment. Moreover, we have discussed some associated results under  $T$ -intuitionistic fuzzy environment. Finally, conclusions and future research directions are drawn.

**Keywords:** Intuitionistic fuzzy sets,  $T^{(+)}$ -characteristic functions,  $T^{(-)}$ -characteristic functions, intuitionistic Fuzzy  $T$ -sets,  $T$ -extension principle.

## On Generalized Double Statistical Convergence of Order $\alpha$ in Intuitionistic Fuzzy $N$ - Normed Spaces

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### Abstract

In the present paper, we introduce and study the notion  $\mathcal{I}$ -double statistical convergence and ideal  $\lambda$ -double statistical convergence of order  $\alpha$  with respect to the intuitionistic fuzzy  $n$ -normed space, briefly  $IFnNS$ , also we examine the relationship between these classes.

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## On Feng Qi-Type Integral Inequalities for Conformable Fractional Integrals

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### Abstract

In the last few decades, much significant development of integral inequalities had been established. Recall the famous integral inequality of Feng Qi type [1, 2, 3]:

$$\int_a^b (f(t))^{n+2} dt \geq \left( \int_a^b f(t) dt \right)^{n+1} \quad (0.0.8)$$

where  $f \in C^n(a, b)$ ,  $f^{(i)} \geq 0$ ,  $0 \leq i \leq n$ ,  $f^{(n)} \geq n!$ ,  $n \in \mathbb{R}$ .

In this study, we establish the generalized Qi-type inequality involving conformable fractional integrals.

Firstly we give a important integral inequality which is generalized Qi inequality. Finally, we obtain several inequalities related these inequalities using the conformable fractional integral [4,5].

**Keywords:** Integral Inequalities, Special Functions, Fractional Calculus, Conformable Fractional Integral.

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