STABILITY ANALYSIS ON EFFECT OF SYSTEM RESTORE ON EPIDEMIC MODEL FOR COMPUTER VIRUSES

MEHMET EMRE ERDOGAN AND KEMAL USLU

Abstract. More than 317 million new pieces of malware computer viruses or other malicious software were created last year. That means nearly one million new threats were released each day. Every year computer viruses cost homes and businesses billions of dollars in lost time and equipment. Computer viruses are continually evolving and their structures increasingly becoming more complex and transmission capabilities are becoming more powerful. So, we consider a SEIS model to demonstrate the system restore has a more effective role than antivirus softwares on virus defense. Also we have investigated the global behavior of the endemic equilibrium and we have supported our results with numerical simulation.

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1. INTRODUCTION

Epidemiology is an area of medicine concerned with the identification of factors and conditions associated with the spread of an infectious process in a community. Because of a virus programs behavior is similar to the infectious process, this areas detects and strategies that may be useful for us [1]. Biological viruses enter their host through an opening after passively being breathed in, swallowed or via direct contact. Virtual viruses also enter their host passively when you insert an infected disk or open an infected e-mail attachment. Similarly to a biological virus which has to have the correct host and tissue specicity to gain a foothold a horse virus would not make a human being sick a computer virus has to be compatible with the system to gain a foothold. The damage these viruses do is also similar. Biological viruses replicate at the cost of the host damage can include pain, suffering and even death. Computer viruses slow down the computer files can become inaccessible and even lost, and sometimes the complete hard disk gets damaged [2]. Community, population, carrier, portal of entry, vector, symptom, modes of transmission, extra-host survival, immunity, susceptibility, sub-clinical, indicator, effective transfer rate, quarantine, isolation, infection, medium and culture are all terms from epidemiology that are useful in understanding and fighting computer viruses [1].
A computer virus is a program that can infect other programs by modifying them to include a possibly evolved version of itself. With this infection property, a virus can spread to the transitive closure of information flow, corrupting the integrity of information as it spreads. Given the widespread use of sharing in current computer systems, the threat of a virus causing widespread integrity corruption is significant [3]. Viruses, worms and trojans are all part of a class of software called malware. Malware or malicious code (malcode) is short for malicious software. It is code or software that is specifically designed to damage, disrupt, steal, or in general inflict some other bad or illegitimate action on data, hosts, or networks. A computer virus is a type of malware that propagates by inserting a copy of itself into and becoming part of another program. It spreads from one computer to another, leaving infections as it travels. Viruses can range in severity from causing mildly annoying effects to damaging data or software and causing denial-of-service conditions. Almost all viruses are attached to an executable file, which means the virus may exist on a system but will not be active or able to spread until a user runs or opens the malicious host file or program. When the host code is executed, the viral code is executed as well. Normally, the host program keeps functioning after it is infected by the virus. However, some viruses overwrite other programs with copies of themselves, which destroys the host program altogether. Viruses spread when the software or document they are attached to is transferred from one computer to another using the network, a disk, file sharing, or infected e-mail attachments.

The proper assessment of computer viruses in the management of information security and integrity depends on estimates of the risk and impact of computer virus incidents and an analysis of how they are influenced by various factors in the computing environment. Mathematical or computer simulation models of the transmission and control of computer viruses can be useful in synthesizing available information and providing a theoretical basis for control strategies [4]. Predicting virus outbreaks is extremely difficult due to human nature of the attacks but more importantly, detecting outbreaks early with a low probability of false alarms seems quiet difficult [5]. By developing models it is possible to characterize essential properties of the attacks [6]. Consequently, anti-virus software has been developed to take precautions. In order to understanding the effectiveness of the antivirus technologies, numbers of mathematical models were suggested to investigate the epidemic behaviors of computer virus. Due to analogical similarity between the computer viruses and infectious diseases biological counterparts, several propagation models of computer viruses have been proposed, and the obtained results indicate that the long-term behavior of computer virus can be predicted. One of the early triumphs of mathematical epidemiology was the formulation of a simple model that predicted behaviour very similar to the behaviour observed in countless epidemics [7]. The Kermack McKendrick model is a compartmental model based on relatively simple assumptions on the rates of flow between different classes of members of the population [8]. Thenceforward many computer viruses modelling studies have been made. These are; SI models [9-11], SIS models [12-14], SIR models [15-20], SIRS models [21-26], SAIC models [27], SEIR models [28], SEIQR-SEIQRS models [29-32], SLBS models [33-36], and some other models [37-41], have been proposed that every compartment which are Susceptible computers, Infected computers including the latent and breaking-out computers based on some
models. Recovered computers including the quarantine computers based on some models are considered to have the same connecting and disconnecting constants, on some models for the latent and breaking-out computers have the same infecting, corrupting and recovered constants. When considering today’s conditions, it is clear that this situation is how inadequate. That’s what we have done in this study, eliminating this deficiency we tried to develop an advanced epidemic model by thinking total of 14 separate constants for connecting, disconnecting, infecting, recovering and corrupting to the each compartment, instead of previous models have been constructed by 4-5 parameters. Furthermore, we optimized our model for the present day including the effect of system restore which is not considered so far.

So, we consider a SEIS model (see also Fig.1), where the compartments are: $S(t)$ susceptible, $E(t)$ exposed, $I(t)$ infective at time $t$, respectively. The parameters of the model are defined as:

D1.: Every computer connects to the Internet with constant rate $\alpha > 0$. Respectively positive constant rates for each compartments are: $\alpha_1$ for Susceptible, $\alpha_2$ for Exposed, $\alpha_3$ for Infective. Let $\alpha = \alpha_1 + \alpha_2 + \alpha_3$.

D2.: Every computer disconnects from the Internet with constant rate $\delta > 0$. Respectively positive constant rates for each compartments are: $\delta_1$ for Susceptible, $\delta_2$ for Exposed and $\delta_3$ for Infective. Let $\delta = \delta_1 + \delta_2 + \delta_3$.

D3.: Every susceptible computer is infected with constant rate $\beta > 0$ by infected removable storage media.

D4.: Every susceptible computer is infected by exposed computers with constant rate $\gamma_1 > 0$ and infective computers with constant rate $\gamma_2 > 0$, where $\gamma = \gamma_1 + \gamma_2$.

D5.: Every exposed computer is transformed into infective computer with constant rate $\eta > 0$, and recovered with constant rate $\theta_1 > 0$.

D6.: Every infective computer is recovered with constant rate $\theta_2 > 0$.

D7.: Every infective computer is returned to susceptible computer with constant rate $\mu_1 > 0$, and returned to exposed computer with constant rate $\mu_2 > 0$ by using system restore. Let $\mu = \mu_1 + \mu_2$.

Figure 1. The SEIS Model
From the definitions, model can be shown by:

\[
S' (t) = \alpha_1 + \theta_1 E(t) + (\theta_2 + \mu_1) I (t) - (\beta + \gamma_1 E (t) + \gamma_2 I (t) + \delta_1) S (t)
\]

(1.1) \quad \begin{align*}
E' (t) &= \alpha_2 + (\beta + \gamma_1 E (t) + \gamma_2 I (t)) S (t) - (\theta_1 + \eta + \delta_2) E (t) + \mu_2 I (t) \\
I' (t) &= \alpha_3 + \eta E (t) - (\theta_2 + \mu_1 + \mu_2 + \delta_3) I (t)
\end{align*}

with the initial conditions \((S(0), E(0), I(0)) \in \mathbb{R}^3_+\). Let \(N(t) = S(t) + E(t) + I(t)\). Summing the system and simplifying, we get \(\frac{dN(t)}{dt} = \alpha - \delta N(t)\), it is easy to get \(\lim_{t \to \infty} N(t) = \frac{\alpha}{\delta}\). Hence, the following system would be obtained by system (1.1):

\[
E' = \alpha_2 + (\beta + \gamma_1 E + \gamma_2 I) (N - E - I) - (\theta_1 + \eta + \delta_2) E + \mu_2 I \\
I' = \alpha_3 + \eta E - (\theta_2 + \mu_1 + \mu_2 + \delta_3) I
\]

(1.2)

with initial conditions \((E(0), I(0)) \in \mathbb{R}^2_+\). It is easy to verify that \(\Delta = \{(E, I)|E, I \geq 0, N \geq E + I\}\) is positively invariant for the system (1.2). As thus, we will consider the global stability of (1.2) on the set \(\Delta\). Let us shortly overview the theory of asymptotically autonomous systems. An ordinary differential equation in \(\mathbb{R}^n\),

\[
x = f (t, x)
\]

(1.3)

is called asymptotically autonomous with limit equation

\[
y = g (y)
\]

(1.4)

if \(f (t, x) \to g (x), t \to \infty\), locally uniformly in \(x \in \mathbb{R}^n\), i.e. for \(x\) in any compact subset of \(\mathbb{R}^n\). For simplicity we assume that \(f (t, x), g (x)\) are continuous functions and locally Lipschitz in \(x\). The \(\omega\)-limit sets, \(\omega (t_0, x_0)\), of forward bounded solutions \(x\) to (1.3), subject to \(x (t_0) = x_0\), \(x \in \omega (t_0, x_0) \iff x = \lim_{t \to \infty} x (t_j)\) for some sequence \(t_j \to \infty\).

**Theorem 1.1.** Let \(n = 2\) and \(\omega\) be the \(\omega\)-limit set of a forward bounded solution \(x\) of the asymptotically autonomous system (1.3). Assume that there exists a neighborhood of \(\omega\) which contains at most finitely many equilibria of system (1.4). Then the following trichotomy holds:

- \(\omega\) consists of an equilibrium of system (1.4).
- \(\omega\) is the union of periodic orbits of system (1.4) and possibly of centers of system (1.4) that are surrounded by periodic orbits of system (1.4) lying in \(\omega\).
- \(\omega\) contains equilibria of system (1.4) that are cyclically chained to each other in \(\omega\) orbits of system (1.4).

It follows from the Thieme’s Theorem and the subsequent study that, for the purpose of understanding the behavior of the original system (1.1), it suffices to investigate its limit system (1.2) [42].

2. Model Analysis

2.1. Equilibrium Point.
Theorem 2.1. Assume \((\gamma_1 + \gamma_2b) \left( \frac{\alpha}{\delta} - a \right) + \mu_2b > (b + 1)(\beta + \gamma_2a) + (\theta_1 + \eta + \delta_2)\). Then the system (1.1) has a unique equilibrium point \(E = (S_*, E_*, I_*)\), where

\[
\begin{align*}
(2.1) & \quad S_* = N_0 - (E_* + I_*) \\
(2.2) & \quad E_* = \frac{-m_1 - \sqrt{m_1^2 - 4m_0m_2}}{2m_0} \\
(2.3) & \quad I_* = \frac{\alpha_3 + \eta E_*}{\theta_2 + \mu + \delta_3} \\
(2.4) & \quad a = \frac{\alpha_3}{\theta_2 + \mu + \delta_3} \\
(2.5) & \quad b = \frac{\eta}{\theta_2 + \mu + \delta_3} \\
(2.6) & \quad m_0 = -(b + 1)(\gamma_1 + \gamma_2b) \\
(2.7) & \quad m_1 = (\gamma_1 + \gamma_2b) \left( \frac{\alpha}{\delta} - a \right) - (b + 1)(\beta + \gamma_2a) - (\theta_1 + \eta + \delta_2) + \mu_2b \\
(2.8) & \quad m_2 = \alpha_2 + (\beta + \gamma_2a) \left( \frac{\alpha}{\delta} - a \right) + \mu_2a
\end{align*}
\]

Furthermore, \(S_* + E_* + I_* > 0\).

Proof. 1. If we solve the system,

\[
\alpha - \delta(S + E + I) = 0
\]

(2.9) \[
\begin{align*}
& \alpha_2 + (\beta + \gamma_1E + \gamma_2I) \left( \frac{\alpha}{\delta} - E - I \right) - (\theta_1 + \eta + \delta_2)E + \mu_2I = 0 \\
& \alpha_3 + \eta E - (\theta_2 + \mu + \delta_3)I = 0
\end{align*}
\]

From the third equation of system (2.9), we can get \(I_* = \frac{\alpha_3 + \eta E_*}{\theta_2 + \mu + \delta_3}\). Substituting this equation into the second equation of system (2.9) and rearranging terms we have,

\[
\left[- (b + 1)(\gamma_1 + \gamma_2b)\right] E_*^2 + \left[ (\gamma_1 + \gamma_2b) \left( \frac{\alpha}{\delta} - a \right) - (b + 1)(\beta + \gamma_2a) - (\theta_1 + \eta + \delta_2) + \mu_2b \right] E_* + \alpha_2 + (\beta + \gamma_2a) \left( \frac{\alpha}{\delta} - a \right) + \mu_2a = 0
\]

where \(a = \frac{\alpha_3}{\theta_2 + \mu + \delta_3} > 0\), \(b = \frac{\eta}{\theta_2 + \mu + \delta_3} > 0\). If we substituting equations (2.6), (2.7), (2.8) into this equation, we have

\[
m_0E_*^2 + m_1E_* + m_2 = 0
\]

(2.10) where \(m_0 = -(b + 1)(\gamma_1 + \gamma_2b) < 0\), and we can get,

\[
m_1 = (\gamma_1 + \gamma_2b) \left( \frac{\alpha}{\delta} - a \right) - (b + 1)(\beta + \gamma_2a) - (\theta_1 + \eta + \delta_2) + \mu_2b > 0,
\]

from our assuming \(m_2 = \alpha_2 + (\beta + \gamma_2a) \left( \frac{\alpha}{\delta} - a \right) \mu_2a > 0\). Let the discriminant of (2.10) be \(\Delta = m_1^2 - 4m_0m_2 > 0\). So, \(E_* = \frac{-m_1 - \sqrt{\Delta}}{2m_0} > 0\). That is shows us \(E = (S_*, E_*, I_*)\) is the unique equilibrium of the system (1.1). It is trivial to verify that \(E_* + I_* > 0\).
2.2. Stability Analysis.

Lemma 1. The equilibrium point $\bar{E}$ is locally asymptotically stable.

Proof. According to Theorem (1.1),

$$N' = \alpha - \delta N,$$

(2.11) $$E' = \alpha_2 + (\beta + \gamma_1 E + \gamma_2 I) (N - E - I) - (\theta_1 + \eta + \delta_2) E + \mu_2 I$$

$$I' = \alpha_3 + \eta E - (\theta_2 + \mu_1 + \mu_2 + \delta_3) I$$

The dynamical system (2.11) has a unique equilibrium point $\bar{E} = (N_*, E_*, I_*)$ and locally asymptotically stable at $\bar{E}$. The Jacobian matrix of the linearized system of system (2.11) evaluated at $\bar{E}$ is

$$J = \begin{pmatrix} -\delta & 0 & 0 \\ j_{21} & \lambda - j_{22} & j_{23} \\ 0 & -\eta & (\theta_2 + \mu + \delta_3) \end{pmatrix}$$

where

$$j_{21} = \beta + \gamma_1 E_* + \gamma_2 I_*,$$

(2.12) $$j_{22} = - (\beta + \gamma_1 E_* + \gamma_2 I_*) + \gamma_1 (N_* - E_* - I_*) - (\theta_1 + \eta + \delta_2),$$

$$j_{23} = - (\beta + \gamma_1 E_* + \gamma_2 I_*) + \gamma_2 (N_* - E_* - I_*) + \mu_2,$$

in order to determine the characteristic equation,

$$\det |\lambda I_n - J| = \begin{vmatrix} \lambda + \delta & 0 & 0 \\ j_{21} & \lambda - j_{22} & j_{23} \\ 0 & -\eta & (\theta_2 + \mu + \delta_3) \end{vmatrix}$$

$$(\lambda + \delta) [(\lambda - j_{22})(\lambda + \theta_2 + \mu + \delta_3) - \eta j_{23}] = 0$$

Thus the characteristic polynomial is

(2.13) $$h(\lambda) = (\lambda + \delta) \left(n_2 \lambda^2 + n_1 \lambda + n_0\right),$$

where

$$n_2 = 1,$$

$$n_1 = \theta_2 + \mu + \delta_3 + (\beta + \gamma_1 E_* + \gamma_2 I_*) - \gamma_1 (N_* - E_* - I_*) + (\theta_1 + \eta + \delta_2),$$

$$n_0 = (\theta_2 + \mu + \delta_3 + \eta)(\beta + \gamma_1 E_* + \gamma_2 I_*) - (\gamma_1 (\theta_2 + \mu + \delta_3) + \eta \gamma_2)(N_* - E_* - I_*)$$

$$+ (\theta_2 + \mu + \delta_3)(\theta_1 + \eta + \delta_2) - \eta \mu_2,$$

it is obvious that

$$\alpha_2 + (\beta + \gamma_1 E_* + \gamma_2 I_*) S_* - (\theta_1 + \eta + \delta_2) E_* + \mu_2 I_* = 0$$

hence

$$\alpha_2 + \beta S_* + \gamma_1 S_* E_* + \gamma_2 S_* \frac{\alpha_3 + \eta E_*}{\theta_2 + \mu + \delta_3} - (\theta_1 + \eta + \delta_2) E_* + \mu_2 \frac{\alpha_3 + \eta E_*}{\theta_2 + \mu + \delta_3} = 0,$$

we have

$$\gamma_1 S_* + \gamma_2 S_* \frac{\eta}{\theta_2 + \mu + \delta_3} < \theta_1 + \eta + \delta_2$$
and
\[ \gamma_1 S_\ast + \gamma_2 S_\ast \frac{\eta}{\theta_2 + \mu + \delta_3} + \mu_2 \frac{\eta}{\theta_2 + \mu + \delta_3} < \theta_1 + \eta + \delta_2 \]
it is,
\[ S_\ast < \frac{(\theta_1 + \eta + \delta_2)(\theta_2 + \mu + \delta_3) - \eta \mu_2}{\gamma_1 (\theta_2 + \mu + \delta_3) + \gamma_2 \eta} \]
from here,
\[ n_1 = \theta_2 + \mu + \delta_3 + (\beta + \gamma_1 E_\ast + \gamma_2 I_\ast) - \gamma_1 S_\ast + (\theta_1 + \eta + \delta_2) \]
\[ > \theta_2 + \mu + \delta_3 + (\beta + \gamma_1 E_\ast + \gamma_2 I_\ast) - \frac{\theta_1 + \eta + \delta_2}{\gamma_1 (\theta_2 + \mu + \delta_3) + \gamma_2 \eta} + \eta \mu_2 + (\theta_1 + \eta + \delta_2) \]
\[ > 0 \]
\[ n_0 = (\theta_2 + \mu + \delta_3 + \eta) (\beta + \gamma_1 E_\ast + \gamma_2 I_\ast) - (\gamma_1 (\theta_2 + \mu + \delta_3) + \eta \gamma_2) S_\ast \]
\[ + (\theta_2 + \mu + \delta_3)(\theta_1 + \eta + \delta_2) - \eta \mu_2, \]
\[ > \eta \left( \beta + \gamma_1 E_\ast + \gamma_2 \frac{\alpha_3 + \eta E_\ast}{\theta_2 + \mu + \delta_3} \right) \]
\[ > 0 \]
it follows from the Hurwitz criterion \[43\], that three roots of (2.13) have negative reel parts. So, the claimed result follows by the Lyapunov theorem \[43\].

As a consequence of Theorem (2.1) to prove the global stability of the endemic equilibrium point \( \vec{E} \) of system (1.1), it suffices to prove the global stability of \( E = (E_\ast, I_\ast) \) for system (1.2). For that purpose, let us establish two lemmas.

**Lemma 2.** (1.2) allows no periodic solution in the interior of \( \Delta \).

**Proof. 3.** Let,
\[ f_1 (E, I) = \alpha_2 + (\beta + \gamma_1 E + \gamma_2 I) \left( \frac{\alpha}{\delta} - E - I \right) - (\theta_1 + \eta + \delta_2) E + \mu_2 I \]
\[ f_2 (E, I) = \alpha_3 + \eta E \left( - \theta_2 + \mu_1 + \mu_2 + \delta_3 \right) I \]
\[ D (E, I) = \frac{1}{I}. \]

Then
\[ \frac{\partial (D f_1)}{\partial E} + \frac{\partial (D f_2)}{\partial I} = -\gamma_1 \left( 1 + \frac{E}{T} - \frac{\alpha}{\delta I} \right) - (\beta + \gamma_1 E + \gamma_2 I) - (\theta_1 + \eta + \delta_2) \]
\[ - \frac{\alpha_3 + \eta E}{I^2} < 0. \]
The claimed result follows from the Bendixson-Dulac criterion \[43\].

**Lemma 3.** System (1.2) allows no periodic solution that passes through a point on the boundary of \( \Delta \).

**Proof. 4.** If there is a periodic solution that passing through a non-corner point on \( \partial \Delta \), then it must be tangent to \( \partial \Delta \) at this point. On the contrary, suppose there is a periodic solution \( \Gamma \) that passes through a non-corner point \( (E, I) \) on \( \partial \Delta \). There are three cases to be considered.
Case-1: $0 < E < \frac{\alpha}{\delta}, I = 0$. Then, $I|_{E, I} = \alpha_3 + \eta E > 0$, implying that $\Gamma$ is not tangent to $\partial \Delta$ at this point, which leads to a contradiction.

Case-2: $0 < I < \frac{\alpha}{\gamma}, E = 0$. Then, $E|_{E, I} = \alpha_2 + (\beta + \gamma_2 I) (\frac{\alpha}{\gamma} - I) + \mu_2 I > 0$, implying that $\Gamma$ is not tangent to $\partial \Delta$ at this point, which leads to a contradiction.

Case-3: $E + I = \frac{\alpha}{\delta}, E \neq 0$ and $I \neq 0$. Then, $\frac{d(E+I)}{dt}|_{E, I} = - (\theta_1 + \delta_2) E - (\theta_2 + \mu_1 + \delta_3) I < 0$, implying that $\Gamma$ is not tangent to $\partial \Delta$ at this point, also a contradiction.

The claimed result follows by combining the above discussions. Hence, the proof is complete.

On this basis, we present

**Theorem 2.2.** The equilibrium point $\bar{E}$ is globally asymptotically stable for system (1.2).

**Proof.** 5. The claimed result follows by combining the generalized Poincaré-Bendixson theorem [43] with lemmas 1-3.

3. Numerical Examples

This section provides numerical examples for illustrating main result and the effects of System Restore and antivirus software on virus spread. In what follows, observe the asymptotic behavior of system (1.2) with varying $\alpha_1, \alpha_2, \alpha_3, \delta_1, \delta_2$, $\delta_3, \beta, \eta, \mu_1, \mu_2, \theta_1, \theta_2, \gamma_1$ and $\gamma_2$.

In Figs. 2, 3, 4, 5 Evolutions of $S(t), E(t), I(t)$ are performed with constant $\alpha_1 = 0.6, \alpha_2 = 0.2, \alpha_3 = 0.3, \beta = 0.32, \eta = 0.65, \delta_1 = 0.02, \delta_2 = 0.03, \delta_3 = 0.04, \gamma_1 = 0.28, \gamma_2 = 0.42$ and initial conditions $(S(0), E(0), I(0)) = (10, 5, 2)$, and varying $\mu_1, \mu_2, \theta_1$ and $\theta_2$ which are recover with antivirus software and system restore parameters.

Whether system restore and a scanning with an antivirus software will not be done, the number of infected computers will be equivalent to the number of the total computer in the system shown by Fig.2 with $\mu_1, \mu_2, \theta_1, \theta_2 = 0$. Also the system of computers equilibrium point $\bar{E}_1 = (S_1, E_1, I_1)$ equivalent to $\bar{E}_1 = (0.0574, 1.1753, 23.3037)$ in Fig.??.

If only a scanning with an antivirus software will be done, the system which is newly formed is shown in Fig.3 with $\mu_1, \mu_2 = 0$ and $\theta_1 = 0.65, \theta_2 = 0.35$. Also just system restore will be done, the system which is newly formed by new parameters is shown in Fig.4 with $\mu_1 = 0.7, \mu_2 = 0.2$ and $\theta_1, \theta_2 = 0$. And also the system of computers equilibrium point $\bar{E}_2 = (S_2, E_2, I_2)$ equivalent to $\bar{E}_2 = (1.2676, 9.1327, 15.7034)$ in Fig.3 and the system of computers equilibrium point $\bar{E}_3 = (S_3, E_3, I_3)$ equivalent to $\bar{E}_3 = (0.8915, 15.2111, 10.7503)$ in Fig.4.

If system restore and a scanning with an antivirus software will be done, one can easily see in the Fig.5 with $\mu_1 = 0.7, \mu_2 = 0.2$ and $\theta_1 = 0.65, \theta_2 = 0.35$ that
Figure 2. Evolutions of S, E, I, N with $\mu_1, \mu_2, \theta_1, \theta_2 = 0$.

Figure 3. Evolutions of S, E, I, N with $\mu_1, \mu_2 = 0$ and $\theta_1 = 0.65, \theta_2 = 0.35$.

quite decreasing of the number of infected computers. And the system of computers equilibrium point $\bar{E}_4 = (S_4, E_4, I_4)$ equivalent to $\bar{E}_4 = (2.3740, 16.6080, 8.5448)$ in Fig.5.

Additionally one can see from Fig. 2, 3, 4, 5 that the equilibrium points $\bar{E}_{1,2,3,4}$ are globally asymptotically stable. From the figures which exhibits solutions $S(t), E(t), I(t)$ are converging to stable state, which is consistent with the main result.

If the System Restore will not done, infected computers will be dominate the system in time. Therefore, total number of computers will be equivalent to infected
Figure 4. Evolutions of S, E, I, N with $\mu_1 = 0.7, \mu_2 = 0.2$ and $\theta_1, \theta_2 = 0$.

Figure 5. Evolutions of S, E, I, N with $\mu_1 = 0.7, \mu_2 = 0.2$ and $\theta_1 = 0.65, \theta_2 = 0.35$.

computers. (Fig. 2).

If the System Restore will done, we will recover non-copy datas in infected computers also total number of exposed and susceptible computers will be increase while infected computers number decreasing. (Fig. 4).

If we use system restore after have a comprehensive virus scanning, we will have prevented the pretty much deterioration of our computer. (Fig. 5).
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5. Reference


HUGLU VOCATIONAL HIGH SCHOOL, SELCUK UNIVERSITY
E-mail address: m_emre448@hotmail.com

DEPARTMENT OF MATHEMATICS, SELCUK UNIVERSITY
E-mail address: kuslu@selcuk.edu.tr