IFSCOM2016 [1] Proceeding Book
No. 1 pp. 84-90 (2016)
ISBN: 978-975-6900-54-3

SOME INTUITIONISTIC FUZZY MODAL OPERATORS OVER INTUITIONISTIC FUZZY IDEALS AND GROUPS

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Abstract. K.T. Atanassov generalized fuzzy sets in to Intuitionistic Fuzzy Sets in 1983[1]. Intuitionistic Fuzzy Modal Operator was firstly defined by same author and the other operators were defined by several researchers[2, 3, 4]. Intuitionistic fuzzy algebraic stuctures and their properties were studied in[5, 6, 7].

In this paper, we studied some intuitionistic fuzzy operators on intuitionistic fuzzy ideals and groups.

Received: 14–July–2016 Accepted: 29–August–2016

1. Introduction

The original concept of fuzzy sets in Zadeh [9] was introduced as an extension of crisp sets by enlarging the truth value set to the real unit interval $[0, 1]$. In fuzzy set theory, if the membership degree of an element x is $\mu(x)$ then the nonmembership degree is $1-\mu(x)$ and thus it is fixed. Intuitionistic fuzzy sets have been introduced by Atanassov in 1983 [1] and form an extension of fuzzy sets by enlarging the truth value set to the lattice $[0, 1] \times [0, 1]$.

Definition 1.1. [1] An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$$

where $\mu_A(x), (\mu_A : X \to [0, 1])$ is called the "degree of membership of x in A ", $\nu_A(x), (\nu_A : X \to [0, 1])$is called the " degree of non- membership of x in A ",and where μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \text{ for all } x \in X.$$

The hesitation degree of x is defined by $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$

Definition 1.2. [1]An IFS A is said to be contained in an IFS B (notation $A \sqsubseteq B$) if and only if, for all $x \in X : \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$.

It is clear that $A = B$ if and only if $A \sqsubseteq B$ and $B \sqsubseteq A$.

[1] 3^{rd} International Intuitionistic Fuzzy Sets and Contemporary Mathemathics Conference

2010 *Mathematics Subject Classification.* 03E72,47S40.

Key words and phrases. Intuitionistic Fuzzy Modal Operator, Intuitionistic Fuzzy Algebraic Structures.

Definition 1.3. [1]Let $A \in IFS$ and let $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ then the above set is callede the complement of A

$$A^c = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$$

Definition 1.4. [2] Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in$

$IFS(X)$.

(1) $\Box A = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}$

(2) $\Diamond A = \{< x, 1 - \nu_A(x), \nu_A(x) >: x \in X\}$

Definition 1.5. [3] Let X be a set and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \in IFS(X)$, for $\alpha, \beta \in I$

(1) $\boxplus(A) = \left\{\left\langle x.\frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2}\right\rangle : x \in X\right\}$
(2) $\boxtimes(A) = \left\{\left\langle x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2}\right\rangle : x \in X\right\}$
(3) $\boxplus_\alpha(A) = \{\langle x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha\rangle : x \in X\}$
(4) $\boxtimes_\alpha(A) = \{\langle x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x)\rangle : x \in X\}$
(5) for $\max\{\alpha, \beta\} + \gamma \in I$, $\boxplus_{\alpha,\beta,\gamma}(A) = \{< x, \alpha\mu_A(x), \beta\nu_A(x) + \gamma >: x \in X\}$
(6) for $\max\{\alpha, \beta\} + \gamma \in I$, $\boxtimes_{\alpha,\beta,\gamma}(A) = \{< x, \alpha\mu_A(x) + \gamma, \beta\nu_A(x) >: x \in X\}$

Definition 1.6. [8] Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$, $\alpha, \beta, \alpha + \beta \in I$

(1) $\boxplus_{\alpha,\beta}(A) = \{\langle x, \alpha\mu_A(x), \alpha\nu_A(x) + \beta\rangle : x \in X\}$
(2) $\boxtimes_{\alpha,\beta}(A) = \{\langle x, \alpha\mu_A(x) + \beta, \alpha\nu_A(x)\rangle : x \in X\}$

The operators $\boxplus_{\alpha,\beta,\gamma}, \boxtimes_{\alpha,\beta,\gamma}$are an extensions of $\boxplus_{\alpha,\beta}, \boxtimes_{\alpha,\beta}$(resp.).

In 2007, the author[4] defined a new operator and studied some of its properties. This operator is named $E_{\alpha,\beta}$ and defined as follows:

Definition 1.7. [4]Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$, $\alpha, \beta \in [0, 1]$. We define the following operator:

$$E_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + 1 - \alpha), \alpha(\beta\nu_A(x) + 1 - \beta) >: x \in X\}$$

If we choose $\alpha = 1$ and write α insteed of β we get the operator $\boxplus$. Similarly, if $\beta = 1$ is chosen and writen insteed of β, we get the operator $\boxtimes_\alpha$.

In2007, Atanassov introduced the operator $\boxdot_{\alpha,\beta,\gamma,\delta}$ which is a natural extension of all these operators in [3].

Definition 1.8. [3]Let Xbe a set, A$\in IFS(X)$, $\alpha, \beta, \gamma, \delta \in [0, 1]$ such that

$$\max(\alpha, \beta) + \gamma + \delta \leqslant 1$$

then the operator $\boxdot_{\alpha,\beta,\gamma,\delta}$ defined by

$$\boxdot_{\alpha,\beta,\gamma,\delta}(A) = \{< x, \alpha\mu_A(x) + \gamma, \beta\nu_A(x) + \delta >: x \in X\}$$

In 2010, the author [4] defined a new operator which is a generalization of $E_{\alpha,\beta}$.

Definition 1.9. [4]Let X be a set and $A \in IFS(X)$, $\alpha, \beta, \omega \in [0,1]$. We define the following operator:

$$Z^{\omega}_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + \omega - \omega.\alpha), \alpha(\beta\nu_A(x) + \omega - \omega.\beta) >: x \in X\}$$

We have defined a new OTMO on IFS, that is generalization of the some OTMOs. $Z^{\omega,\theta}_{\alpha,\beta}$ defined as follows:

Definition 1.10. [4]Let X be a set and $A \in IFS(X)$, $\alpha, \beta, \omega, \theta \in [0,1]$. We define the following operator:

$$Z^{\omega,\theta}_{\alpha,\beta}(A) = \{< x, \beta(\alpha\mu_A(x) + \omega - \omega.\alpha), \alpha(\beta\nu_A(x) + \theta - \theta.\beta) >: x \in X\}$$

The operator $Z^{\omega,\theta}_{\alpha,\beta}$ is a generalization of $Z^{\omega}_{\alpha,\beta}$,and also, $E_{\alpha,\beta}, \boxplus_{\alpha,\beta}, \boxtimes_{\alpha,\beta}$.

Definition 1.11. [5]Let G be a groupoid, $A \in IFS(G)$.If for all $x, y \in G$,

$$A(xy) \geq \min(A(x), A(y))$$

then A called an intuitionistic fuzzy subgrupoid over G.

Definition 1.12. [6]Let G be a grupoid, $A \in IFS(G)$. If for all $x, y \in G$,

$$A(xy) \geq \max(A(x), A(y))$$

then A called an intuitionistic fuzzy ideal over G, shortly $IFI(G)$.

Definition 1.13. [6]Let G be a grup and $A \in IFS(G)$ a grupoid. If for all $x \in G$,

$$A(x^{-1}) \geq A(x)$$

then A called an intuitionistic fuzzy subgroup over G,shortly $IFG(G)$.

2. Main Results

Theorem 2.1. *Let G be a groupoid and $A \in IFS(G)$.*

(1) If $A \in IFI(G)$ then $\Box A \in IFI(G)$
(2) If $A \in IFI(G)$ then $\Diamond A \in IFI(G)$

Proof. (1)For $x, y \in G$,

$$\mu_{\Box A}(xy) = \mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$$

and

$$\begin{aligned} \nu_{\Box A}(xy) &= 1 - \mu_A(xy) \leq (1 - \mu_A(x)) \wedge (1 - \mu_A(y)) \\ &= \nu_{\Box A}(x) \wedge \nu_{\Box A}(y) \end{aligned}$$

So,

$$\Box A(xy) \geq \Box A(x) \vee \Box A(y)$$

□

Theorem 2.2. *Let G be a groupoid and $A \in IFS(G)$.*

(1) If $A \in IFI(G)$ then $\boxplus(A) \in IFI(G)$
(2) If $A \in IFI(G)$ then $\boxtimes(A) \in IFI(G)$

Proof. (1)For $x, y \in G$,

$$\begin{aligned}\mu_{\boxplus(A)}(xy) &= \frac{\mu_A(xy)}{2} \geq \frac{\mu_A(x)}{2} \vee \frac{\mu_A(y)}{2} \\ &= \mu_{\boxplus(A)}(x) \vee \mu_{\boxplus(A)}(y)\end{aligned}$$

and

$$\begin{aligned}\nu_{\boxplus(A)}(xy) &= \frac{\nu_A(xy)+1}{2} \leq \frac{\nu_A(x)+1}{2} \wedge \frac{\nu_A(y)+1}{2} \\ &= \nu_{\boxplus(A)}(x) \wedge \nu_{\boxplus(A)}(y)\end{aligned}$$

So,

$$\boxplus(A)(xy) \geq \boxplus(A)(x) \vee \boxplus(A)(y)$$

□

Theorem 2.3. *Let G be a groupoid and $A \in IFS(G)$.*

(1) If $A \in IFI(G)$ then $\boxplus_\alpha(A) \in IFI(G)$
(2) If $A \in IFI(G)$ then $\boxtimes_\alpha(A) \in IFI(G)$

Proof. (1)For $x, y \in G$,

$$\begin{aligned}\mu_{\boxtimes_\alpha(A)}(xy) &= \alpha\mu_A(xy) + 1 - \alpha \geq (\alpha\mu_A(x) + 1 - \alpha) \vee (\alpha\mu_A(y) + 1 - \alpha) \\ &= \mu_{\boxtimes_\alpha(A)}(x) \vee \mu_{\boxtimes_\alpha(A)}(y)\end{aligned}$$

and

$$\begin{aligned}\nu_{\boxtimes_\alpha(A)}(xy) &= \alpha\nu_A(xy) \leq (\alpha\nu_A(x)) \wedge (\alpha\nu_A(y)) \\ &= \nu_{\boxtimes_\alpha(A)}(x) \vee \nu_{\boxtimes_\alpha(A)}(y)\end{aligned}$$

So,

$$\boxtimes_\alpha(A)(xy) \geq \boxtimes_\alpha(A)(x) \vee \boxtimes_\alpha(A)(y)$$

□

Theorem 2.4. *Let G be a groupoid and $A \in IFS(G)$.*

(1) If $A \in IFI(G)$ then $\boxplus_{\alpha,\beta}(A) \in IFI(G)$
(2) If $A \in IFI(G)$ then $\boxtimes_{\alpha,\beta}(A) \in IFSI(G)$
(3) If $A \in IFI(G)$ then $\boxplus_{\alpha,\beta,\gamma}(A) \in IFI(G)$
(4) If $A \in IFI(G)$ then $\boxtimes_{\alpha,\beta,\gamma}(A) \in IFI(G)$

Proof. For $x, y \in G$,

$$\begin{aligned}\mu_{\boxplus_{\alpha,\beta,\gamma}(A)}(xy) &= \alpha\mu_A(xy) \geq \alpha\mu_A(x) \vee \alpha\mu_A(y) \\ &= \mu_{\boxplus_{\alpha,\beta,\gamma}(A)}(x) \vee \mu_{\boxplus_{\alpha,\beta,\gamma}(A)}(y)\end{aligned}$$

and

$$\begin{aligned}\nu_{\boxplus_{\alpha,\beta,\gamma}(A)}(xy) &= \beta\nu_A(xy) + \gamma \leq (\beta\nu_A(x) + \gamma) \wedge (\beta\nu_A(y) + \gamma) \\ &= \nu_{\boxplus_{\alpha,\beta,\gamma}(A)}(x) \wedge \nu_{\boxplus_{\alpha,\beta,\gamma}(A)}(y)\end{aligned}$$

So,

$$\boxplus_{\alpha,\beta,\gamma}(A)(xy) \geq \boxplus_{\alpha,\beta,\gamma}(A)(x) \vee \boxplus_{\alpha,\beta,\gamma}(A)(y)$$

The other properties can proof with same way. □

Theorem 2.5. *Let G be a groupoid and $A \in IFS(G)$ an ideal then $E_{\alpha,\beta}(A) \in IFS(G)$ is an ideal.*

Proof. For $x, y \in G$,

$$\begin{aligned}\mu_{E_{\alpha,\beta}(A)}(xy) &= \beta(\alpha\mu_A(xy)+1-\alpha) \geq \beta(\alpha\mu_A(x)+1-\alpha) \vee \beta(\alpha\mu_A(y)+1-\alpha)\\ &= \mu_{E_{\alpha,\beta}(A)}(x) \vee \mu_{E_{\alpha,\beta}(A)}(y)\end{aligned}$$

and

$$\begin{aligned}\nu_{E_{\alpha,\beta}(A)}(xy) &= \alpha(\beta\nu_A(xy)+1-\beta) \leq \alpha(\beta\nu_A(x)+1-\beta) \wedge \alpha(\beta\nu_A(y)+1-\beta)\\ &= \nu_{E_{\alpha,\beta}(A)}(x) \wedge \nu_{E_{\alpha,\beta}(A)}(y)\end{aligned}$$

So,

$$E_{\alpha,\beta}(A)(xy) \geq E_{\alpha,\beta}(A)(x) \vee E_{\alpha,\beta}(A)(y)$$

□

Theorem 2.6. *Let G be a groupoid and $A \in IFS(G)$ an ideal then $\boxdot_{\alpha,\beta,\gamma,\delta}(A) \in IFS(G)$ is an ideal.*

Proof. For $x, y \in G$,

$$\begin{aligned}\mu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(xy) &= \alpha\mu_A(xy)+\gamma \geq (\alpha\mu_A(x)+\gamma) \vee (\alpha\mu_A(y)+\gamma)\\ &= \mu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(x) \vee \mu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(y)\end{aligned}$$

and

$$\begin{aligned}\nu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(xy) &= \beta\nu_A(xy)+\delta \leq (\beta\nu_A(x)+\delta) \wedge (\beta\nu_A(y)+\delta)\\ &= \nu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(x) \wedge \nu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(y)\end{aligned}$$

So, $\boxdot_{\alpha,\beta,\gamma,\delta}(A)(xy) \geq \boxdot_{\alpha,\beta,\gamma,\delta}(A)(x) \vee \boxdot_{\alpha,\beta,\gamma,\delta}(A)(y)$ □

Theorem 2.7. *Let G be a groupoid and $A \in IFS(G)$ an ideal then $Z^{\omega,\theta}_{\alpha,\beta}(A) \in IFS(G)$ is an ideal.*

Proof. For $x, y \in G$,

$$\begin{aligned}\mu_{Z^{\omega,\theta}_{\alpha,\beta}(A)}(xy) &= \beta(\alpha\mu_A(xy)+\omega-\omega.\alpha) \geq \beta(\alpha\mu_A(x)+\omega-\omega.\alpha) \vee \beta(\alpha\mu_A(y)+\omega-\omega.\alpha)\\ &= \mu_{Z^{\omega,\theta}_{\alpha,\beta}(A)}(x) \vee \mu_{Z^{\omega,\theta}_{\alpha,\beta}(A)}(y)\end{aligned}$$

and

$$\begin{aligned}\nu_{Z^{\omega,\theta}_{\alpha,\beta}(A)}(xy) &= \alpha(\beta\nu_A(xy)+\theta-\theta.\beta) \leq \alpha(\beta\nu_A(x)+\theta-\theta.\beta) \wedge \alpha(\beta\nu_A(y)+\theta-\theta.\beta)\\ &= \nu_{Z^{\omega,\theta}_{\alpha,\beta}(A)}(x) \wedge \nu_{Z^{\omega,\theta}_{\alpha,\beta}(A)}(y)\end{aligned}$$

Therefore, we obtain $Z^{\omega,\theta}_{\alpha,\beta}(A)(xy) \geq Z^{\omega,\theta}_{\alpha,\beta}(A)(x) \vee Z^{\omega,\theta}_{\alpha,\beta}(A)(y)$. □

Theorem 2.8. *Let G be a group and $A \in IFS(G)$.*

(1) If $A \in IFG(G)$ then $\Box A \in IFG(G)$.
(2) If $A \in IFG(G)$ then $\Diamond A \in IFG(G)$.

Proof. It is clear that, if $A \in IFG(G)$ then it means $A \in IFI(G)$ and for all $x \in G$, $A(x^{-1}) \geq A(x)$.

So, it will be enough to prove the correctness of the second condition.

(2)For $x \in G$

$$\mu_{\Diamond A}(x^{-1}) = 1-\nu_A(x^{-1}) \geq 1-\nu_A(x) = \mu_{\Diamond A}(x)$$

and

$$\nu_{\Diamond A}(x^{-1}) = \nu_A(x^{-1}) \leq \nu_A(x) = \nu_{\Diamond A}(x)$$

The other property can be proved same way. □

Theorem 2.9. *Let G be a group and $A \in IFS(G)$.*

(1) If $A \in IFG(G)$ then $\boxplus(A) \in IFG(G)$
(2) If $A \in IFG(G)$ then $\boxtimes(A) \in IFG(G)$

Proof. (2)For $x, y \in G$,If $A \in IFG(G)$ then $\boxtimes(A) \in IFI(G)$. So, $\boxtimes(A)(xy) \geq \boxtimes(A)(x) \wedge \boxtimes(A)(y)$.

Now,

$$\mu_{\boxtimes(A)}(x^{-1}) = \frac{\mu_A(x^{-1}) + 1}{2} \geq \frac{\mu_A(x) + 1}{2} = \mu_{\boxtimes(A)}(x)$$

and

$$\nu_{\boxtimes(A)}(x^{-1}) = \frac{\nu_A(x^{-1})}{2} \leq \frac{\nu_A(x)}{2} = \nu_{\boxtimes(A)}(x)$$

Therefore,

$$\boxtimes(A)(x^{-1}) \geq \boxtimes(A)(x)$$

□

Theorem 2.10. *Let G be a group and $A \in IFS(G)$.*

(1) If $A \in IFG(G)$ then $\boxplus_\alpha(A) \in IFG(G)$
(2) If $A \in IFG(G)$ then $\boxtimes_\alpha(A) \in IFG(G)$

Proof. (1)For $x, y \in G$, it is clear that $\boxplus_\alpha(A)(xy) \geq \boxplus_\alpha(A)(x) \wedge \boxplus_\alpha(A)(y)$. On the other hand,

$$\mu_{\boxplus_\alpha(A)}(x^{-1}) = \alpha\mu_A(x^{-1}) \geq \alpha\mu_A(x) = \mu_{\boxplus_\alpha(A)}(x)$$

and

$$\nu_{\boxplus_\alpha(A)}(x^{-1}) = \alpha\nu_A(x^{-1}) + 1 - \alpha \leq \alpha\nu_A(x) + 1 - \alpha = \nu_{\boxplus_\alpha(A)}(x)$$

So,

$$\boxplus_\alpha(A)(x^{-1}) \geq \boxplus_\alpha(A)(x)$$

□

Theorem 2.11. *Let G be a group and $A \in IFS(G)$.*

(1) If $A \in IFG(G)$ then $\boxplus_{\alpha,\beta}(A) \in IFG(G)$
(2) If $A \in IFG(G)$ then $\boxtimes_{\alpha,\beta}(A) \in IFG(G)$
(3) If $A \in IFG(G)$ then $\boxplus_{\alpha,\beta,\gamma}(A) \in IFG(G)$
(4) If $A \in IFG(G)$ then $\boxtimes_{\alpha,\beta,\gamma}(A) \in IFG(G)$

Proof. For $x, y \in G$,

$$\mu_{\boxtimes_{\alpha,\beta,\gamma}(A)}(x^{-1}) = \alpha\mu_A(x^{-1}) + \gamma \geq \alpha\mu_A(x) + \gamma = \mu_{\boxtimes_{\alpha,\beta,\gamma}(A)}(x)$$

and

$$\nu_{\boxtimes_{\alpha,\beta,\gamma}(A)}(x^{-1}) = \beta\nu_A(x^{-1}) \leq \beta\nu_A(x) = \nu_{\boxtimes_{\alpha,\beta,\gamma}(A)}(x)$$

So,

$$\boxtimes_{\alpha,\beta,\gamma}(A)(x^{-1}) \geq \boxtimes_{\alpha,\beta,\gamma}(A)(x)$$

The other properties can proof with same way. □

Theorem 2.12. *Let G be a group and $A \in IFS(G)$. If A is an intuitionistic fuzzy subgroup on G then $E_{\alpha,\beta}(A) \in IFG(G)$.*

Proof. It is clear that for $x, y \in G$, $E_{\alpha,\beta}(A)(xy) \geq E_{\alpha,\beta}(A)(x) \wedge E_{\alpha,\beta}(A)(y)$.

$$\begin{aligned} \mu_{E_{\alpha,\beta}(A)}(x^{-1}) &= \beta(\alpha\mu_A(x^{-1}) + 1 - \alpha) \geq \beta(\alpha\mu_A(x) + 1 - \alpha) \\ &= \mu_{E_{\alpha,\beta}(A)}(x) \end{aligned}$$

and

$$\begin{aligned} \nu_{E_{\alpha,\beta}(A)}(x^{-1}) &= \alpha(\beta\nu_A(x^{-1}) + 1 - \beta) \leq \alpha(\beta\nu_A(x) + 1 - \beta) \\ &= \nu_{E_{\alpha,\beta}(A)}(x) \end{aligned}$$

So, $E_{\alpha,\beta}(A) \in IFG(G)$. □

Theorem 2.13. *Let G be a group and $A \in IFS(G)$ an intuitionistic fuzzy group then $\boxdot_{\alpha,\beta,\gamma,\delta}(A) \in IFS(G)$ is an intuitionistic fuzzy subgroup.*

Proof. For $x \in G$,

$$\begin{aligned} \mu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(x^{-1}) &= \alpha\mu_A(x^{-1}) + \gamma \geq \alpha\mu_A(x) + \gamma \\ &= \mu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(x) \end{aligned}$$

and

$$\begin{aligned} \nu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(x^{-1}) &= \beta\nu_A(x^{-1}) + \delta \leq \beta\nu_A(x) + \delta \\ &= \nu_{\boxdot_{\alpha,\beta,\gamma,\delta}(A)}(x) \end{aligned}$$

Therefore $\boxdot_{\alpha,\beta,\gamma,\delta}(A) \in IFG(G)$. □

Theorem 2.14. *Let G be a group and $A \in IFS(G)$. If A is an intuitionistic fuzzy subgroup on G then $Z^{\omega,\theta}_{\alpha,\beta}(A) \in IFG(G)$.*

Proof. It can shown easily. □

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