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**SECTION II  
ABSTRACTS**

## Fracral Reconstruction of Dilaton Field

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### Abstract

Numerous papers have been presented[1,2,3,4,5] to implement the dynamics of scalar field describing nature of the dark energy by establishing a connection between the pilgrim/new agegraphic/Ricci/ghost/holographic energy density and a scalar field definition. These works showed that the analytical form of potential in terms of the scalar field cannot be obtained due to the complexity of the involved equations. On the other hand, writing a meaningful quantum gravity theory is one of the tough puzzles in modern theoretical physics[6,7]. In the quantum gravity theories, the universe is described as a dimensional flow and one can discuss whether and how these attractive features are connected with the ultraviolet-divergence problem[8]. That's why, such important points motivated us to reconstruct the potential and dynamics of the dilaton scalar field model[9] according to the evolutionary behavior of the extended holographic energy description[10] in fractal geometry.

### Referances

- [1] Sahni, V., *Dark Matter and Dark Energy*, Lect. Notes Phys. Vol.653, 141-180, 2004.
- [2] Chiba, T., Okabe, T. and Yamaguchi, M., *Kinetical Driven Quintessence*, Phys. Rev. D, Vol.62, No.2, 023511-02351, 2000.
- [3] Feng, B., Wang, X.L. and Zhang, X.M., *Dark energy constraints from the cosmic age and supernova*, Phys. Lett. B, Vol.607, Nos.1-2, 35-41, 2005.
- [4] Caldwell, R.R., *A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state*, Phys. Lett. B, Vol.545, Nos.1-2, 23-29, 2002.
- [5] Copeland, E.J., Sami, M. and Tsujikawa, S., *Dynamics of dark energy*, Int. J. Mod. Phys. D, Vol.15, No.11, 1753-1935, 2006.
- [6] Calcagni, G., *Fractal Universe and Quantum Gravity*, Phys. Rev. Lett. Vol.104, No.25, 251301, 2010.
- [7] Calcagni, G., *Quantum field theory, gravity and cosmology in a fractal universe*, JHEP, Vol.2010(3), No.120, 1-38, 2010.
- [8] Salti, M. and Aydogdu, A., *Holographic Tachyon in Fractal Geometry*, Math. Comput. Appl. Vol. 21, No.21, 1-12, 2016.

[9] Piazza, F. and Tsujikawa, S., *Dilatonic ghost condensate as dark energy*, JCAP, Vol.2004(7), No.004, 1-28, 2004.

[10] Miao, L., Xiao-Dong, L., Shuang, W. and Yi, W., *Dark Energy*, Communications in Theoretical Physics, Vol.56, No.3, 525-604, 2011.

## The Cauchy Problem for Complex Intuitionistic Fuzzy Differential Equations

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### Abstract

In this paper, we discuss the existence of a solution to the Cauchy problem for complex intuitionistic fuzzy differential equations. We first propose definitions of complex intuitionistic fuzzy sets and discuss entailed results which parallel those of complex fuzzy sets.

**Keywords :** complex intuitionistic fuzzy sets, complex intuitionistic fuzzy differential equations.

### References

- [1] K. Atanassov, (1986), *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20, 87-96.
- [2] K. Atanassov, (1999), *Intuitionistic Fuzzy Sets : Theory and Applications*, Springer Physica-Verlag, Heidelberg.
- [3] K. Atanassov, T., Vassilev, P. M., and Tsvetkov, R. T. (2013), *Intuitionistic Fuzzy Sets*, Mea-sures and integrals Bulgarian Academic Monographs (12), Professor Marin Drinov Academic Publishing House, Sofia.
- [4] R. Ettoussi, S. Melliani, M. Elomari and L. S. Chadli, *Solution of intuitionistic fuzzy differential equations by successive approximations method*, 19th Int. Workshop on IFSs, Burgas, 4-6 June 2015 Notes on Intuitionistic Fuzzy Sets ISSN 1310-4926 Vol. 21, 2015, No. 2, 51-62.
- [5] M. Elomari, S. Melliani, R. Ettoussi and L. S. Chadli, *Intuitionistic fuzzy semigroup*, 19th Int. Workshop on IFSs, Burgas, 4-6 June 2015 Notes on Intuitionistic Fuzzy Sets ISSN 1310-4926 Vol. 21, 2015, No. 2, 43-50.

- [6] S. Melliani, M. Elomari and R. Ettoussi, L. S. Chadli, *Intuitionistic fuzzy metric space*, Notes on Intuitionistic Fuzzy Sets ISSN 1310-4926 Vol. 21, 2015, No. 1, 43-53.
- [7] J. Nieto, *The Cauchy problem for continuous fuzzy differential equations*, Fuzzy Sets Syst. 102 (1999) 259-262.
- [8] D. Ramot, R. Milo, M. Friedman, A. Kandel, *Complex fuzzy sets*, IEEE Trans. Fuzzy Syst. 10 (2002) 171-186.
- [9] D.E. Tamir, L. Jin, A. Kandel, *A new interpretation of complex membership grade*, Int. J. Intell. Syst. 26 (2011) 285-312.

## Numerical Solution of Intuitionistic Fuzzy Differential Equations by Runge-Kutta Method of Order Four

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### Abstract

This paper presents solution for first order fuzzy differential equation by Runge-Kutta method of order four. This method is discussed in detail and this is followed by a complete error analysis. The accuracy and efficiency of the proposed method is illustrated by solving an intuitionistic fuzzy initial value problem.

**Keywords :** intuitionistic fuzzy Cauchy problem, Runge-Kutta method of order four.

### Referances

- [1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems **20**(1986), pp. 87-96.
- [2] K. Atanassov, Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.
- [3] J.J Buckley , and Feuring T., Fuzzy differential equations, Fuzzy Sets and Systems, **110**(2000), pp. 43-54.
- [4] B. Ben Amma, S. Melliani and L.S. Chadli, Numerical Solution Of Intuitionistic Fuzzy Differential Equations By Euler and Taylor Method (submitted on Notes on Intuitionistic Fuzzy Sets)**21**(2)(2016).

- [5] C. Duraisamy , and Usha B., Numerical Solution of Differential Equation, by Runge-Kutta Method of Order Four, European Journal of Scientific Research, **67**(2012), pp. 324-337.
- [6] M. Friedman , and Kandel A., Numerical solution of fuzzy differential equations, Fuzzy Sets and Systems, **105**(1999), pp. 133-138.
- [7] S. Melliani, M. Elomari, L.S. Chadli and R. Ettoussi,, Intuitionistic Fuzzy Metric Space, Notes on Intuitionistic Fuzzy sets, **21**(1) (2015), pp. 43-53.
- [8] S. Melliani, M. Elomari, M. Atraoui and L. S. Chadli, Intuitionistic fuzzy differential equation with nonlocal condition, Notes on Intuitionistic Fuzzy Sets, **21**(4)(2015), pp. 58-68.
- [9] R. Ettoussi, S. Melliani and L.S. Chadli, Solution of Intuitionistic Fuzzy Differential Equations(submitted on Notes on Intuitionistic Fuzzy Sets)**21**(2)(2016).
- [10] Zadeh, L.A., Fuzzy sets, Inf. Control, **8**(1965), pp. 338-353.

## Fractional Differential Equations with Intuitionistic Fuzzy Data

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### Abstract

The purpose of this paper is to study the existence and uniqueness of solution for fractional differential equation with intuitionistic fuzzy data where the intuitionistic fuzzy fractional derivatives and integral are considered in the Riemann-Liouville sense. Finally we give an example.

**Keywords :** intuitionistic fuzzy number, Fractional differential equations.

### Referances

- [1] K. Atanassov , Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986), pp. 87-96.
- [2] K. Atanassov , Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, (1999).
- [3] I. M. Gelfand and G. E. Shilvov, Generalized Functions, 1(1958), Moscow.
- [4] A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier Science B.V, Amsterdam, 2006.
- [5] G. E. Shilove, Generalized functions and partial differential equations, in Mathematics and its Applications, Science publishers, Inc., 1968.

- [6] A. J. Turski, B. Atamaniuk and E. Turska, On the appearance of the fractional derivative in the behavior of real materials, *J. Appl. Mechanics*, 51 (1984), 294-298.
- [7] S. Melliani, M. Elomari, L. S. Chadli and R. Ettoussi, Intuitionistic Fuzzy metric spaces, *Notes on intuitionistic Fuzzy sets*, 21(1)(2015), pp. 43-53.
- [8] S. Melliani, M. Elomari, L. S. Chadli and R. Ettoussi, Intuitionistic fuzzy Fractional differential equations, *Notes on Intuitionistic Fuzzy Sets*, 21(4) (2015), pp. 76-89.
- [9] D. Junsheng, A. Jianye and Xu Mingyu; Solution of system of fractional differential equations by Adomian Decomposition Method, *Appl. Math. J. Chinese Univ. Ser. B* 2007, 22: 7-12.
- [10] V. Lakshmikantham and J. Vasundhara Devi; Theory of fractional differential equations in a Banach Space. *European Journal of Pure and Applied Mathematics*, 1(1)(2008), pp. 38-45.

## Solving Second Order Intuitionistic Fuzzy Initial Value Problems with Heaviside Function

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### Abstract

In this work, we examined the solution of the following second order intuitionistic fuzzy initial value problem through intuitionistic Zadeh's Extension Principle [17]:

$$y''(x) + \bar{a}_1^i y'(x) + \bar{a}_2^i y(x) = \sum_{j=1}^r \bar{b}_j^i g_j(x); \quad (0.0.1)$$

$$y(0) = \bar{\gamma}_0^i; \quad (0.0.2)$$

$$y'(0) = \bar{\gamma}_1^i. \quad (0.0.3)$$

Here  $\bar{a}_1^i$ ,  $\bar{a}_2^i$ ,  $\bar{\gamma}_0^i$ ,  $\bar{\gamma}_1^i$  and  $\bar{b}_j^i$  ( $j=1,2,\dots,r$ ) are intuitionistic fuzzy numbers and  $g_i(x)$  ( $i=1,2,\dots,r$ ) are continuous functions on the interval  $[0, \infty)$ . We reformulated the approach in [2] and [3] for finding an analytical form of alpha and beta cuts for the solution of intuitionistic fuzzy initial value problem for the second order differential equation with the help of Heaviside step function. Firstly we reformulated the general solution of the crisp differential equation corresponding to Eq. 0.1 and applied intuitionistic Zadeh's Extension Principle to intuitionistically fuzzify the solution. Then, we obtained the analytical form

of  $(\alpha, \beta)$ -cuts of the solution of the fuzzy initial value problem by using interval operations and Heaviside step function. Finally, we have illustrated some examples by using this algorithm.

**Keywords :** Intuitionistic Fuzzy Initial Value Problem, Intuitionistic Zadeh's Extension Principle, Heaviside Function.

### Referances

- [1] L. Atanassova, *On Intuitionistic Fuzzy Versions of L. Zadeh's Extension Principle*, NIFS, vol. 13. pp. 33–36, 2007.
- [2] O.Akin, T.Khaniyev, S.Bayeg, B. Turken *Solving a Second Order Fuzzy Initial Value Problem using, Turk. J. Math. Comput. Sci.,4(2016) 16–25,2015.*
- [3] Buckley, J. J.,Feuring, T.,*Fuzzy initial value problem for N-th order linear differential equations*, Fuzzy Sets and Systems, 121, 247–255, 2001.
- [4] ...

## Equivalence Among Three 2-Norms on the Space of $p$ -Summable Sequences

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### Abstract

There are two known 2-norms defined on the space of  $p$ -summable sequences of real numbers. The first 2-norm is a special case of Gähler's formula [Mathematische Nachrichten, 1964], while the second is due to Gunawan [Bulletin of the Australian Mathematical Society, 2001]. The aim of this paper is to define a new 2-norm on  $\ell^p$  and prove the equivalence among these three 2-norms.

**Keywords :** 2-normed spaces; the space of  $p$ -summable sequences; completeness; norm equivalence.

### Referances

- [1] Gähler, S., *2-metrische räume und ihre topologische struktur*, Mathematische Nachrichten, Vol. **26**, 115-148, 1963.

- [2] Gozali, S.M., Gunawan H. and Neswan, O., *On  $n$ -norms and bounded  $n$ -linear functionals in a Hilbert space*, Annals of Functional Analysis, Vol. **1**, 72-79, 2010.
- [3] Gunawan, H., *The space of  $p$ -summable sequences and its natural  $n$ -norms*, Bulletin of the Australian Mathematical Society, Vol. **64**, 137-147, 2001.
- [4] Idris, M., Ekariani, S. and Gunawan, H., *On the space of  $p$ -summable sequences*, Matematicki Vesnik, Vol. **65** No. 1, 58-63, 2013.
- [5] Konca, S., Idris, M., Gunawan, H. and Basarir, M.  *$p$ -Summable sequence spaces with 2-inner products*, Contemporary Analysis and Applied Mathematics, Vol. **3** No. 2, 213-226, 2015.
- [6] Konca, S., Idris, M. and Gunawan, H. *A new 2-inner product on the space of  $p$ -summable sequences*, Journal of Egyptian Mathematical Society, Vol. **24** No. 2, 244-249, 2016.
- [7] Konca, S., Gunawan, H. and Başarır, M., *Some remarks on  $\ell^p$  as an  $n$ -normed space*, Mathematical Sciences and Applications E-Notes, Vol. **2** No. 2, 45-50, 2014.
- [8] Misiak, A.,  *$n$ -inner product spaces*, Mathematische Nachrichten, Vol. **140**, 299-319, 1989.
- [9] Mutaqin, A. and Gunawan, H., *Equivalence of  $n$ -norms on the space of  $p$ -summable sequences*, Journal of the Indonesian Mathematical Society, Vol. **16**, 2010.
- [10] Wibawa-Kusumah, R.A. and Gunawan, H. *Two equivalent  $n$ -norms on the space of  $p$ -summable sequences*, Periodica Mathematica Hungarica, Vol. **67** No. 1, 63-69, 2013.

## Some Properties of Soft Mappings on Soft Metric Spaces

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### Abstract

In this study we define the soft topology generated by the soft metric and show that every soft metric space is a soft normal space. We also investigate some properties of soft continuous mappings on soft metric spaces and finally we give a few examples of soft contraction mapping on soft metric spaces.

**Keywords:** soft metric space, soft normal space, soft continuous mapping, soft contraction mapping.



### Referances

- [1] Yazar, M.I., Gunduz(Aras),C. and Bayramov, S., *Fixed point theorems of soft contractive mappings*, Filomat, Vol. **30**, No.2, 269-279,2016.
- [2] Bayramov, S. and Gunduz(Aras), C., *Soft locally compact and soft para-compact spaces*, Journal of Mathematics and System Science, ,Vol. **3**, 122-130, 2013.
- [3] Das, S. and Samanta, S. K., *Soft metric*, Ann. Fuzzy Math. Inform., ,Vol. **6**, No. 1, 77-94, 2013.
- [4] Gunduz (Aras), C. ,Sonmez, A. and Çakallı, H., *On Soft Mappings*, arXiv:1305.4545v1 [math.GM] 16 May 2013.
- [5] Long-Guang, H. and Xian,Z., *Cone metric spaces and fixed point theorems of contractive mappings*, J. Math. Anal. Appl.,Vol. **332**, 1468–1476, 2007.
- [6] Majumdar, P. and Samanta, S. K. , *On soft mappings*, Comput. Math. Appl. ,Vol. **60**, 2666-2672, 2010.
- [7] Molodtsov D., *Soft set theory-first results*, Comput. Math. Appl., Vol. **37**,19-31, 1999.
- [8] Shabir, M. and Naz, M., *On soft topological spaces*, Comput. Math. Appl., Vol. **61**, 1786-1799, 2011.
- [9] Rhoades, B.E., *A comparison of various definition of contractive mappings*, Trans. Amer. Math. Soc., Vol. **226**, 257-290, 1977.

## Soft Totally Bounded Spaces in Soft Metric Spaces

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### Abstract

In this study we define the soft topology generated by the soft metric and show that every soft metric space is a soft normal space. We also investigate some properties of soft continuous mappings on soft metric spaces and finally we give a few examples of soft contraction mapping on soft metric spaces.

**Keywords:** Soft set, soft metric space, soft sequential compact, soft totally bounded sets.

### Referances

- [1] Das, S. and Samanta, S. K., *Soft metric*, Ann. Fuzzy Math. Inform. ,Vol. **6**, No. 1, 77-94, 2013.
- [2] Molodtsov D., *Soft set theory-first results*, Comput. Math. Appl., Vol. **37**,19-31, 1999.
- [3] Shabir, M. and Naz, M., *On soft topological spaces*, Comput. Math. Appl., Vol. **61**, 1786-1799, 2011.
- [4] Bayramov, S. and Gunduz(Aras), C., *Soft locally compact and soft para-compact spaces*, Journal of Mathematics and System Science, ,Vol. **3**, 122-130, 2013.

## Some Generalized Fixed Point Type Theorems on an $S$ -Metric Space

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### Abstract

In this talk, we give new contractive mappings on an  $S$ -metric space. We investigate some generalizations of the Banach's contraction principle and new fixed point type theorems using the notion of periodic index on an  $S$ -metric space.

**Keywords:**  $S$ -metric, Banach's contraction principle, periodic index.

### Referances

- [1] Chang, S.S. and Zhong, Q.C., *On Rhoades' open questions*, Proc. Amer. Math. Soc., Vol. **109**, No. 1, 269-274, 1990.
- [2] Gupta, A., *Cyclic contraction on  $S$ -metric space*, International Journal of Analysis and Applications, Vol. **3**, No. 2, 119-130, 2013.
- [3] Hieu,N.T., Ly, N.T. and Dung, N.V., *A generalization of Ciric quasi-contractions for maps on  $S$ -metric spaces*, Thai Journal of Mathematics, Vol. **13**, No. 2, 369-380, 2015.
- [4] Özgür, N.Y. and Taş, N., *Some generalizations of fixed-point theorems on  $S$ -metric spaces*, Essays in Mathematics and Its Applications in Honor of Vladimir Arnold, New York, Springer, 2016.
- [5] Özgür, N.Y. and Taş, N., *Some fixed point theorems on  $S$ -metric spaces*, submitted for publication.
- [6] Özgür, N.Y. and Taş, N., *Some new contractive mappings on  $S$ -metric spaces and their relationships with the mapping (S25)*, submitted for publication.

- [7] Rhoades, B.E., *A comparison of various definitions of contractive mappings*, Trans. Amer. Math. Soc., Vol. **226**, 257-290, 1977.
- [8] Sedghi, S., Shobe, N. and Aliouche, A., *A generalization of fixed point theorems in  $S$ -metric spaces*, Mat. Vesnik, Vol. **64**, No. 3, 258-266, 2012.
- [9] Sedghi, S. and Dung, N.V., *Fixed point theorems on  $S$ -metric spaces*, Mat. Vesnik, Vol. **66**, No. 1, 113-124, 2014.

## A New Generalization of Soft Metric Spaces

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### Abstract

In this talk, we describe the notion of a soft  $S$ -metric as a generalization of a soft metric. We investigate some basic and topological properties of this new metric. Also we give some existence and uniqueness conditions of fixed-point theorems on a complete soft  $S$ -metric space. We verify our results with some examples.

**Keywords:** Soft  $S$ -metric space, fixed point, topological properties

### References

- [1] Das, S. and Samanta, S. K., *Soft real sets, soft real numbers and their properties*, J. Fuzzy Math., Vol. **20**, No. 3, 551-576, 2012.
- [2] Das, S. and Samanta, S. K., *Soft metric*, Ann. Fuzzy Math. Inform., Vol. **6**, No. 1, 77-94, 2013.
- [3] Maji, P. K., Biswas, P. and Roy, A. R., *Soft set theory*, Comput. Math. Appl., Vol. **45**, 555-562, 2003.
- [4] Molodtsov, D., *Soft set theory - first results*, Comput. Math. Appl., Vol. **37**, 19-31, 1999.
- [5] Özgür, N.Y. and Taş, N., *A note on "application of fuzzy soft sets to investment decision making problem"*, Journal of New Theory, **1**, No. 7, 1-10, 2015.
- [6] Özgür, N.Y. and Taş, N., *Some generalizations of fixed-point theorems on  $S$ -metric spaces*, Essays in Mathematics and Its Applications in Honor of Vladimir Arnold, New York, Springer, 2016.
- [7] Özgür, N.Y. and Taş, N., *Some generalizations of the Banach's contraction principle on complete soft  $S$ -metric spaces*, submitted for publication.

- [8] Sedghi, S., Shobe, N. and Aliouche, A., *A generalization of fixed point theorems in S-metric spaces*, Mat. Vesnik, Vol. **64**, No. 3, 258-266, 2012.
- [9] Sedghi, S. and Dung, N.V., *Fixed point theorems on S-metric spaces*, Mat. Vesnik, Vol. **66**, No. 1, 113-124, 2014.

## Some Separation Axioms in Fuzzy Soft Topological Spaces

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### Abstract

Molodtsov (1999) proposed a completely new concept called soft set theory to model uncertainty, which associates a set with a set of parameters. Pei and Miao (2005) showed that soft sets are a class of special information systems. Later, Maji et al. (2001) introduced the concept of a fuzzy soft set which combines a fuzzy set and a soft set. From then on, many authors have contributed to (fuzzy) soft set theory in the different fields such as algebra, topology and etc. Soft topology is a relatively new and promising domain which can lead to the development of new mathematical models and innovative approaches that will significantly contribute to the solution of complex problems in natural sciences. Separation is an essential part of topology, on which a lot of work has been done. The aim of this work is to generalize some low-level separation axioms in fuzzifying topology and fuzzy topology to the fuzzifying soft topology and fuzzy soft topology by considering parametrization. So, we obtain some fundamental properties and characterizations of proposed separations.

**Keywords:** Fuzzy soft set, fuzzy soft topology, separation axiom

### Referances

- [1] Ahmad, B. and Kharal, A., *On fuzzy soft sets*, Advances in Fuzzy Systems, Article ID 586507, 2009.
- [2] Aygünoğlu, A., Çetkin, V., and Aygün, H., *An introduction to fuzzy soft topological spaces*, Hacettepe Journal of Mathematics and Statistics, **48**, No. 2, 197-208, 2014.
- [3] Çetkin, V., and Aygün, H., *On convergence of fuzzy soft filters*, 3rd International Eurasian Conference on Mathematical Sciences and Applications, Vienna, 25-28 August 2014.

- [4] Çetkin, V., and Aygün, H., *Lowen functors in fuzzy soft topological spaces*, the 4th International Fuzzy Systems Symposium, Istanbul, 5-6 November 2016.
- [5] Hong-Yan, L., Fu-Gui, S., *Some separation axioms in I-fuzzy topological spaces*, Fuzzy Sets and Systems, 159, 573-587, 2008.
- [6] Kharal,, A., Ahmad, B., *Mappings on fuzzy soft classes*, Advances in Fuzzy Systems, Article ID 407890, 2009.
- [7] Maji, P. K., Biswas, R., Roy, A. R., *Fuzzy soft sets*, Journal of Fuzzy Mathematics, **9**, No. 3, 589-602, 2001
- [8] Molodtsov, D., *Soft set theory: First results*, Computers and Mathematics with Applications, **37**, No. 4/5, 19-31, 1999.
- [9] Pei, D., Miao, D., *From soft sets to informations systems*, Granular Computing, 2005 IEEE International Conference on (2), 2005, pp. 617-621.
- [10] Yueli, Y., Jinming, F., *On separation axioms in I-fuzzy topological spaces*, Fuzzy Sets and Systems, 157, 780-793,2006.

## Fuzzy Equilibrium Analysis of a Transportation Network Problem

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### Abstract

In this paper, we focused on the solution process of a fuzzy transportation network equilibrium problem. This problem aims to minimize the total travel time of vehicles on traffic flows between specified origin and destination points. The link travel time for a vehicle is taken as a linear function of link flow (the number of vehicles on that link). Thus, the objective function can be formulated in terms of link flows and link travel times in a quadratic form while satisfying the flow conservation constraints. The parameters of this problem are path lengths, number of lanes, average velocity of a vehicle, vehicle-length, clearance, spacing, link capacity and free flow travel time. Considering a road network, path lengths and number of lanes are taken as crisp numbers. The average velocity of a vehicle and the vehicle-length are imprecise in nature, so these are taken as triangular fuzzy numbers. Since the remaining parameters, that are clearance, spacing, link capacity and free flow travel time, are determined by the average velocity of a vehicle and vehicle-length, all of them will be triangular

fuzzy numbers. Finally, the original fuzzy transportation network problem is converted to a fuzzy quadratic programming problem, and it is solved with an existing approach from the literature. A numerical experiment is illustrated.

**Keywords:** Fuzzy transportation network equilibrium problem, fuzzy quadratic programming, triangular fuzzy numbers.

### Referances

- [1] Leblanc, L.J, Morlok, E.K and Pierskalla, W.P., *An Efficient Approach to Solving the Road Network Equilibrium Traffic Assignment Problem*, Transportation Research, Vol. **9**, 309-318, 1975.
- [2] Leblanc, L.J., *The Use of Large Scale Mathematical Programming Models in Transportation Systems*, Transportation Research, Vol. **10**, 419-421, 1976.
- [3] Branston, D., *Link Capacity Functions: A Review*, Transportation Research, Vol. **10**, 233-236, 1976.
- [4] Daganzo, C.F., *On the Traffic Assignment Problem with Flow Dependent Costs-I*, Transportation Research, Vol. **11**, 433-437, 1977.
- [5] Sheffi, Y., *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*, Prentice-Hall, USA, 1985.

## A Special Type of Sasakian Finsler Structures on Vector Bundles

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### Abstract

Sasakian Finsler structures can be obtained on horizontal and vertical distributions of vector bundles. In this paper, Sasakian Finsler structures satisfying  $R(X^H, Y^H)C^* = 0$  on horizontal distribution of vector bundles are examined where  $R$  is Riemann curvature tensor,  $C^*$  is quasi-conformal curvature tensor and  $X^H, Y^H$  are elements of family of vector fields on horizontal distribution. In this regard some structure theorems are examined.

**Keywords:** Quasi-conformal curvature tensor, Sasakian Finsler structure.

### References

- [1] Antonelli P.L., *Handbook of Finsler Geometry*, Kluwer Academic Publishers, ISBN-10:1402015550, 1437 p., 2003.
- [2] Besse A.L., *Einstein manifolds*, Series: Classics in Mathematics, ISBN 978-3-540-74120-6, doi 10.1007/978-3-540-74311-8, 516 p. 2002.
- [3] De U.C., Jun J.B. and Gaz A.K., *Sasakian manifolds with quasi-conformal curvature tensor*, Bull. Korean Math. Soc., Vol. **45**, No. 2, 313-319, 2008.
- [4] Tripathi M.M., Gupta P. and Kim J.S., *Index of quasi-conformally symmetric semi-Riemannian manifolds*, Int. J. Math. Math. Sci., Vol. **2012**, Article ID 461383, 14 p., 2012.
- [5] Yano K. and Kon M., *Structures on manifolds*, Series in Pure Mathematics, Vol. **3**, World Scientific-Singapore, 508 p., 1984.
- [6] Yalnız A.F. and Çalışkan N., *Sasakian Finsler manifolds*, Turk. J. Math., Vol. **37**, No. 2, 319-339, 2013.

## On Some Properties and Applications of the Quasi-Resolvent Operators of the Infinitesimal Operator of a Strongly Continuous Linear Representation of the Unit Circle Group in a Complex Banach Space

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### Abstract

Let  $A$  be the infinitesimal operator of a strongly continuous linear representation of the unit circle group on a complex Banach space  $H$ . In this talk, the quasi-resolvent operator of  $A$  which is denoted by  $R_\lambda$  is defined by the spectrum of  $A$ . Some properties and inter relations of operators  $R_\lambda$  are introduced, and by using them, some theorems on existence of periodic solutions to the non-linear equations  $\phi(A)x = f(x)$  are stated and proven, where  $\phi(A)$  is a polynomial of  $A$  and  $f$  is a continuous mapping of  $H$  into itself.

## Existence and Nonexistence for Nonlinear Problems with Singular Potential

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### Abstract

Let  $\Omega \subset \mathbb{R}^N$  be a bounded regular domain of  $\mathbb{R}^N$  we consider the following class of elliptic problem

$$\begin{cases} -\Delta u = \frac{u^q}{d^2} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $0 < q \leq 2^* - 1$ . We investigate the question of existence and nonexistence of positive solutions depending on the range of the exponent  $q$ .

## Intuitionistic Fuzzy Soft Generalized Superconnected

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### Abstract

Shabir and Naz [7] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also studied some of basic concepts of soft topological spaces. In the present study, we introduce some new concepts in intuitionistic fuzzy soft topological spaces such



as intuitionistic fuzzy soft generalized superconnected. We also give characterizations and properties of this notion.

**Keywords:** intuitionistic fuzzy soft set, Intuitionistic fuzzy soft topology, intuitionistic fuzzy soft mapping.

### References

- [1] K. Atanassov, On Intuitionistic Fuzzy Sets Theory, Springer, (2012) Berlin
- [2] A. Császár, Generalized open sets in generalized topologies , Acta math. Hungar., 106 (2005) 53–66.
- [3] A. Kharal and B. Ahmad, Mappings on soft classes, to appear in New Math. Nat. Comput.
- [4] K. Kannan, Soft Generalized Closed Sets In Soft Topological Spaces, J. Theoret. Appl. Inf. Tech., 37 (2012) 17–21.
- [5] P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, J. Fuzzy Mathematics, 9 (3) (2001) 677–693.
- [6] D. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications, 37 (1999) 19–31.
- [7] M. Shabir and M. Naz , On soft topological spaces , Comput. Math. Appl., 61 (2011) 1786-1799.
- [8] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform., 3 (2) (2012) 171–185.
- [9] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform., 3 (2) (2012) 171–185.

## Vietoris Topology in the Context of Soft Set

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### Abstract

In the present paper, we study the notion of a Vietoris topology by using soft sets. We obtain some properties related to the first countability of soft Vietoris topology. Then, we focus on second countability of it.

**Keywords:** Soft set, Soft Vietoris topology, Soft first countability, Soft second countability.

### References

- [1] Akdağ, M. and Erol, F., *On hyperspaces of soft sets*, Journal of New Theory, Vol. **7**, 86-97, 2015.
- [2] Andrijevic, D., Jelic, M. and Mrsevic, M., *Some properties of hyperspaces of Cech closure spaces*, Filomat, Vol. **24**, 53-61, 2010.
- [3] Aygünöglu, A. and Aygün, H., *Some notes on soft topological spaces*, Neural Comput. Appl., Vol. **21**, 113-119, 2012.
- [4] Demir, İ. and Özbakır, O.B., *Soft Hausdorff spaces and their some properties*, Annals of Fuzzy Mathematics and Informatics, Vol. **8**, No. 5, 769-783, 2014.
- [5] Di Maio, G. and Hola, L., *On hit-and-miss hyperspace topologies*, Rend. Accad. Sci. Fis. Mat. Napoli, Vol. **62**, 103-124, 1995.
- [6] Hola, L. and Levi, S., *Decomposition properties of hyperspace topologies*, Set-valued Analysis, Vol. **5**, 309-321, 1997.
- [7] Hussain, S. and Ahmad, B., *Soft separation axioms in soft topological spaces*, Hacettepe Journal of Mathematics and Statistics, Vol. **44**, 559-568, 2015.
- [8] Nazmul, S. and Samanta, S.K., *Neighbourhood properties of soft topological spaces*, Ann. Fuzzy Math. Inform., Vol. **6**, 1-15, 2013.
- [9] Shabir, M. and Naz, M., *On soft topological spaces*, Comput. Math. Appl., Vol. **61**, 1786-1799, 2011.
- [10] Shakir, Q. R., *On Vietoris soft topology I*, J. Sci. Res., Vol. **8**, 13-19, 2016.
- [11] Smithson, R. E., *First countable hyperspaces*, Proc. Amer. Math. Soc., Vol. **36**, 325-328, 1976.
- [12] Zsilinszky, L., *Baire spaces and hyperspace topologies*, Proc. Amer. Math. Soc., Vol. **124**, 2575-2584, 1996.

## On Totally Umbilical and Minimal Cauchy Riemannian Lightlike Submanifolds of an Indefinite Kaehler Manifold

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### Abstract

In this talk, we survey Cauchy Riemannian lightlike submanifolds of an indefinite Kaehler manifold. Firstly, we mention definition of Cauchy Riemannian (CR, SCR and GCR) lightlike submanifolds of an indefinite Kaehler manifold. Then, we investigate minimal and totally umbilical Cauchy Riemannian lightlike submanifolds and give some examples for these classes.

### Referances

- [1] Bejancu A., *Geometry of CR-Submanifolds*, Kluwer Academic, (1986).
- [2] Chen B.Y., *CR-Submanifolds of a Kähler Manifold, I-II*, J. Differential Geometry, 16 (1981), 305-322, 493-509.
- [3] Duggal K.L. and Bejancu A., *Lightlike submanifolds of semi-Riemannian Manifolds and Applications*, Kluwer Academic, (1996).
- [4] Kupeli D.N., *Singular Semi-Riemannian Geometry*, Kluwer Acad. Publishers, Dordrecht, (1996).
- [5] Duggal K.L. and Sahin B., *Screen Cauchy Riemann Lightlike Submanifolds*, Acta Math. Hungar., 106(1-2), (January 2005).
- [6] Duggal K.L. and Sahin B., *Generalized Cauchy Riemann Lightlike Submanifolds*, Acta Math. Hungar., 112(1-2), (2006).
- [7] Duggal K.L. and Sahin B., *Differential Geometry of Lightlike Submanifolds*, Birkhauser, (2010).
- [8] Sachdeva R., Kumar R. and Bhatia S.S., *Totally umbilical hemi-slant lightlike submanifolds*, New York J. Math., 21 (2015), 191-203.
- [9] Sahin B., *CR-lightlike Altmanifoldlar  $\mathbb{A}^{\pm n}$  Geometrisi* *Äæzerine*, Doktora Tezi, İnönü University Fen Bil. Ens., (2000).
- [10] Yano K. and Kon M., *CR-submanifolds of Kählerian and Sasakian Manifolds*, Birkhauser, (1983).

## Algebraic Properties of Dual Quasi-Quaternions

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In this work, we consider the algebra of dual quasi-quaternion and give some algebraic properties of this algebra.

### References

- [1] Ercan Z., Yuce S., *On properties of the Dual Quaternions*, European J. of Pure and Appl. Math., Vol. 4, No. 2, 142-146, 2011.
- [2] Majernik V., *Quaternion Formulation of the Galilean Space-Time Transformation*, Acta Phy. Slovaca, Vol. 56, No. 1, 9-14, 2006.
- [3] Rosenfeld B.A., *Geometry of Lie Groups*, Kluwer Academic Publishers, Dordrecht, 1997.

[4] Jafari M., *On the Properties of Quasi-quaternion Algebra*, Commun. Fac. Sci. Univ. Ank. Series A1 Vol. **63**, No. 1, 1-10, 2014.

## Fixed Intuitionistic Fuzzy Point Theorem in Hausdorff Intuitionistic Fuzzy Metric Spaces

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### Abstract

Atanassov (see [2,3]) introduced and studied the concept of intuitionistic fuzzy sets (i-fuzzy set, for short) and later there has been much progress in the study of i-fuzzy sets by many authors (see [1,4,5,10]). Using the idea of i-fuzzy set, Park [10] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [7] and proved some known results of metric spaces for intuitionistic fuzzy metric space. In 2001 Estruch and Vidal [6] introduced the concept of intuitionistic fuzzy mapping (i-fuzzy mapping, for short) and gave an intuitionistic version of Heilpern's mentioned theorem (see [9]). After that, Gregori et al [8] defined Hausdorff intuitionistic fuzzy metric on a family of non-empty compact subsets of a given intuitionistic fuzzy metric space.

In this study we modify concept of Hausdorff intuitionistic fuzzy metric using i-fuzzy sets and obtain fixed i-fuzzy point results for i-fuzzy mappings.

**Keywords:** i-fuzzy mapping, i-fuzzy point.

### Referances

- [1] Alaca, C., Turkoglu, D. and Yildiz, C., *Fixed points in intuitionistic fuzzy metric spaces*, Chaos, Solitons & Fractals, **29(5)**, 1073-1078, 2006.
- [2] Atanassov, K., *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20**, 87-96, 1986.
- [3] Atanassov, K., *Review and new results on intuitionistic fuzzy sets*, Preprint IM-MFAIS-1-88, 1988.
- [4] Çoker, D., *An introduction to Intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems, **88**, 81-89, 1997.
- [5] Efe, H. and Yildiz, C., *On The Hausdorff Intuitionistic Fuzzy Metric Spaces*, International Journal of Pure and Applied Mathematics, **31(2)**, 143-165, 2006.

- [6] Estruch, V.D. and Veeramani, P., *A note on fixed fuzzy points for fuzzy mappings*, Rend. Istit. Mat. Univ. Trieste, **32**, 39-45, 2001.
- [7] George, A. and Vidal, A., *On some results in fuzzy metric spaces*, Fuzzy Sets and Systems, **64**, 395-399, 1994.
- [8] Gregori, V., Romaguera, S. and Veeramani, P., *A note on intuitionistic fuzzy metric spaces*, Chaos, Solitons & Fractals, **28**, 902-905, 2006.
- [9] Heilpern, S., *Fuzzy mappings and fixed point theorem*, J. Math. Anal. Appl., **83(2)**, 566-569, 1981.
- [10] Park, J.H., *Intuitionistic fuzzy metric spaces*, Chaos, Solitons & Fractals, **22**, 1039-1046, 2004.

## Orthonormal Systems in Spaces of Number Theoretical Functions

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### Abstract

In this paper we regard some (for number theory important) examples of set algebras  $\mathcal{A}$  on  $\mathbb{N}$ . In each example we obtain the measure space  $\Omega := (\beta\mathbb{N}, \sigma(\overline{\mathcal{A}}), \bar{\delta})$  by the model of Indlekofer which is based on the Stone-Cech compactification of  $\mathbb{N}$ .

Let  $\mathcal{E}(\mathcal{A})$  be the set of simple functions on  $\mathcal{A}$  and let  $\mathcal{L}^{*\alpha}(\mathcal{A})$  be  $\|\cdot\|_\alpha$  the closure of  $\mathcal{E}(\mathcal{A})$  with

$$\|f\|_\alpha := \left\{ \limsup_{x \rightarrow \infty} \frac{1}{x} \sum_{n \leq x} |f(n)|^\alpha \right\}^{\frac{1}{\alpha}}, \quad 1 \leq \alpha < \infty.$$

Now our aim was to give a description of a complete orthonormal system for  $L^{*2}(\mathcal{A})$  in each regarded case where  $L^{*2}(\mathcal{A})$  is denoted the quotient space  $\mathcal{L}^{*2}(\mathcal{A})$  modulo null-functions.

**Keywords:** Stone-Cech compactification, function spaces, complete orthonormal systems.

### References

- [1] Indlekofer, K.-H., *A new method in probabilistic number theory*, Probability Theory and Applications, Math. Appl., **80**, 299-308, 1992.

- [2] Indlekofer, K.-H., *New approach to probabilistic number theory-compactifications and integration*, *Advanced Studies in Pure Mathematics*, **49**, 133-170, 2005.
- [3] Indlekofer, K.-H., *Some remarks on almost-even and almost-periodic functions*, *Arch. Math.*, **37**, 353-358, 1981.
- [4] Indlekofer, K.-H., Y.-W. Lee and R. Wagner, *Mean behaviour of uniformly summable  $q$ -multiplicative functions*, *Ann. Univ. Sci. Budapest.*, **25**, 171-294, 2005.
- [5] Schwarz, W. and Spilker J., *Arithmetical Functions: An Introduction to Elementary and Analytic Properties of Arithmetic Functions and to Some of Their Almost-Periodic Properties*, Cambridge University Press, Cambridge, 388p. 1994.

## A Step Size Strategy for Numerical Integration of the Hurwitz Stable Differential Equation Systems

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### Abstract

For the efficiency of the numerical integration of the Cauchy problems, it is not practical to use constant step size. There are some studies in the literature about the variable step size for numerical integration (for example see; [3,4,5]). One of these studies has given in [1,2]. On the region  $D = (t, X) : |t - t_0| \leq T, |x_j - x_{j0}| \leq b_j$ , in [1,2] the step size strategy for the Cauchy Problem

$$X' = AX, X(t_0) = X_0$$

has proposed such that the local error  $\|LE_i\| \leq \delta_L$ , where  $\delta_L$  is the error level that is determined by user. Here  $X(t) = (x_j(t))$ ,  $X_0 = (x_{j0})$ ;  $x_{j0} = x_j(t_0)$ ,  $A \in R^{N \times N}$ ,  $X(t)$ ,  $X_0$  and  $b = (b_j) \in R^N$ .

In this study, we aimed to develop the step size strategy in [1,2] for the system

$$X'^{N \times N} - \text{Hurwitz stable matrix.}$$

A step size strategy and an algorithm for the Hurwitz systems which calculate the step sizes based on the given strategy and numerical solutions are given. The numerical solutions obtained with the new strategy and algorithm are compared with the results in [1,2]. The given strategy and algorithm are applied to some industrial problems.

**Keywords:** Variable step size, Hurwitz stable differential equation systems, numerical integration, step size strategy

### References

- [1] Çelik Kızıllkan, G., *Step size strategies on the numerical integration of the systems of differential equations*, Ph.D. Thesis, Selcuk University Graduate Natural and Applied Sciences, Konya (in Turkish), 2009.
- [2] Çelik Kızıllkan, G., Aydın, K., *Step size strategies based on error analysis for the linear systems*, SDU Journal of Science, Vol. **6**, No. 2, 149-159, 2011.
- [3] Holsapple, R., Iyer, R., Doman, D., *Variable Step-Size Selection Methods for Implicit Integration Schemes for ODEs*, IJNAM, Vol. **4**, No. 2, 212-242, 2007.
- [4] Ramos, H., Vigo-Aguiar, J., *Variable Stepsize Störmer-Cowell Methods*, Math Comput Model, Vol. **42**, 837-846, 2005.
- [5] Shampine, L.F., Allen, R. C., Pruess, S., *Fundamentals of Numerical Computing*, John Wiley Sons, INC, New York, 1996.

## Schur Stabilitiy in Floating Point Arithmetic: Systems with Constant Coefficients

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### Abstract

The representation of the numbers  $\mathbb{F} = \mathbb{F}(\gamma, p_-, p_+, k) = \{0\} \cup \{z | z = \pm \gamma^{p(z)} m_\gamma(z)\}$ , which are called floating point numbers [1,9,13,14]. Computers use floating point numbers for computing. These numbers are also called computer numbers or machine numbers [3,10,15,17]. If the results of arithmetic operations are elements of set  $\mathbb{F}$ , they are directly stored in the memory. Otherwise they are stored with error [4,7,11,12,16,18].

The linear difference equation in order of  $N$  as  
 $y(n + N) = a_N y(n + N - 1) + \dots + a_1 y(n)$

can be transform to one order system as

$$x(n+1) = Ax(n), n\text{-integer number,}$$

where matrix  $A$  ( $N \times N$  dimension) is the companion matrix. It is well known that the solution of the Cauchy problem

$$x(n+1) = Ax(n), x(0) = x_0$$

is  $x(n) = A^n x_0$  (see, [1,8]). In [2], an algorithm has given which have computed power of the companion matrix.

The matrix  $fl(A^n)$  is the computed companion matrix  $A^n$  in floating point arithmetic. The effects of floating point arithmetic in the computation of the companion matrix  $A^n$  were investigated [5,6]. Error bounds were obtained for  $\|A^n - fl(A^n)\|$ , where  $A \in M_N(D)$ . Additionally, Schur stability were investigated according to floating point arithmetic. The obtained results were supported with numerical examples.

**Keywords:** floating point arithmetics, difference equation, error analysis, companion matrix.

### Referances

- [1] Akın, O. and Bulgak H., *Linear difference equations and stability theory*, Selçuk University Research Centre of Applied Mathematics, No. 2, Konya (in Turkish), 1998.
- [2] Aydın, K., *On a power of the companion matrix*, SDU Journal of Science (E-Journal), Vol. 3, No. 2. 230-235, 2008.
- [3] Behrooz, P., *Computer Architecture: From Microprocessors to Supercomputers*, Oxford University Press, 2005.
- [4] Çibikdiken, A.O., *Error estimation of elementary matrix operations*, Master Thesis, Selçuk University Graduate Natural and Applied Sciences, Konya (in Turkish), 2002.
- [5] Çibikdiken, A.O., *The effect of floating point arithmetic on fundamental matrix computation of periodic linear difference equation system*, Ph.D. Thesis, Selçuk University Graduate Natural and Applied Sciences, Konya (in Turkish), 2008.
- [6] Çibikdiken, A.O., Aydın, K., *Computation of monodromy matrix on floating point arithmetic with Godunov Model*, Konuralp Journal of Mathematics, Volume 4, No 1, 2016.
- [7] Dahlquist, G., Björck, A., *Numerical Mathematics and Scientific Computation*, SIAM, Vol. 1, 2008.
- [8] Elaydi, S.N., *An Introduction to Difference Equations*, Second Edition, Springer-Verlag, New York, 1999.
- [9] Godunov, S.K., Antonov, A.G., Kiriluk, O.P., Kostin, V.I., *Guaranteed Accuracy in Mathematical Computations*, Prentice-Hall, Englewood Cliffs, NJ, 1993.
- [10] Goldberg, D., *What every computer scientist should know about floating-point arithmetic*, J. ACM Comput. Surv., Vol. 23, No. 1, 5-48, 1991.
- [11] G.H. Golub, C.F. Van Loan, *Matrix Computations*, (third ed.)The Johns Hopkins University Press, Baltimore (1996).



- [12] Higham, N.J., *Accuracy and Stability of Numerical Algorithms*, SIAM, 1996.  
 [13] Kulisch, U.W., *Mathematical foundation of computer arithmetic*, IEEE Trans. Comput., C-26 (7), 610-621, 1977.  
 [14] Kulisch, U.W., Miranker, W.L., *Computer Arithmetic in Theory and Practice*, Academic Press Inc., 1981.  
 [15] Overton, M.L., *Numerical Computing with IEEE Floating Point Arithmetic*, SIAM, 2001.  
 [16] Shampine, L.F., Allen, R.C., Pruess, S., *Fundamentals of Numerical Computing*, John Wiley - Sons Inc., 1997.  
 [17] Sterbenz, P.H., *Floating-Point Computation*, Prentice-Hall Inc., Englewood Cliffs, NJ, 1974.  
 [18] Wilkinson, J.H., *Rounding Errors in Algebraic Processes*, Prentice-Hall Inc., 1963.

## The Numerical Solution of Some SIR Epidemic Models with Variable Step Size Strategy

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### Abstract

Selection of step size is one of the most important concepts in numerical integration of differential equation systems. Even to use constant step size, it must be investigated how should be selected the step size in the first step of numerical integration. Because, if the selected step size is large in numerical integration, computed solution can diverge from the exact solution. And if the chosen step size is small; calculation time, number of arithmetic operations, the calculation errors start to increase. So, it will be sensible to use small step sizes in the region where the solution changes rapidly and to use bigger step size in the region where the solution changes slowly. So, it is not practical to use constant step size in numerical integration. In literature, step size strategies have been given for the numerical integration. One of these strategies is given for the Cauchy problem

$$X'(t) = AX(t) + \varphi(t, X), X(t_0) = X_0 \quad (0.0.4)$$

where  $A = (a_{ij}) \in R^{N \times N}$ ,  $X \in R^N$  and  $\varphi \in C^1([t_0 - T, t_0 + T] \times R^N)$  in [3,4].

Many dynamical system models are represented by non-linear differential equation systems as in (0.0.4). The epidemic models is one of these systems also attracted attention in recent years (for example see, [1,2,5,6,7]). The classical epidemic model is SIR model.

In this study, we have aimed to investigate the effectiveness of the variable step size strategy for some SIR epidemic models. We have applied the variable step size strategy to the SIR model and its modifications.

**Keywords:** Step size strategy, variable step size, epidemic model, SIR model, system of non-linear differential equations.

### Referances

- [1] Chauhan, S., Misra, O.P., Dhar, J., *Stability analysis of SIR model with vaccination*, Am. J. Comput. Appl. Math., Vol. 4, No. 1, 17-23, 2014.
- [2] Chinviriyasit, S., Chinviriyasit, W., *Numerical modelling of on SIR epidemic model with diffusion*, Appl. Math. Comput., Vol. 216, 395-409, 2010.
- [3] Çelik Kızıllan, G., *Step size strategies on the numerical integration of the systems of differential equations*, Ph.D. Thesis, Selcuk University Graduate Natural and Applied Sciences, Konya (in Turkish), 2009.
- [4] Çelik Kızıllan, G., Aydın, K., *Step size strategies for the numerical integration of systems of differential equations*, J. Comput. Appl. Math., Vol. 236, No. 15, 3805-3816, 2012.
- [5] Harko, T., Lobo, S.N.F., Mak, M.K., *Exact analytical solutions of the susceptible- infected- recovered (SIR) epidemic model and of the SIR model with equal death and birt dates*, Appl. Math. Comput., Vol. 236, 84-94, 2014.
- [6] Keeling, J.M., Rohani, P., *Modeling infectious diseases in humans and animals*, Princeton University Press, New Jersey, 2008.
- [7] Murray, J. D., *Mathematical biology: I. An introduction*, Third Edition, Springer, New York, 2002.

## When is an Archimedean $f$ -Algebra Finite Dimensional?

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Abstract

In this note, we give necessary and sufficient conditions for an Archimedean  $f$ -algebra to be of finite dimensional. As an application, we give a positive answer to a question raised by Bresar in [1].

### Referances

[1] Brešar, M., *Finite dimensional zero product determined algebras are generated by idempotents*, Expo Math, Vol. **34**, No.1, 130-143, 2016.

## By Calculating for Some Linear Positive Operators to Compare of the Errors in the Approximations

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### Abstract

In mathematics, we investigate a Korovkin-type approximation theorem for sequences of positive linear operators on the space of all continuous real valued functions defined on  $[a, b]$  in "Approximation theory". In this paper, we get some approximation properties for sequences of positive linear operators constructed by means of the Bernstein operator and give a Korovkin-type approximations properties for them. We research convergence and approximation properties for type generalized Stancu operators and Bernstein operator to give some examples. We also made a comparison between the approximations obtained by them with calculating the errors in the approximations for different continuous functions. Recently, some authors draw graphics of some modified operators and calculating the errors in approximations [1,2]. Figures of these kind of operator are very difficult because these operators have many of properties such that integrals, summations etc. Consequently figures and numerical results verify the theoretical results in the view of different aspects.

**Keywords:** Approximation, Positive linear operators, Korovkin-type theorem, Comparison, Errors, Figures.

### Referances

- [1] Buyukyazici, İ., *Approximation by Stancu-Chlodowsky polynomials*, Computers and Mathematics with Applications 59 (2009) 274-282 Berens H., Lorentz G.G., *Inverse theorems for Bernstein polynomials*, Indiana Univ. Math. J., 21(8)(1972), 693-708.
- [2] Buyukyazici, İ., *A Korovkin Type Approximation Theorem and Its Application*, Fasciculi Mathematici, Nr 45, 2010,
- [3] Cheney E. W., Sharma A., *Bersntein power series*, Canad. J. Math. 16, 241-253 (1960).
- [4] D.D.Stancu, *Approximation of functions by a new class of linear polinomial operators*, REv. Roumaine Math Pures Appl. 13(1968) 1113-1194.
- [5] Gadjiev, A.D., *On Korovkin Type Theorems*, Math. Notes, 20,(5-6), 996-998, 1976.
- [6] Korovkin P. P., *Linear operators and approximation theory*, Hindustan Publish Co., Delphi (1960).

## Intuitionistic Fuzzy Soft Neighborhoods

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### Abstract

In this paper, we introduce the concept of intuitionistic fuzzy soft point as a generalization of intuitionistic fuzzy point and study some basic properties. We consider the neighborhood structures of an intuitionistic fuzzy point and generate an intuitionistic fuzzy soft topology by using the systems of neighborhood.

**Keywords:** Intuitionistic fuzzy soft set, intuitionistic fuzzy soft point, intuitionistic fuzzy soft topology, neighborhood.

### Referances

- [1] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 64(2)87-96, 1986.
- [2] Z. Li and R. Cui, *On the topological structure of intuitionistic fuzzy soft set*, Annals of Fuzzy Mathematics and Informatics, 5(1), 229-239, 2013.
- [3] P.K. Maji, A.R. Roy and R. Biswas, *On intuitionistic fuzzy soft sets*, Journal of Fuzzy Mathematics 12(3), 669-683, 2004.

- [4] D. Molodtsov, *Soft set theory-First results*, Computers and Mathematics with Application, 37(4/5), 19-31, 1999.
- [5] B.Pazar Varol, A. Aygünoğlu and H. Aygün, *Neighborhood structures of fuzzy soft topological spaces*, Journal of Intelligent and Fuzzy Systems, 27(4), 2127-2135, 2014.
- [6] J. L. Seok and P.L. Eun, *The category of intuitionistic fuzzy topological spaces*, Bull. Korean Math. Soc. 37(1), 63-76, 2000.
- [7] K.Serkan and M. Akdağ, *On intuitionistic fuzzy soft continuous mappings*, Journal of New Results in Science, 4, 55-70, (2014).
- [8] K. Serkan and I. M. Akif, *Connectedness on intuitionistic fuzzy soft topological spaces*, Journal of New Theory, 1, 02-16, 2015.
- [9] Y.Yin, H. Li and Y.B. Jun, *On algebraic structures of intuitionistic fuzzy soft sets*, Computers and Mathematics with Applications 61, 2896-2911, 2012.
- [10] L.A. Zadeh, *Fuzzy sets*, Information and Control, 8, 338 - 353, 1965.

## On Nonlightlike Offset Curves in Minkowski 3-Space

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### Abstract

In this study, the offset curves of nonlightlike curves are investigated in three different cases. Then the curvature, torsion and arclength of a given offset curve are expressed in terms of the curvature, torsion of the main curve and constants A and B for each case. Moreover, it is proved that the offset curve constitutes another Bertrand curve.

**Keywords:** Bertrand Curve, Offset Curve, Minkowski Space.

### References

- [1] Arrondo, E., Sendra, J., Sendra, R., *Genus formula for generalized offset curves*, Journal of Pure and Applied Algebra, Vol. **136**, 199-209, 1999.
- [2] Babaarslan, M., Yaylı, Y., *On Space-like constant slope surfaces and Bertrand curves in Minkowski 3-space*, arXiv:1112.1504v6., 2014.
- [3] Ding, Q., Inoguchi, J., *Schrödinger flows, binormal motion for curves and second AKNS-hierarchies*, Chaos Solitons and Fractals, Vol. **21**, 669-677, 2004.
- [4] Inoguchi, J., *Biharmonic curves in Minkowski 3-space*, International Journal of Mathematics and Mathematical Sciences, Vol. **21**, 1365-1368, 2003.

- [5] Maekawa, T., *An overview of offset curves and surfaces*, Computer-Aided Design, Vol. **31**, 165-173, 1999.
- [6] Pham, B., *Offset curves and surfaces: a brief survey*, Computer-Aided Design, Vol. **24**, 223-239, 1992.
- [7] Schief, W. K., *On the integrability of Bertrand curves and Razzaboni surfaces*, Journal of Geometry and Physics, Vol. **45**, 130-150, 2003.

## Weierstrass Representation, Degree and Classes of the Surfaces in the Four Dimensional Euclidean Space

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### Abstract

In this talk, we present on a minimal surface using Weierstrass representation in the four dimensional Euclidean space. We compute implicit equations, degree and class of the surface.

**Keywords:** 4-space, Weierstrass representation, minimal surface, degree, class.

### Referances

- [1] Eisenhart, L.P. *A Treatise on the Differential Geometry of Curves and Surfaces*, Dover Publications, N.Y. 1909.
- [2] Hoffman, D.A., Osserman, R. *The Geometry of the Generalized Gauss Map*. Memoirs of the AMS, 1980.
- [3] Nitsche, J.C.C. *Lectures on Minimal Surfaces. Vol. 1. Introduction, Fundamentals, Geometry and Basic Boundary Value Problems*. Cambridge Un. Press, Cambridge, 1989.
- [4] Osserman, R. *A Survey of Minimal Surfaces*. Van Nostrand Reinhold Co., New York-London-Melbourne, 1969.
- [5] Weierstrass, K. *Untersuchungen über die flächen, deren mittlere Krümmung überall gleich Null ist*. Monatsber d Berliner Akad., 612-625, 1866.
- [6] Weierstrass, K., *Mathematische Werke*. Vol. 3, Mayer & Muller, Berlin, 1903.

## A New Type Graph and Their Parameters

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### Abstract

The graph theory has been improved fastly since it has applications in different fields of science. In this paper a special algebraic graph has been defined then some parameters of this graph have been studied.

**Keywords:** Graph, Graph Parameter.

### Referances

- [1] Gross, J.L. and Yellen, J. , *Handbook of graph theory* , Chapman Hall, CRC Press, 2004.
- [2] Das, K.Ch. , Akgüneş, N. and Çevik, A.S. , *On a graph of monogenic semigroups* , Journal of Inequalities and Applications , 2013, 2013:44.
- [3] Klotz, W. and Sander, T. , *Some properties of unitary cayley graphs* , The Electronic Journal of Combinatorics 14, 2007, R45.

## The Dot Product Graph of Monogenic Semigroup

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### Abstract

We can identify  $S$  as a cartesian product of finite times a finite semigroup  $S_M^n$  which has some elements like  $\{0, x, x^2, \dots, x^n\}$  Let  $\Gamma(S)$  be a dot product graph whose vertices are the nonzero elements of  $S$ . In this study we are going to analyze some parameters of  $\Gamma(S)$ .

**Keywords:** Dot Product, Monogenic Semigroup, Graph.

### Referances

- [1] Gross, J.L. and Yellen, J. , *Handbook of graph theory* , Chapman Hall, CRC Press, 2004.
- [2] Das, K.Ch. , Akgüneş, N. and Çevik, A.S. , *On a graph of monogenic semigroups* , Journal of Inequalities and Applications , 2013, 2013:44.
- [3] Badawi, A. , *On the dot product graph of a commutative ring* , Communications in Algebra 43, 2015, 43-50.

## Some Number Theoretical Results Related to the Suborbital Graphs for the Congruence Subgroup $\Gamma_0\left(\frac{n}{h}\right)$

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### Abstract

In this work, we study the congruence subgroup  $\Gamma_0\left(\frac{n}{h}\right)$  of the Modular group  $\Gamma$  acting transitively on the subset  $\bar{Q}(h)$ . From the suborbital graph  $F(1, n)$  we obtain some interesting number theoretical results, for instance, for all  $n \in \mathbb{N}$ , the numbers  $n(n-4)b^2 - 4$  are not squares.

**Keywords:** Graph Theory, Number Theory.

### Referances

- [1] G. A. Jones, D. Singerman, and K. Wicks, *The Modular Group and Generalized Farey Graphs*, London Mathematical Society Lecture Note Series, Cambridge University Press, **160**, 1991.

## Schur Stabilitiy in Floating Point Arithmetic: Systems with Periodic Coefficients

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### Abstract

The representation of the numbers

$$\mathbb{F} = \mathbb{F}(\gamma, p_-, p_+, k) = \{0\} \cup \left\{ z \mid z = \pm \gamma^{p(z)} m_\gamma(z) \right\}, \quad (0.0.5)$$

which are called floating point numbers [2, 13, 17, 18]. Computers use floating point numbers for computing. These numbers are also called computer numbers or machine numbers [6, 14, 19, 21]. If the results of arithmetic operations are elements of set  $\mathbb{F}$ , they are directly stored in the memory. Otherwise they are stored with error [7, 11, 15, 16, 20, 22].

Let  $A_n$  be an  $N$ -dimensional periodic matrix ( $T$ -period) and difference equation system  $x_{n+1} = A_n x_n$ . The matrix  $X_n$  is called the fundamental matrix of the system, and the matrix  $X_T$  is called the monodromy matrix of the system [1, 3, 4, 5, 12]. The matrix  $Y_n = fl(A_{n-1} Y_{n-1})$  is the computed fundamental matrix  $X_N$  in floating point arithmetic. Cauchy problem of difference equation system can be written

$$fl(A_{n-1} Y_{n-1}) = Y_n = A_{n-1} Y_{n-1} + \phi_N; Y_0 = I, \quad (0.0.6)$$

where  $\phi_N$  is computation error of  $(A_{n-1} Y_{n-1})$ .

The effects of floating point arithmetic in the computation of the fundamental matrix  $X_N$  were investigated. Error bounds were obtained for  $\|X_N - Y_N\|$ , where  $A_N \in M_N(D)$ . The obtained results were investigated for Schur stability of the system [8, 9, 10]. These results were supported with numerical examples.

**Keywords:** floating point arithmetic, difference equation, error analysis, Schur stability.

### References

- [1] Agarwal R.P., *Difference Equations and Inequalities*, Pure and Applied Mathematics Series (second ed.), vol. 228, National University of Singapore, 2000.
- [2] Akın, O. and Bulgak H., *Linear difference equations and stability theory*, Selçuk University Research Centre of Applied Mathematics, No. 2, Konya, 1998 (in Turkish).
- [3] Aydın K., *The condition number for the asymptotic stability of the periodic ordinary differential systems*, Ph.D. Thesis, Selçuk University Graduate Natural and Applied Sciences, Konya, (1996) (in Turkish).

- [4] Aydın K., Bulgak H., Demidenko G.V., *Numeric characteristics for asymptotic stability of solutions to linear difference equations with periodic coefficients*, Sib. Math. J., 41 (6), pp. 1005-1014, 2000.
- [5] Aydın K., Bulgak H., Demidenko G.V., *Asymptotic stability of solutions to perturbed linear difference equations with periodic coefficients*, Sib. Math. J., 43 (3), pp. 389-401, 2002.
- [6] Behrooz P., *Computer Architecture: From Microprocessors to Supercomputers*, Oxford University Press (2005).
- [7] Çıbıkdiken A.O., *Error estimation of elementary matrix operations*, Master Thesis, Selçuk University Graduate Natural and Applied Sciences, Konya, 2002 (in Turkish).
- [8] Çıbıkdiken A.O., *The effect of floating point arithmetic on fundamental matrix computation of periodic linear difference equation system*, Ph.D. Thesis, Selçuk University Graduate Natural and Applied Sciences, Konya, 2008 (in Turkish).
- [9] Çıbıkdiken A.O., Aydın K., *Computation of the monodromy matrix in floating point arithmetic with the Wilkinson Model*, Computers & Mathematics with Applications, Volume 67, Issue 5, Pages 1186-1194, 2014.
- [10] Çıbıkdiken A.O., Aydın K., *Computation of monodromy matrix on floating point arithmetic with Godunov Model*, Konuralp Journal of Mathematics, Volume 4, No 1, 2016.
- [11] Dahlquist G., Björck A., *Numerical Mathematics and Scientific Computation*, Vol. 1, SIAM (2008).
- [12] Elaydi S.N., *An Introduction to Difference Equations*, Springer-Verlag, New York 1996.
- [13] Godunov, S.K., Antonov, A.G., Kiriluk, O.P., V.I. Kostin, *Guaranteed Accuracy in Mathematical Computations*, Prentice-Hall, Englewood Cliffs, NJ (1993).
- [14] Goldberg D., *What every computer scientist should know about floating-point arithmetic*, J. ACM Comput. Surv., 23 (1) (1991), pp. 5-48.
- [15] Golub G.H., Van Loan C.F., *Matrix Computations*, (third ed.)The Johns Hopkins University Press, Baltimore (1996).
- [16] Higham N.J., *Accuracy and Stability of Numerical Algorithms*, SIAM (1996).
- [17] Kulisch, U.W., *Mathematical foundation of computer arithmetic*, IEEE Trans. Comput., C-26 (7) (1977), pp. 610-621.
- [18] Kulisch U.W., Miranker W.L., *Computer Arithmetic in Theory and Practice*, Academic Press Inc. (1981).
- [19] Overton M.L., *Numerical Computing with IEEE Floating Point Arithmetic*, SIAM (2001).
- [20] Shampine L.F., Allen Jr. R.C., Pruess S., *Fundamentals of Numerical Computing*, John Wiley & Sons Inc. (1997).
- [21] Sterbenz P.H., *Floating-Point Computation*, Prentice-Hall Inc., Englewood Cliffs, NJ (1974).
- [22] Wilkinson J.H., *Rounding Errors in Algebraic Processes*, Prentice-Hall Inc. (1963).

## $\mathcal{I}$ -Limit Inferior and $\mathcal{I}$ -Limit Superior of Sequences of Sets

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### Abstract

In this talk, we extend the concepts of  $\mathcal{I}$ -Limit superior and  $\mathcal{I}$ -Limit inferior for real number sequences to  $\mathcal{I}$ -Limit superior and  $\mathcal{I}$ -Limit inferior for sequences of sets, study their certain properties and establish some basic theorems.

**Keywords:** Statistical convergence,  $I$ -convergence and  $I^*$ -convergence, sequence of sets, Wijsman convergence.

### References

- [1] Baronti, M. and Papini, P. (1986). Convergence of sequences of sets. In methods of functional analysis in approximation theory, ISNM 76, Birkhauser, Basel, 133-155.
- [2] Beer, G. (1985). *On convergence of closed sets in a metric space and distance functions*, Bull. Austral. Math. Soc., **31**, 421-432.
- [3] Fast, H. (1951). *Sur la convergence statistique*, Coll. Math., **2**, 241-244.
- [4] Kişİ, Ö. and Nuray, F. (2013). *New Convergence Definitions for Sequence of Sets*, Abstract and Applied Analysis Volume 2013, Article ID 852796, 6 pages <http://dx.doi.org/10.1155/2013/852796>
- [5] Kostyrko, P., Šalát, T. and Wilezyński, W. (2000),  $\mathcal{I}$ -Convergence, Real Analysis Exchange, **26**, 669-680.
- [6] Demirci, H. (2001).  $\mathcal{I}$ -limit superior and  $\mathcal{I}$ -limit inferior, Mathematical Communications, **6**, 165-172.
- [7] Wijsman, R. A. (1965). *Convergence of sequences of convex sets, cones and functions*, Bull. Amer. Math. Soc., **70**, 186-188.

## A Ditopological Fuzzy Structural View of Inverse Systems and Inverse Limits

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### Abstract

One of the theories defined to develop some complement-free concepts is the *texture (fuzzy structure) theory* and it was constructed in [1] as a point-based setting for the study of classical sets and fuzzy sets. Add to that, the notion of *ditopology* described on a texture is essentially a topology for which there is no a priori relation between the open and closed sets, and thus ditopological fuzzy structures [1] were conceived as a unified setting for the study of fuzzy topology. Especially, some useful relationships with fuzzy topology may be found in [3].

In the previous study [2], the foundations of a corresponding theory of *inverse (projective) systems* and their limits, called *inverse limits* were laid in the category **ifPTex** of plain textures which are special types of textures, and point functions satisfying a compatibility condition, named *w*-preserving. Therefore firstly, a detailed analysis of inverse systems and inverse limits was presented in [2] insofar as the category of plain textures is concerned. Evidently, this theory was constituted as an analogue of the inverse system theory in the classical categories **Set**, **Top** and **Rng** in algebra, simultaneously.

As the main theme of this presentation, a suitable theory of inverse systems and their limits is established for some subcategories of the category **ifPDitop** topological over **ifPTex**, whose objects are ditopological fuzzy structures which have plain texturing and morphisms are bicontinuous **ifPTex**-morphisms. In addition, many useful properties of inverse limits in **ifPDitop** are studied via examples in the context of ditopological fuzzy structures, as natural counterparts of the classical cases.

**Keywords:** Inverse Limit, Fuzzy Topology, Category, Texture, Ditopology.

### Referances

- [1] Brown, L. M., *Ditopological fuzzy structures I*. Fuzzy Systems and A.I. Magazine, 3 (1993)
- [2] Yıldız, F., *Inverse Systems and Inverse Limits in the Category of Plain Textures*, Topology and Its Applications , Vol. 201, pp. 217-234, 2016.
- [3] Yıldız, F. and Brown, L.M., *Extended Real Dicomactness and an Application to Hutton Spaces*, Fuzzy Sets and Systems, 227, 74–95, 2013.

## On Nullnorms on Bounded Lattices

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### Abstract

t-operators and nullnorms were introduced in [9], [7] respectively, which are also generalizations of the notions of t-norms and t-conorms. And then in [10], it is pointed out that nullnorms and t-operators are equivalent since they have the same block structures in  $[0, 1]^2$ . Namely, if a binary operator  $F$  is a nullnorm then it is also a t-operator and vice versa.

**Definition 0.0.1** *Let  $(L, \leq, 0, 1)$  be a bounded lattice. A commutative, associative, non-decreasing in each variable function  $F : L^2 \rightarrow L$  is called a nullnorm if there is an element  $a \in L$  such that  $F(x, 0) = x$  for all  $x \leq a$ ,  $F(x, 1) = x$  for all  $x \geq a$ .*

In this study, given a bounded lattice  $L$  and a nullnorm on it, taking into account the properties of nullnorms, we investigate an order induced by nullnorms and equivalence relation on bounded lattice. In this way, we obtain that interesting results.

**Keywords:** Nullnorm, Bounded lattice, Partial order.

**Acknowledgement:** The work on this study was supported by the Research Fund of Karadeniz Technical University, project number FBB-2015-5218.

### References

- [1] Aşıcı, E. and Karaçal, F., *On the T-partial order and properties*, Inf.Sci., Vol. **267**, 323-333, 2014.
- [2] Aşıcı, E. and Karaçal, F., *Incomparability with respect to the triangular order*, Kybernetika, Vol. **52**, 15-27, 2016.
- [3] Birkhoff, G., *Lattice Theory*, 3 rd edition, Providence, 1967.
- [4] Drewniak, J., Drygaś, P. and Rak, E., *Distributivity between uninorms and nullnorms*, Fuzzy Sets Syst., Vol. **159**, 1646-1657, 2008.
- [5] Drygaś, P., *A characterization of idempotent nullnorms*, Fuzzy Sets Syst., Vol. **145**, 455-461, 2004.
- [6] Drygaś, P. and E. Rak, *Distributivity equation in the class of 2-uninorms*, Fuzzy Sets Syst. Vol. **291**, 82-97, 2016.
- [7] Calvo, T., De Baets, B. and Fodor J., *The functional equations of Frank and Alsina for uninorms and nullnorms*, Fuzzy Sets Syst. Vol. **120**, 385-394, 2001.
- [8] Karaçal, F. and Aşıcı, E., *Some notes on T-partial order*, J. Inequal. Appl. **2013**, 219, 2013.
- [9] Mas, M., Mayor, G. and Torrens, J., *T-operators*, Int. J. Uncertain. Fuzz. Knowl.-Based Syst. Vol. **7**, 31-50, 1999.

[10] Mas, M., Mayor, G. and Torrens, J., *The distributivity condition for uni-norms and  $t$ -operators*, Fuzzy Sets Syst. Vol. **128**, 209-225, 2002.

## Ideal Version of Weighted Lacunary Statistical Convergence of Sequences of Order $\alpha$

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### Abstract

In this work, we are interested in ideal version of weighted lacunary statistical convergence of sequences of order  $\alpha$  and we examine some inclusion relations.

**Keywords:**  $I$ -convergence;  $I$ -statistical convergence; weighted lacunary  $I$ -statistical convergence of order  $\alpha$ ; sequence space

### References

- [1] Lahiri, Kumar B. and Das, P.  *$I$  and  $I^*$ -convergence in topological spaces*, Mathematica Bohemica, Vol. **130**, No. 2, 153–160, 2005.
- [2] Kostyrko, P., Macaj, M., Salat T. and Sleziak, M.  *$I$ -convergence and extremal  $I$ -limit points*, Mathematica Slovaca, Vol. **55** No. 4 (2005) 443–464.
- [3] Çolak, R. *Statistical convergence of order  $\alpha$* , Modern Methods in Analysis and Its Applications, New Delhi, India: Anamaya Pub., 121–129, 2010.
- [4] Bhunia, S., Das, P. and Pal, S. *Restricting statistical convergence*, Acta Mathematica Hungarica, **134** No. 1-2, 153–161, 2011.
- [5] Et, M., Çınar M. and Karakas, M. *On  $\lambda$ -statistical convergence of order  $\alpha$  of sequences of function*, Journal of Inequalities and Applications, Vol. **2013** No. 1 (2013) 1–8.
- [6] Şengül, H. and Et, M. *On lacunary statistical convergence of order  $\alpha$* , Acta Mathematica Scientia Series, Vol. **34** No. 2 (2014) 473–482.
- [7] Başarır, M. and Konca, Ş. *On some spaces of lacunary convergent sequences derived by Nörlund-type mean and weighted lacunary statistical convergence*, Arab Journal of Mathematical Sciences, Vol. **20** No. 2 (2014) 250–263.
- [8] Başarır, M. and Konca, Ş. *Weighted lacunary statistical convergence in locally solid Riesz spaces*, Filomat Vol. **28** No. 10 (2014) 2059–2067.

## $AK(S)$ and $AB(S)$ Properties of a $K$ -Space

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### Abstract

A typical sum  $s$  on a  $K$ -space  $S$  has often the representation,

$$s(z) = \lim_{\gamma \in \Gamma} \sum_k u_{\gamma k} z_k, \quad z = (z_k) \in S \quad (0.0.7)$$

where  $\Gamma$  is a directed index set and  $u_\gamma = (u_{\gamma k}) \in \phi$ , the space of finitely non-zero sequences, for each  $\gamma \in \Gamma$ . Let a  $K$ -space  $S$  be equipped with a sum (0.0.7). Then, for each  $x = (x_k)$  and  $\gamma \in \Gamma$ , the sequence  $P_\gamma(x) = \sum_k u_{\gamma k} x_k \delta^k$ , ( $\gamma \in \Gamma$ ) is called the  $\gamma$ th  $S$ -section of  $x$  [2]. Here  $\delta^k$  is the sequence whose  $k$ th component is 1 all the others are 0.

If  $\lambda \supset \phi$  is a  $K$  space, then Boos and Leiger defined the spaces  $\lambda_{AB(S)}$  and  $\lambda_{AK(S)}$  in [?] as

$$\lambda_{AB(S)} = \{x \in \omega | (P_\gamma(x))_{\gamma \in \Gamma} \text{ is a bounded net in } \lambda\},$$

and

$$\lambda_{AK(S)} = \{x \in (\lambda_{AB(S)} \cap \lambda) | \lim_\gamma P_\gamma(x) \text{ exists in } \lambda\}.$$

In this work, we investigate some properties of these spaces and give some theorems related to the duals.

**Keywords:**  $K$ - spaces,  $n$ -th section of a sequence,  $\beta$ -,  $\gamma$ -,  $f$ -duality.

### References

- [1] J. Boos, T. Leiger, Dual pairs of sequence spaces, *Int. J. Math. Math. Sci.*, **28**(2001): 9-23.
- [2] J. Boos, T. Leiger, Dual pairs of sequence spaces II, *Proc. Estonian Acad. Sci. Phys. Math.*, **51**(2002): 3-17.
- [3] J. Boos, T. Leiger, Dual pairs of sequence spaces III, *J. Math. Anal. Appl.*, **324** (2006): 1213-1227.
- [4] J. Boos, Classical and Modern Methods in Summability, *Oxford University Press. New York, Oxford*, 2000.
- [5] M. Buntinas, Convergent and bounded Cesàro sections in FK-spaces, *Math. Zeitschr.*, **121** (1971): 191-200.
- [6] M. Buntinas, On Toeplitz sections in sequence spaces, *Math. Proc. Camb. Phil. Soc.*, **78** (1975), 451-460.
- [7] W. H. Ruckle, An abstract concept of the sum of a numerical series, *Can. J. Math.*, 1970, **22**: 863-874.

# On the Second Homology of the Schützenberger Product of Monoids

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## Abstract

For two finite monoids  $S$  and  $T$ , we prove that the second integral homology of the Schützenberger product  $S \diamond T$  is equal to

$$H_2(S \diamond T) = H_2(S) \times H_2(T) \times (H_1(S) \otimes_{\mathbb{Z}} H_1(T))$$

as the second integral homology of the direct product of two monoids.

This is joint work with Hayrullah Ayık and Leyla Bugay.

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**Keywords:** Monoid, Schützenberger product, Second integral homology.

## Referances

- [1] Çevik, A.S., *Minimal but inefficient presentations of the semi-direct products of some monoids*, Semigroup Forum, **66**, 1-17, 2003.
- [2] Çevik, A.S., *The  $p$ -Cockcroft property of the semi-direct products of monoids*, Internat. J. Algebra Comput. **13**, 1-16, 2003.
- [3] Campbell, C.M., Mitchell, J.D., Ruskuc, N., *On defining groups efficiently without using inverses*, Math. Proc. Camb. Phil. Soc. **133**, 31-36, 2002.
- [4] Squier, C., *Word problems and a homological finiteness condition for monoids*, Journal Pure Appl. Algebra, **49**, 201-217, 1987.
- [5] Johnson D.L., *Presentations of Groups*, Cambridge Univ. Press, Cambridge, 1990.
- [6] Ayık, H., Campbell, C.M., O'Connor, J.J., *On the efficiency of the direct products of Monogenic Monoids*, Algebra Colloq. **14**, 279-284, 2007.
- [7] Ayık, H., Campbell, C.M., O'Connor, J.J., Ruskuc, N., *Minimal presentations and efficiency of semigroups*, Semigroup Forum, **60**, 231-242, 2000.
- [8] Ayık, H., Campbell, C.M., O'Connor, J.J., Ruskuc, N., *On the efficiency of finite simple semigroups*, Turkish Journal of Math. **24**, 129-146, 2000.
- [9] Howie, J.M., *Fundamentals of Semigroup Theory*, New York, Oxford University Press, 1995.
- [10] Howie, J.M., Ruskuc, N., *Constructions and presentations for monoids*, Comm. Algebra, **49**, 6209-6224, 1994.



- [11] Guba, V.S., Pride, S.J., *Low dimensional (co)homology of free Burnside monoids*, Journal Pure Appl. Algebra, **108**, 61-79, 1996.  
 [12] Guba, V.S., Pride, S.J., *On the left and right cohomological dimension of monoids*, Bull London Math. Soc. **30**, 391-396, 1998.

## New Sequence Spaces with Respect to a Sequence of Modulus Functions

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### Abstract

In this talk, we introduce the notion of  $A^{\mathcal{I}}$ -invariant statistical convergence,  $A^{\mathcal{I}}$ -lacunary invariant statistical convergence with respect to a sequence of modulus functions. We establish some inclusion relations between these spaces under some conditions.

**Keywords:** Lacunary invariant statistical convergence; Invariant statistical convergence, ideal convergence, modulus function.

### References

- [1] H. Nakano, *Concave modulars*, J. Math. Soc. Jpn. 5 (1953) 29-49.  
 [2] T. Bilgin, *Lacunary strong  $A$ -convergence with respect to a modulus*, Mathematica XLVI(4) (2001) 39-46.  
 [3] P. Kostyrko, M. Macaj, T. Salat,  $\mathcal{I}$ -convergence, Real Anal. Exchange, 26 (2) (2000) 669-686.  
 [4] E. Savaş, F. Nuray, *On  $\sigma$ -statistically convergence and lacunary  $\sigma$ -statistically convergence*. Math. Slovaca, 43 (3), 1993, 309-315.  
 [5] E. Savaş, P. Das, *A generalized statistical convergence via ideals*, App. Math. Lett. 24, 2011, 826-830.

## TF-Type Hypersurfaces in 4-Space

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### Abstract

We study on translation and factorable hypersurfaces in the four dimensional Euclidean space. We calculate implicit algebraic equations of the hypersurfaces.

**Keywords:** 4-space, translation hypersurface, factorable hypersurface, algebraic equation.

### Referances

- [1] Dillen, F., Goemans, W., Van de Woestyne, I. *Translation surfaces of Weingarten type in 3-space*. Bull. Transilv. Univ. Braşov Ser. III 1 (50) 109-122, 2008.
- [2] Do Carmo, M.P. , *Differential Geometry of Curves and Surfaces*, Prentice-Hall, Englewood Cliffs, 1976.
- [3] Liu, H. *Translation surfaces with dependent Gaussian and mean curvature in 3-dimensional spaces*. (Chinese) J. Northeast Univ. Tech. 14 (1), 88-93, 1993.
- [4] Verstraelen, L., Walrave, J., Yaprak, S. *The minimal translation surfaces in Euclidean space*. Soochow J. Math. 20 (1), 77-82, 1994.

## A Note on $q$ -Binomial Coefficients

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### Abstract

The  $q$ -calculus has been developing fast. In the present work we study on a  $q$ -extension of binomial coefficients. The infinite sum of  $q$ -extension of binomial coefficients is obtained. Then, by using its infinite sum, we obtain Volkenborn integral value of  $q$ -extension of binomial coefficients.

**Keywords:**  $p$ -adic number, Indefinite sum,  $q$ -analogue of the binomial coefficients, Volkenborn integral.

### Referances

- [1] Araci, S. and Açıkgöz, M., *A note on the values of weighted  $q$ -Bernstein polynomials and weighted  $q$ -Genocchi numbers*, Advances in Differences Equations, 1-9, 2015.
- [2] Foata, D. and Han G., *The  $q$ -series in combinatorics; Permütation Statistics*, Lecture Note, 2004.
- [3] Schikhof, W. H., *Ultrametric Calculus: An Introduction to  $p$ -adic Analysis*, Cambridge University Pres, 1984.
- [4] Volkenborn A., *Ein  $p$ -adisches Integral und seine Anwendungen. I*, Manuscripta Math. 7-4, 341–373, 1972.
- [5] Volkenborn A., *Ein  $p$ -adisches Integral und seine Anwendungen II*, Manuscripta Math. 12, 17-46, 1974.

## On Statistical Convergence of Sequences of $p$ -Adic Numbers

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### Abstract

Let  $p$  be a fixed prime number. By  $\mathbb{Q}_p$  we denote the field of  $p$ -adic numbers, the completion of the rational numbers field  $\mathbb{Q}$  with respect to the  $p$ -adic norm  $|\cdot|_p$ . The concept of statistical convergence was introduced by H. Fast (1951) [1] and R. C. Buck (1953) [2] independently for real or complex sequences. This concept was studied by T.Salat (1980) [3], J. A. Fridy (1985) [4] and many authors. We note that the field of  $p$ -adic numbers  $\mathbb{Q}_p$  is non-Archimedean, means that the ultrametric inequality is valid

$$|x + y|_p \leq \max \left\{ |x|_p, |y|_p \right\}$$

for all  $x, y \in \mathbb{Q}_p$ . In the present work we define the concept of statistical convergence of sequences for  $p$ -adic numbers and give some its properties.

**Keywords:**  $p$ -adic number, statistical convergence of sequence of  $p$ -adic numbers, statistical Cauchy sequence of  $p$ -adic numbers.

### Referances

- [1] H.Fast, Sur la convergence statistique, Colloq. Math., 241-244 (1951).
- [2] R. C Buck, Generalized Asymptotic Density, Amer. J. Math. 75 335–346 (1953)

- [3] T.Salat ,On statistically convergent sequences of real numbers, *Mathematica Slovaca*,vol.30 issue 2, pp. 139-150 (1980).  
 [4] Fridy, J.A., On statistical convergence, *Analysis* 5 (1985),301-313.  
 [5] Schikhof, W. H. , *Ultrametric Calculus*, Cambridge Univ. Press, New York, 2006.

## Lightlike Hypersurface of an Indefinite Kaehler Manifold with a Complex Semi-Symmetric Metric Connection

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### Abstract

In this paper, we study lightlike hypersurface of an indefinite Kaehler manifold admitting a complex semi-symmetric metric connection. We get the equations of Gauss and Codazzi. Then, we give some characterizations of lightlike hypersurface in an indefinite complex space form with a complex semi-symmetric metric connection. Finally, we show that the Ricci tensor of lightlike hypersurface of an indefinite Kaehler manifold with complex semi-symmetric metric connection is not symmetric.

**Keywords:** Lightlike Hypersurface, Indefinite Complex space form, Complex Semi-Symmetric Metric Connection, Levi-Civita connection, Ricci tensor.

## Intuitionistic Fuzzy Fractional Evolution Problem

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### Abstract

We introduce the generalized intuitionistic fuzzy derivative, this concept used in order to give a generalized intuitionistic fuzzy Caputo fractional derivative. And we discuss the intuitionistic fuzzy fractional evolution problem.

**Keywords:** Generalized intuitionistic fuzzy Hukuhara difference, Generalized intuitionistic fuzzy derivative, generalized intuitionistic fuzzy Caputo-derivative, intuitionistic fuzzy fractional evolution problem.

### References

- [1] T. Allahviranloo, A. Armand and Z. Gouyandeh, Fuzzy fractional differential equations under generalized fuzzy Caputo derivative, *J. Intell. Fuzzy Systems*, 26 (2014) 1481–1490.
- [2] T. Allahviranloo, A. Armand, Z. Gouyandeh, H. Ghadiri, Existence and uniqueness of solutions for fuzzy fractional Volterra-Fredholm integro-differential equations, *J. Fuzzy Set Valued Analysis*, 2013 (2013) 1–9.
- [3] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy sets and Systems*, 20(1) (1986) 87–96.
- [4] V. Barbu, *Nonlinear differential equations of monotone types in Banach spaces*. Springer, Monographs in mathematics.
- [5] B. Bede and L. Stefanini, Generalized differentiability of fuzzy-valued functions, *Fuzzy Sets and Systems*, 230 (2013) 119–141.
- [6] H. Brezis and A. Pazy. *Convergence and Approximation of Semigroups of Nonlinear Operators in Banach Spaces*. *J. Funct. Ana.*, 9 (1972) 63–74.
- [7] G. Debreu, *Integration of correspondences*, in *Proc. Fifth Berkeley Symp. Math. Stat. Probab.*, Vol. 2, Part 1 (Univ. California Press, Berkeley, CA, 1967) 351–372.
- [8] M. Elomari, S. Melliani, R. Ettoussi, L. S. Chadli, Intuitionistic fuzzy semigroups, *Notes on Intuitionistic Fuzzy Sets*, 21(1) (2015) 43–50.
- [9] V. Lakshmikantham, S. Leela, *Nonlinear Differential Equations in Abstract Spaces*, Pergamon Press, New York, (1981).
- [10] O. Kaleva, Nonlinear iteration semigroup of fuzzy Cauchy problems, *Fuzzy sets and systems*, 209 (2012) 104–111.
- [11] O. Kaleva, On the convergence of fuzzy sets, *Fuzzy Sets and Systems*, 17 (1985) 53–65.
- [12] O. Kaleva, The Cauchy problem for fuzzy differential equations, *Fuzzy Set Systems*, 35 (1990) 366–389.
- [13] O. Kaleva, Fuzzy differential equations, *Fuzzy Sets Systems*, 24 (1987) 301–317.
- [14] T. Kato. *Nonlinear semigroup and evolution equations*, *J. Math. Soc. Japan.*, 19 (4) (1967).
- [15] N. Kosmatov, Integral equations and initial value problems for nonlinear differential equations of fractional order, *Nonlinear Anal.*, 70 (2009) 2521–2529.
- [16] Y. Komura, Nonlinear semigroup in Hilbert space, *J. Math. Soc. Japan*, 21(1969), 375–507.

- [17] S. Melliani, M. Elomari, L. S. Chadli, R. Ettoussi, Intuitionistic fuzzy metric space, *Notes on Intuitionistic Fuzzy Sets*, 21(1) (2015) 43–53.
- [18] S. Melliani, M. Elomari, L. S. Chadli, R. Ettoussi, Extension of Hukuhara difference in intuitionistic fuzzy set theory, *Notes on Intuitionistic Fuzzy Sets*, 21(4) (2015) 34–47.
- [19] S. Melliani, M. Elomari, L. S. Chadli, R. Ettoussi, Intuitionistic fuzzy metric space, *Notes on Intuitionistic Fuzzy Sets*, 21 (1) (2015) 43–53.
- [20] A. Pazy. *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer-Verlag, 1983.
- [21] M. L. Puri and D.A. Ralescu, Differentials for fuzzy functions, *J. Math. Anal. Appl.* 91 (1983) 552–558.
- [22] M. L. Puri and D.A. Ralescu, Fuzzy random variables, *J. Math. Anal. Appl.* 114 (1986) 409–422.
- [23] M. L. Puri and D.A. Ralescu, Fuzzy random variables, *J. Math. Anal. Appl.* 114 (1986) 409–422.
- [24] K. Roshdi, M. Al Horani, A. Yousef, M. Sababheh, A new Definition Of Fractional Derivative, *J. Comput. Appl. Math.*, 264 (2014) 65–70.

## A General Tableaux Method for Contact Logics Interpreted over Intervals

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### Abstract

In this paper, we focus our attention on tableau methods for contact logics interpreted over intervals on the reals. Contact logics provide a natural framework for representing and reasoning about regions in several areas of computer science such as geological information systems, artificial intelligence and etc. In this paper, we focus our attention on tableau methods for contact logics interpreted over intervals on the reals. Contact logics provide a natural framework for representing and reasoning about regions in several areas of computer science such as geological information systems, artificial intelligence and etc.[1,3,4]. However,

while various tableau methods have been developed for classical logic, modal logics and intuitionistic logic, not much work has been done on tableau methods for contact logics [1,2]. We develop a general tableau method for contact logic interpreted over intervals. In this paper we give sound and complete tableaux-based decision procedures for contact logics. Developing such tableaux-based decision procedures, we obtain new decidability/complexity results.

### References

- [1] Balbiani, Philippe, Tinko Tinchev, and Dimiter Vakarelov. "Modal logics for region-based theories of space." *Fundamenta Informaticae* 81.1-3 (2007): 29-82.
- [2] Galton, Antony. "The mereotopology of discrete space." *International Conference on Spatial Information Theory*. Springer Berlin Heidelberg, 1999.
- [3] Galton, A. "Qualitative Spatial Change". Oxford University Press." (2000).
- [4] Wolter, Frank, and Michael Zakharyashev. "Spatial representation and reasoning in RCC-8 with Boolean region terms." *Proceedings of the 14th European Conference on Artificial Intelligence (ECAI 2000)*.

## Solving Intuitionistic Fuzzy Differential Equations with Linear Differential Operator by Adomain Decomposition Method

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### Abstract

In this paper we presented intuitionistic fuzzy differential equation with linear differential operator which can be of any order and it also involves nonlinear functional. So our solution procedure gives the solutions of a large area of problems involving intuitionistic fuzzy differential equations. Adomain decomposition method (ADM) has been used to find the approximate solution. Note that we used ADM which gives solution even for some nonlinear problems that can't be solved by classical methods. We have given two numerical examples and by comparing the numerical results obtain from ADM with the exact solution, we have studied their accuracy.

## References

- [1] G. Adomian, Stochastic systems analysis, in “Applied Stochastic Processes”, Academic Press, New York (1980) 1-18.
- [2] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers, Dordrecht, 1994.
- [3] G. Adomian, A review of the decomposition method in applied mathematics, Journal of Mathematical Analysis and Applications 135 (1988) 501-544.
- [4] G. Adomian, Convergent series solution of nonlinear equations, Journal of Computational and Applied Mathematics 11 (1984) 225-230.
- [5] G. Adomian, On Green’s function in higher order stochastic differential equations, Journal of Mathematical Analysis and Applications 88 (1982) 604-606.
- [6] E. Babolian, H. Sadeghi, Sh. Javadi, Numerically solution of fuzzy differential equations by Adomian method, Applied Mathematics and Computation 149 (2004) 547-557.
- [7] M. Paripour, E. Hajilou, A. Hajilou, H. Heidari, Application of Adomian decomposition method to solve hybrid fuzzy differential equations, Journal of Taibah University for Science 9 (2015) 95-103.
- [8] L. Wang and S. Guo, Adomian method for second-order fuzzy differential equation, World Academy of Science, Engineering and Technology Vol: 5 (2011) 04-23.
- [9] L. Zadeh, Toward a generalized theory of uncertainty (GTU) – an outline, Information Sciences 175 (2005) 1-40.
- [10] O. Kaleva, Fuzzy differential equations, Fuzzy sets and Systems 24 (1987) 301-317.
- [11] B. Bede, I. J. Rudas, A. L. Bencsik, First order linear fuzzy differential equations under generalized differentiability, Information Sciences 177 (2007) 1648-1662.
- [12] Y. Chalco-Cano, H. Roman-Flores, Comparison between some approaches to solve fuzzy differential equations, Fuzzy Sets and Systems 160 (2009) 1517-1527.
- [13] Z. Ding, M. Ma, A. Kandel, Existence of the solutions of fuzzy differential equations with parameters, Information Sciences 99 (1997) 205-217.
- [14] S. Seikkala, On the fuzzy initial value problem, Fuzzy Sets and Systems 24 (1987) 319-330.
- [15] M. T. Mizukoshi, L. C. Barros, Y. Chalco-Cano, H. Roman-Flores, R. C. Bassanezi, Fuzzy differential equations and extension principle, Information Sciences 177 (2007) 3627-3635.
- [16] T. Allahviranloo, N. A. Kiani, N. Motamedi, Solving fuzzy differential equations by differential transformation method, Information Sciences 179 (2009) 956-966.
- [17] R. Rodriguez-Lopez, Monotone method for fuzzy differential equations, Fuzzy Sets and Systems 159 (2008) 2047-2076.
- [18] B. Ghazanfari, A. Shakerami, Numerical Solutions of fuzzy differential equations by extended Runge-Kutta-like formulae of order 4, Fuzzy Sets and Systems



189 (2011) 74-91.

- [19] C. Wu, S. Song, E. S. Lee, Approximate solutions, existence and uniqueness of the Cauchy problem of fuzzy differential equations, *Journal of Mathematical Analysis and Applications* 202 (1996) 629- 644.
- [20] J. J. Buckley, Thomas Feuring, Fuzzy differential equations, *Fuzzy sets and Systems* 110 (2000) 43-54.
- [21] C. Wu, S. Song, Existence theorem to the Cauchy problem of fuzzy differential equations under compactness- type conditions, *Information Sciences* 108 (1998) 123-134.
- [22] A. Khastan, R. Rodriguez-Lopez, On periodic solutions to first order linear fuzzy differential equations under differential inclusions' approach, *Information Sciences* 322 (2015) 31-50.
- [23] S. Song, C. Wu, Existence and uniqueness of Solutions to the Cauchy problem of fuzzy differential equations, *Fuzzy Sets and Systems* 110 (2000) 55-67.
- [24] M. Friedman, M. Ma, A. Kandel, Numerical solutions of fuzzy differential and integral equations, *Fuzzy Sets and Systems* 106 (1999) 35-48.
- [25] M. Mosleh, M. Otadi, Approximate solution of fuzzy differential equations under generalized differentiability, *Applied Mathematical Modelling* 39 (2015) 3003-3015.
- [26] V. M. Cabral, L. C. Barros, Fuzzy differential equation with completely correlated parameters. *Fuzzy Sets and Systems* 265 (2015) 86-98.
- [27] K. T. Atanassov, Intuitionistic fuzzy sets. VII ITKR's session, Sofia (deposited in Central Science and Technical Library of the Bulgarian Academy of Sciences 1697/84) (1983).
- [28] K. T. Atanassov, Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* 20 (1986) 87–96.
- [29] K. T. Atanassov, Operators over interval valued intuitionistic fuzzy sets. *Fuzzy Sets Syst.* 64(2) (1994) 159–174.
- [30] M. Nikolova, N. Nikolov, C. Cornelis, G. Deschrijver, Survey of the research on intuitionistic fuzzy sets. *Adv. Stud. Contempor. Math.* 4(2) (2002) 127–157.
- [31] S. K. De, R. Biswas, , A. R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis. *Fuzzy Sets Syst.* 117(2) (2001) 209–213.
- [32] M. H. Shu, C. H. Cheng, J. R. Chang, Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly. *Microelectron. Reliab.* 46(12) (2006) 2139–2148.
- [33] D. F. Li, C. T. Cheng, New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. *Pattern Recognit. Lett.* 23 (2002) 221–225.
- [34] J. Ye, Multicriteria fuzzy decision-making method based on a novel accuracy function under interval valued intuitionistic fuzzy environment. *Expert Syst. Applicat.* 36 (2009) 6899–6902.
- [35] D. F. Li, Multiattribute decision making models and methods using intuitionistic fuzzy sets. *J. Comput. Syst. Sci.* 70 (2005) 73–85.
- [36] A. Kharal, Homeopathic drug selection using intuitionistic fuzzy sets. *Homeopathy* 98(1) (2009) 35–39.

- [37] M. Oberguggenberger, S. Pittschmann, Differential equations with fuzzy parameters. *Math. Mod. Syst.* 5 (1999) 181–202.
- [38] J. Casanovas, F. Rossell, Averaging fuzzy biopolymers. *Fuzzy Sets Syst.* 152 (2005) 139–158.
- [39] M .Z. Ahmad, B. De Baets, A predator–prey model with fuzzy initial populations. In: *Proceedings of the 13th IFSA World Congress and 6th European Society of Fuzzy Logic and Technology Conference, IFSA-EUSFLAT (2009)*.
- [40] L. C. Barros, R. C. Bassanezi, P. A. Tonelli, Fuzzy modelling in population dynamics. *Ecol. Model.* 128 (2000) 27–33.
- [41] M. S. El Naschie, From experimental quantum optics to quantum gravity via a fuzzy Khler manifold. *Chaos Solitons Fractals* 25 (2005) 969–977.
- [42] S. P. Mondal, T. K. Roy, First order linear non homogeneous ordinary differential equation in fuzzy environment. *Math. Theory Model.* 3(1) (2013) 85–95.
- [43] Z. Hassan, A.V. Kamyad, A. A. Heydari, Fuzzy modeling and control of hiv infection. *Comput. Math. Methods Med.* (2012). doi:10.1155/2012/893474
- [44] S. P. Mondal, S. Banerjee, T.K. Roy, First order linear homogeneous ordinary differential equation in fuzzy environment. *Int. J. Pure Appl. Sci. Technol.* 14(1) (2013) 16–26.
- [45] A. Bencsik, B. Bede, J. Tar, J. Fodor, Fuzzy differential equations in modeling hydraulic Differential servo cylinders. In: *Third Romanian-Hungarian joint symposium on applied computational intelligence (SACI), Timisoara, Romania (2006)*.
- [46] V. Nirmala and S. C. Pandian, Numerical Approach for Solving Intuitionistic Fuzzy Differential Equation under Generalised Differentiability Concept, *Applied Mathematical Sciences*, Vol. 9, 2015, no. 67, 3337 – 3346.
- [47] R. Ettoussi, S. Melliani, M. Elomari and L. S. Chadli, Solution of intuitionistic fuzzy differential equations by successive approximations method, *Notes IFS* 21 (2015) No. 2,51–62.
- [48] S. Melliani, M. Elomari, M. Atraoui and L. S. Chadli, Intuitionistic fuzzy differential equation with nonlocal condition, *Notes on Intuitionistic Fuzzy Sets* 21(2015) No. 4, 58–68.
- [49] Sankar Prasad Mondal and Tapan Kumar Roy, System of Differential Equation with Initial Value as Triangular Intuitionistic Fuzzy Number and its Application, *Int. J. Appl. Comput. Math* 1 (2015) 449– 474.

## Involution Matrices of $\frac{1}{4}$ –Quaternions

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**Abstract**

In this work, we consider the  $\frac{1}{4}$ -quaternion algebra and give the matrix representations of the involution and anti-involution maps obtained by this algebra.

**References**

- [1] Bekar M., and Yayli Y., Involution Matrices of Real Quaternions, Caspian J. of Math. Sci. Vol. 5, No. 1, 7-16, 2016.
- [2] Jafari M. , On The Properties of Quasi-Quaternion Algebra, Commun. Fac. Sci. Univ. Ank. Series A1 Vol. 63, No. 1, 1-10, 2014.
- [3] Rosenfeld B.A., Geometry of Lie Groups, Kluwer Academic Publishers, Dordrecht , 1997.
- [4] Ercan Z., Yuce S., On properties of the Dual Quaternions, European J. of Pure and Appl. Math., Vol. 4, No. 2, 142-146, 2011.

## Topological Full Groups of Cantor Minimal Systems

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**Abstract**

In this work, we study topological full groups of Cantor minimal systems. In recent years, this subject has been very popular since it supplies a connection between dynamical systems and group theory. We will investigate the relationship between conjugation of dynamical systems and isomorphism of their topological full groups. Moreover, topological full groups provide the first examples of finitely generated, simple and amenable groups. We will survey the ideas behind the proofs of these facts.

**Keywords:** Cantor Space, Topological Full Group, Simple Grup, Amenable Group.

**References**

- [1] Matui Hiroki. Some remarks on topological full groups of Cantor minimal systems, *Internat.J. Math.*, (2006), 231-251.
- [2] S. Bezuglyi and K. Medynets. Full Groups, Flip Conjugacy, and Equivalence of Cantor Minimal Systems, *Colloquium Mathematicum*, (2008).
- [3] K. Juschenko, N. Monod. Cantor systems, piecewise translations and simple amenable, *Annals of Mathematics* (2012).

## Global Stability to Nonlinear Neutral Differential Equations of First Order

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### Abstract

In this paper, we study globally asymptotically stability of zero solution to a nonlinear neutral differential of first order. The technique of the proof involves the fixed point method. By this way, we extend and improve some recent works in the literature.

**Keywords:** Fixed point theorem, globally asymptotically stability, neutral differential equation, first order.

### References

- [1] R. P. Agarwal and S. R. Grace, Asymptotic Stability of Differential systems of neutral type, *Appl.Math Letter* 13 (2000), 9-15.
- [2] C. Tunc, Exponential stability to a neutral differential equation of first order with delay, *Ann. Differential Equations* 29 (2013), 253-256.
- [3] Y. N Raffoul, Stability in neutral nonlinear differential equations with functional delays using fixed-point theory. *Math Comput Model* 40 (2004), 691-700.
- [4] T. A. Burton, *Stability by fixed point theory for functional differential equation*. New York: Dover, 2006.

## A Study on the Cartesian Product of a Special Graphs

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### Abstract

Our main aim in this presentation is to extend these studies over  $\Gamma(S_M)$  to the cartesian product. In here,  $\Gamma(S_M)$  is a graph of monogenic semigroup  $S_M = \{x, x^2, x^3, \dots, x^n\}$  with zero. In detail, we will investigate some important graph parameters for the cartesian product of any two (not necessarily different) graphs  $\Gamma(S_{M_1})$  and  $\Gamma(S_{M_2})$ .

**Keywords:** Monogenic semigroup graph, Graph product.

### Referances

- [1] N. Akgunes, K. C. Das, and A. S. Cevik, Some properties on the tensor product of graphs obtained by monogenic semigroups. *Applied Mathematics and Computation* 235, 2014, pp. 352-357.  
 [2] K. C. Das, N. Akgunes, A. S. Cevik, On a graph of monogenic semigroup, *Journal of Inequalities and Applications*. 2013:44, 2013, pp.1-13.

## Unit Dual Lorentzian Sphere and Tangent Bundle of Lorentzan Unit 2-Sphere

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### Abstract

The purpose of this study is two-fold, firstly to recall some basic concepts and notions of unit dual Lorentzian sphere. Secondly, to define a one-to-one relationship between the unit dual Lorentzian sphere and tangent bundle of Lorentzian unit 2-sphere.

### Referances

- [1] Yayli, Y., Caliskan, A. and Ugurlu, H. H., The E. Study Maps of Circles on Dual Hyperbolic and Lorentzian Unit Spheres  $H^2$  and  $S^2$ , *Mathematical Proceedings of the Royal Irish Academy*, Vol. 102, No. A1, 37-47, 2002.

[2] Hathout, F., Bekar, M. and Yayli, Y., N-legendre and N-slant curves in the unit tangent bundle of surfaces, Kuwait Journal of Science, (Accepted on 2015-In Press).

## On the Inverse Problem for a Sturm-Liouville Equation with Discontinuous Coefficient

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### Abstract

In this study, it is investigated nonhomogenous boundary value problem. During the solution it is encountered Sturm-Liouville problem with piecewise continuous coefficients and that contained eigenvalue parameter. One transmission condition, which given by as relations between the right and left hand limit of the solution at the point of discontinuity are added to the boundary conditions. We examined some spectral properties of the problem. The numeric solutions of eigenvalues are obtained. According to the spectral datas the inverse problem are researched.

**Keywords:** Discontinuous Sturm-Liouville Problem, Inverse Problem, Transmission Condition.

### Referances

- [1] A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Nauka, Moscow (1977).  
[2] A.M.Akhtyamov, On the uniqueness of the solution of an inverse spectral problem. Diferential Equations, v.39, No.8, 2003, pp.1061-1066.

## Calculation and Analysis of Electronic Parameters of Electroluminescent Device Cells Through I-V Based Modeling

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### Abstract

Light-emitting electrochemical cells (LECs) is one of the simplest kinds of electroluminescent devices. LEC is constituted to an organic single layer structure that was sandwiched between a cathode and an anode. In this study we calculated theoretically of the electronic parameter of LECs device through I-V based modeling. The LEC diode electronic parameters as the ideality factor  $n$  and barrier height  $\phi_b$  were obtained using a method developed by Cheung and confirmed by Werner. The net current of a LEC device is due to the thermionic emission and it can be expressed as

$$I = I_0 \exp\left(\frac{q(V - IR_s)}{nkT}\right)$$

where  $V$  is applied voltage and saturation current  $I_0$  is defined as

$$I_0 = AA^*T^2 \exp\left(-\frac{q\phi_b}{kT}\right).$$

**Keywords:** LECs device, Electronic parameters, I-V modeling.

### References

- [1] Cheung, S. and Cheung, N., *Extraction of Schottky diode parameters from forward current-voltage characteristic*, Applied Physics Letters, 49, 85, 1986.  
[2] Werner, J. H., *Schottky barrier and pn-junction I/V plots — Small signal evaluation*, Applied Physics A, 47, 3, 291–300, 1988.

## On Some Properties of Sum Spaces

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### Abstract

A sum is a continuous linear functional  $s$  defined on a  $K$ -space  $\lambda \supset \phi$  (space of finitely non-zero sequences) such that,  $s(z) = \sum_k z_k$ ,  $\forall z = (z_k) \in \phi$ . A  $K$ -space

$\lambda$  is called a sum space if and only if  $\lambda \supset \phi$  and  $\lambda^f = \lambda^\lambda$ , where  $\lambda^f = \{(f(\delta^k)) : f \in \lambda'\}$  and  $\lambda^\lambda$  is the set of all sequences  $x$  such that  $xy \in \lambda$ ,  $\forall y \in \lambda$  [4,6]. Here  $\delta^k$  is the sequence whose  $k$ th component is 1 all the others are 0,  $xy = (x_k y_k)$  for  $x = (x_k), y = (y_k)$  and  $\lambda'$  is the space of continuous linear functionals on  $\lambda$ .

An FK space  $\lambda \supset \phi$  is generalized semiconservative FK space if  $\lambda^f \subset \lambda^{\lambda^2}$ , where  $\lambda^{\lambda^2} = \lambda^{\lambda\lambda} = (\lambda^\lambda)^\lambda$ .

In this work, we give some definitions and theorems related with sum spaces and generalized semiconservative FK spaces.

**Keywords:** FK spaces,  $\beta$ - dual,  $f$ - dual, Semiconservative FK spaces.

### References

- [1] J. Boos, T. Leiger, Dual pairs of sequence spaces. *Int. J. Math. Math. Sci.*, **28**(2001): 9-23.
- [2] J. Boos, T. Leiger, Dual pairs of sequence spaces II, *Proc. Estonian Acad. Sci. Phys. Math.*, **51**(2002): 3-17.
- [3] J. Boos, T. Leiger, Dual pairs of sequence spaces III, *J. Math. Anal. Appl.*, **324** (2006): 1213-1227.
- [4] J. Boos, Classical and Modern Methods in Summability, *Oxford University Press. New York, Oxford*, 2000.
- [5] D. J. H. Garling, The  $\beta$ - and  $\gamma$ -duality of sequence spaces, *Proc. Camb. Phil. Soc.*, **63** (Jan. 1967), 963-981.
- [6] W. H. Ruckle, An abstract concept of the sum of a numerical series, *Can. J. Math.*, 1970, **22**: 863-874.

## On $q^\lambda$ and $q_0^\lambda$ Invariant Sequence Spaces

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### Abstract

Invariant sequence spaces are very helpful for investigations of the duality of sequence spaces. For instance, if the sequence space  $X$  satisfies the condition  $\ell_\infty.X = X$  then its  $\alpha$ -,  $\beta$ - and  $\gamma$ - duals are same [4]. Garling [1] investigated  $B$ - and  $B_0$ - invariant sequence spaces and Buntinas [2] introduced and investigated  $q$ - and  $q_0$ - invariant sequence spaces and recently, Grosse- Erdmann [3] studied on  $\ell_1$  invariant sequence spaces.

In this work, we define  $q^\lambda$  and  $q_0^\lambda$  invariant sequence spaces,  $X$  with  $q^\lambda.X = X$  and  $q_0^\lambda.X = X$ , respectively. and give some related theorems.



**Keywords:** K- spaces,  $\lambda$ -boundedness and  $\lambda$ -convergence of a sequence,  $\beta$ -,  $\gamma$ -,  $f$ - duality.

### References

- [1] D. J. H. Garling, *On topological sequence spaces*, Proc. Camb. Phil. Soc., 63 (1967), 997-1019.
- [2] M. Buntinas, *Convergent and bounded Cesro sections in FK-spaces*, Math. Zeitschr., 121 (1971), 191-200.
- [3] K.-G. Grosse-Erdmann, *On  $\ell_1$  invariant sequence spaces*, J. Math. Anal. Appl., **262**(2001), 112-132.
- [4] J. Boos, *Classical and Modern Methods in Summability*, Oxford University Press. New York, Oxford, 2000.

## Some New Results on a Graph of Monogenic Semigroup

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### Abstract

In [2], it has been defined a new graph  $\Gamma(S_M)$  on monogenic semigroups  $S_M$  (with zero) having elements  $\{0, x, x^2, x^3, \dots, x^n\}$ . Many researchers have been working on this area after that work, for example [1,3,4]. As a continues study of these studies, in this paper, it will be investigated define some new graph parameters (such as covering number, accessible number, Zagreb indices, ect.) for monogenic semigroup graph  $\Gamma(S_M)$ .

### References

- [1] N. Akgunes, K. C. Das, A. S. Cevik, Topological indices on a graph of monogenic semigroups, Topics in Chemical Graph Theory. Topics in Chemical Graph Theory, Mathematical Chemistry Monographs, University of Kragujevac and Faculty of Science Kragujevac, Kragujevac, No.16a, 2014, pp. 1-21.
- [2] K. C. Das, N. Akgunes, A. S. Cevik, On a graph of monogenic semigroup, Journal of Inequalities and Applications. 2013:44, 2013, pp.1-13.

- [3] K. C. Das, Proof of conjectures on the distance signless Laplacian eigenvalues of graphs. *Linear Algebra and its Applications*, 467, 2015, pp. 100-115.
- [4] K. C. Das, N. Akgunes, M. Togan, A. Yurttas, I. N. Cangul, A. S. Cevik, On the first Zagreb index and multiplicative Zagreb coindices of graphs, *Analele Stiintifice ale Universitatii Ovidius Constanta*, 24(1), 2016, pp. 153-176.

## Computational Solution of Katugampola Conformable Fractional Differential Equations Via RBF Collocation Method

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### Abstract

In conjunction with the development of fractional calculus, conformable derivatives and integrals has been widely used a number of scientific areas. In this talk, we provide a numerical scheme to solve Katugampola conformable fractional differential equations via radial basis function (RBF) collocation technique. In order to confirm our numerical scheme, we present some numerical experiments results.

### Referances

- [1] U. Katugampola, A new fractional derivative with classical properties, *ArXiv:1410.6535v2*.
- [2] E. J. Kansa, Multiquadrics a scattered data approximation scheme with applications to computational fluid-dynamics. i. surface approximations and partial derivative estimates, *Computers and Mathematics with Applications*, 19(8-9), 1990, 127-145.
- [3] C. Franke, R. Schaback, Solving partial differential equations by collocation using radial basis functions, *Applied Mathematics and Computation*, 93, 1998, 73-82.

## Some Integral Inequalities Via Conformable Calculus

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### Abstract

The purpose of this talk is making generalization of Gronwall, Volterra and Pachpatte type inequalities for conformable differential equations. Then we provide some upper or lower bound for fractional derivatives and integrals with the help of Katugampola definition for conformable calculus. These results are extensions of some existing Gronwall, Volterra and Pachpatte type inequalities in the previous studies.

### References

- [1] U. Katugampola, A new fractional derivative with classical properties, ArXiv:1410.6535v2.
- [2] T. Abdeljawad, On conformable fractional calculus, Journal of Computational and Applied Mathematics 279 (2015) 57{66.
- [3] M.Z. Sarikaya, Gronwall type inequality for conformable fractional integrals, 2016, preprint.

## Weighted Ostrowski, Chebyshev and Grüss Type Inequalities for Conformable Fractional Integrals

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### Abstract

In this presentation, we have obtained weighted versions of Ostrowski, Čebysev and Grüss type inequalities for conformable fractional integrals. In accordance with this purpose we have used Katugompola type conformable fractional integrals. The present study confirms previous findings and contributes additional evidence that provide the bounds for more general functions.

### Referances

- [1] U. Katugampola, A new fractional derivative with classical properties, ArXiv:1410.6535v2.
- [2] D. R. Anderson, Taylor's formula and integral inequalities for conformable fractional derivatives, Contributions in Mathematics and Engineering, in Honor of Constantin Caratheodory, Springer, to appear.
- [3] A. M. Ostrowski, Über die absolutabweichung einer di erentiebaren funktion von ihrem integralmittelwert, Comment. Math. Helv. 10(1938), 226-227..e 1 No. 1 pp. 48-53, 2013.
- [4] P. L. Ceby sev, Sur less expressions approximatives des integrales de nies par les autres prisesentre les memes limites, Proc. Math. Soc. Charkov, 2, 93-98, 1882.
- [5] G. Gruss, Über das maximum des absoluten Betrages von  $\frac{1}{b-a} \int_a^b f(x)g(x)dx - \frac{1}{(b-a)^2} \int_a^b f(x)dx \int_a^b g(x)dx$ , Mat.Z., 39, 215-226, 1935.

## Bifurcation and Stability Analysis of a Discrete-Time Predator-Prey Model

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### Abstract

We consider discrete-time Leslie Model. We first determine its non-negative fixed point. Later on, we study local stability of the fixed point and determine the conditions on the parameters to show the existence of flip bifurcation by taking the step-size as a bifurcation parameter. Analytical results are also supported by some numerical simulation. Moreover, using Center Manifold Theory, we show the existence of flip bifurcation and its properties.

## Applications of Hermite-Hadamard Inequalities for $\mathbb{B}$ -Convex Functions and $\mathbb{B}^{-1}$ -Convex Functions

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### Abstract

Hermite-Hadamard Inequality that is expressed in the following form

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(t) dt \leq \frac{1}{2}(f(a) + f(b))$$

was proven by Hermite in [1] and then, ten years later, Hadamard rediscovered in [2] (for the historical consideration see also [3]). Next, Hermite-Hadamard Inequalities for different kinds of functions were examined in numerous article [4,5].

$\mathbb{B}$ -convex functions were introduced and studied in [6].  $\mathbb{B}^{-1}$ -convex functions were defined and examined in [7]. Hermite-Hadamard Inequalities for  $\mathbb{B}$ -convex Functions and  $\mathbb{B}^{-1}$ -convex Functions were introduced in [8].

In this work, we give the applications of Hermite-Hadamard Inequalities for  $\mathbb{B}$ -convex functions and  $\mathbb{B}^{-1}$ -convex functions.

**Keywords:** Hermite-Hadamard Inequalities,  $\mathbb{B}$ -convex functions,  $\mathbb{B}^{-1}$ -convex functions.

### References

- [1] Ch. Hermite (1883). Sur deux limites d'une integrale define, *Mathesis.* 3, 82.
- [2] J. Hadamard (1893). Etude sur les proprietes des fonctions entieres et en particulier d'une fonction consideree par Riemann, *Journal des Mathematiques Pures et Appliquees*, 58, 171-215.
- [3] S.S. Dragomir, C.E.M. Pearce (2000). *Selected Topics on Hermite-Hadamard Inequalities and Applications*, RGMIA Monographs, Victoria University.
- [4] H. Kavurmaci, M. Avci, M.E. Ozdemir (2011). New Inequalities of Hermite-Hadamard Type for Convex Functions with Applications, *Journal of Inequalities and Applications*, 2011:86, doi: 10.1186/1029-2011-86.

- [5] M.E. Ozdemir, E. Set, M.Z. Sarikaya (2011). Some New Hadamard Type Inequalities for Co-Ordinated  $m$ -Convex and  $(\alpha, m)$ -Convex Functions, *Hacettepe Journal of Mathematics and Statistics*, 40, no. 2, 219-229.
- [6] Kemali, S., Yesilce, I., Adilov, G.,  $\mathbb{B}$ -convexity,  $\mathbb{B}^{-1}$ -convexity and Their Comparison, *Numerical Functional Analysis and Optimization*, Vol. 36, No. 2, pp. 133-146, 2015.
- [7] G. Adilov and I. Yesilce (2017).  $\mathbb{B}^{-1}$ -convex Functions. *Journal of Convex Analysis*. 24(2).
- [8] Adilov, G., Yesilce, I., The Hermite-Hadamard Inequalities for  $\mathbb{B}$ -convex and  $\mathbb{B}^{-1}$ -convex Functions, (submitted).

## Operations and Extension Principle under T-Intuitionistic Fuzzy Environment

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### Abstract

In this paper, we introduce necessary definitions and related theorems on intuitionistic fuzzy T-set theory. In existing crisp set theory, characteristic functions, defined by two valued logic, can take values: zero and one only. In fuzzy set theory, introduced by Zadeh (1965), membership functions can take any value in closed unit interval . And, in intuitionistic fuzzy set theory, introduced by K. T. Atanassov (1986), both membership functions and non-membership functions can take suitable values in closed unit interval . But, we may observe that in those existing theories, we have to assign same membership value unity to elements, even if belongingness of one element is more certain than other to the subsets. In other words, certainly belongingness and more certainly belongingness of elements to subsets of universal sets are treated at par in all these existing theories, including classical crisp set theory. Similarly, in non-membership functions of intuitionistic fuzzy sets, zero is assigned as non-membership values to elements, both not belonging and not belonging certainly to subsets of universal sets. In order to overcome these limitations of existing theories, in 2015, we proposed intuitionistic fuzzy T-set theory, in which real numbers are suitably assigned to membership and non-membership functions. In this paper, we further introduce necessary definitions and related theorems

on intuitionistic fuzzy  $T$ -set theory. Those may be considered as generalizations of existing definitions and theorems from existing fuzzy and/or intuitionistic fuzzy set theory. In particular, we have generalized the concepts of extension principle and alpha, beta cut under existing intuitionistic fuzzy environment to  $T$ -intuitionistic fuzzy environment. Moreover, we have discussed some associated results under  $T$ -intuitionistic fuzzy environment. Finally, conclusions and future research directions are drawn.

**Keywords:** Intuitionistic fuzzy sets,  $T^{(+)}$ -characteristic functions,  $T^{(-)}$ -characteristic functions, intuitionistic Fuzzy  $T$ -sets,  $T$ -extension principle.

## On Generalized Double Statistical Convergence of Order $\alpha$ in Intuitionistic Fuzzy $N$ - Normed Spaces

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### Abstract

In the present paper, we introduce and study the notion  $\mathcal{I}$ -double statistical convergence and ideal  $\lambda$ -double statistical convergence of order  $\alpha$  with respect to the intuitionistic fuzzy  $n$ -normed space, briefly  $IFnNS$ , also we examine the relationship between these classes.

### Referances

- [1] Al. Narayanan, S. Vijayabalaji, Fuzzy  $n$ -normed linear space, Int. J. Math. Sci., 24, 3963-3977, 2005.
- [2] Al. Narayanan, S. Vijayabalaji, N. Thillaigovindan, Intuitionistic fuzzy bounded linear operators, Iran. J. Fuzzy Systems, 4, 89-101, 2007.
- [3] N. Thillaigovindan, S. A. Shanthi, Y. B. Jun, On lacunary statistical convergence in intuitionistic fuzzy  $n$ -normed linear space, Annals of Fuzzy Math. and Inf., 1(2), 119-131, 2011.
- [4] E. Savas,  $\lambda$ -statistical convergence in intuitionistic fuzzy 2-normed space, Appl. Math. Inf. Sci., 9(1), 501-505, 2015

## On Feng Qi-Type Integral Inequalities for Conformable Fractional Integrals

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### Abstract

In the last few decades, much significant development of integral inequalities had been established. Recall the famous integral inequality of Feng Qi type [1, 2, 3]:

$$\int_a^b (f(t))^{n+2} dt \geq \left( \int_a^b f(t) dt \right)^{n+1} \quad (0.0.8)$$

where  $f \in C^n(a, b)$ ,  $f^{(i)} \geq 0$ ,  $0 \leq i \leq n$ ,  $f^{(n)} \geq n!$ ,  $n \in \mathbb{R}$ .

In this study, we establish the generalized Qi-type inequality involving conformable fractional integrals.

Firstly we give a important integral inequality which is generalized Qi inequality. Finally, we obtain several inequalities related these inequalities using the conformable fractional integral [4,5].

**Keywords:** Integral Inequalities, Special Functions, Fractional Calculus, Conformable Fractional Integral.

### Referances

- [1] F. Qi, Several integral inequalities, J. Inequal. Pure and Appl. Math. 1 (2000) Art. 19.
- [2] A. Akkurt and H. Yildirim, Genelleştirilmiş Fractional Integraller İçin Feng Qi Tipli Integral Eşitsizlikleri Üzerine, Fen Bilimleri Dergisi, 1(2), (2014).
- [3] Q.A. Ngô, D.D. Thang, T.T. Dat and D.A. Tuan, Note on an integral inequality, J. Inequal. Pure Appl. Math., 7(4) (2006), Art. 120.
- [4] T. Abdeljawad, On conformable fractional calculus, Journal of Computational and Applied Mathematics 279 (2015) 57–66.
- [5] R. Khalil, M. Al horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, Journal of Computational Applied Mathematics, 264 (2014), 65-70.