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NEW INTUITIONISTIC FUZZY LEVEL SETS

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Abstract. The concept of Intuitionistic Fuzzy Sheet $t-$Cut Set and Intuitionistic Fuzzy $\alpha - t$ Block Cut Set are introduced. The differences between $C_{\alpha,\beta}$ level set and new intuitionistic fuzzy sets is shown.

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1. Introduction

The function $\mu : X \to [0,1]$ is called a fuzzy set over $X(FS(X))$[**?**]. For $x \in X$, $\mu(x)$ is the membership degree of x and the non-membership degree is $1 - \mu(x)$.Intuitionistic fuzzy sets have been introduced by Atanassov [2], as an extension of fuzzy sets. If X is a universal then a intuitionistic fuzzy set A, the membership and non-membership degree for each $x \in X$ respectively, $\mu_A(x)(\mu_A : X \to [0,1])$ and $\nu_A(x)(\ \nu_A{:}X \to [0,1])$ such that $0 \leq \mu_A(x)+\nu_A(x) \leq 1$. The class of intuitionistic fuzzy sets on X is denoted by $IFS(X)$.

Definition 1.1. [2] An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$$

where $\mu_A(x), (\mu_A : X \to [0,1])$ is called the "degree of membership of x in A ", $\nu_A(x), (\nu_A : X \to [0,1])$is called the " degree of non- membership of x in A ",and where μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \text{ for all } x \in X.$$

Definition 1.2. [1] An intuitionistic fuzzy set A is said to be contained in an intuitionistic fuzzy set B if and only if, for all $x \in X$: $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$. If fuzzy set B contains fuzz set A then it is shown by $A \sqsubseteq B$.

It is clear that $A = B$ if and only if $A \sqsubseteq B$ and $B \sqsubseteq A$.

Definition 1.3. [2]Let $A \in IFS(X)$ and let $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ then the set

$$A^c = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$$

is called the complement of A.

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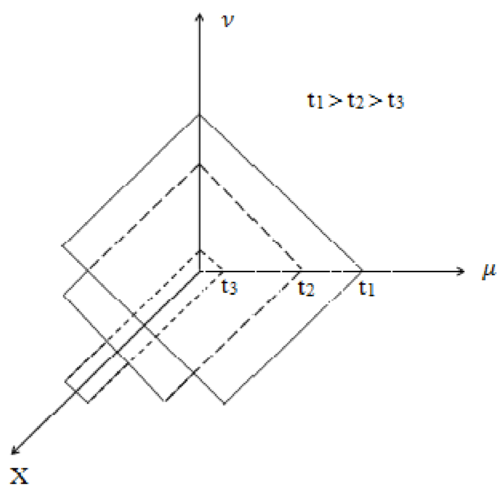

FIGURE 1

The intersection and the union of two IFSs A and B on X are defined by

$$A \sqcap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: x \in X\}$$

$$A \sqcup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: x \in X\}$$

Some special Intuitionistic Fuzzy Sets on X are defined as following;

$$\begin{array}{rcl} O^* & = & \{\langle x, 0, 1\rangle : x \in X\} \\ U^* & = & \{\langle x, 0, 0\rangle : x \in X\} \end{array}$$

Definition 1.4. [4] Let $A \in IFS(X)$.Then $(\alpha, \beta)-$cut of A is a crisp subset $C_{\alpha,\beta}(A)$ of the IFS A is given by

$$C_{\alpha,\beta}(A) = \{x : x \in X \text{ such that } \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$$

where $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$.

2. Sheet and Block Cut Intuitionistic Fuzzy Level Sets

Definition 2.1. Let X be a set and $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\} \in IFS(X)$. If $t \in [0, 1]$ then sheet $t-$cut of A defined as following

$$A(t) = \{\langle x, \mu_A(x), \nu_A(x)\rangle : \mu_A(x) + \nu_A(x) = t, x \in X\}$$

Proposition 2.1. *Let X be a set and $A, B \in IFS(X)$. For every $t \in [0, 1]$,*

(1) $(A \sqcup B)(t) = A(t) \sqcup B(t)$
(2) $A(t) \sqcap B(t) = (A \sqcap B)(t)$
(3) $(A^c(t))^c = A(t)$

Proof. (1)

$$A(t) \sqcup B(t) = \left\{ \begin{array}{c} \langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle : \\ \mu_A(x) + \nu_A(x) = t \wedge \mu_B(x) + \nu_B(x) = t, x \in X \end{array} \right\}$$

If $\mu_A(x) \geq \mu_B(x)$ then from $\mu_A(x) + \nu_A(x) = t = \mu_B(x) + \nu_B(x)$ we obtain $\nu_A(x) \leq \nu_B(x)$.

$\max(\mu_A(x), \mu_B(x)) + \min(\nu_A(x), \nu_B(x)) = \mu_A(x) + \nu_A(x) = t$

If $\mu_A(x) \leq \mu_B(x)$ then from $\mu_A(x) + \nu_A(x) = t = \mu_B(x) + \nu_B(x)$ we obtain $\nu_A(x) \leq \nu_B(x)$.

$\max(\mu_A(x), \mu_B(x)) + \min(\nu_A(x), \nu_B(x)) = \mu_B(x) + \nu_B(x) = t$

Thence, $(A \sqcup B)(t) = A(t) \sqcup B(t)$.

(2)

$$A(t) \sqcap B(t) = \left\{ \begin{array}{c} \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle : \\ \mu_A(x) + \nu_A(x) = t \wedge \mu_B(x) + \nu_B(x) = t, x \in X \end{array} \right\}$$

If $\mu_A(x) \geq \mu_B(x)$ then from $\mu_A(x) + \nu_A(x) = t = \mu_B(x) + \nu_B(x)$ we obtain $\nu_A(x) \leq \nu_B(x)$.

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$\min(\mu_A(x), \mu_B(x)) + \max(\nu_A(x), \nu_B(x)) = \mu_A(x) + \nu_A(x) = t$

Therefore, we obtain that $A(t) \sqcap B(t) = (A \sqcap B)(t)$.

(3) It is clear. □

Remark 2.1. Let X be a set and $A \in IFS(X).A(t)$ is a fuzzy set on $[0, t]$.

Proposition 2.2. *Let X be a set and $A \in IFS(X)$. If $t, s \in [0, 1]$ then*

$$\text{Either } A(t) \sqcap A(s) = O^* \text{ or } t = s$$

Proof. If $A(t) \sqcap A(s) \neq O^*$ and $t \neq s$ then there exists $x \in X$,

$$\begin{aligned} \mu_A(x) + \nu_A(x) &= t \text{ and } \mu_A(x) + \nu_A(x) = s \\ &\Rightarrow t = s \end{aligned}$$

□

Corollary 2.1. *There exist an equivalence relation on X such that the sheet $t-$cuts are equivalence class of that relation.*

Definition 2.2. Let X be a set and $A \in IFS(X)$. If $t \in [0, 1]$ and $\alpha \in [0, t]$ then

$$A(t)_\alpha = \{x : x \in X,\ A(t)(x) \geq (\alpha, t - \alpha)\}$$

is called $\alpha - t$ block cut of A.

From definitions, it is easily seen that for every $t \in [0, 1], A(t) \in FS(X)$. Because $A(t) : X \to [0, t]$ and $[0, t] \sim [0, 1]$. For short notation, if $A(t) : X \to [0, t]$ then $A(t)$ will be called $t-$fuzzy set on $X(A(t) \in FS_t(X))$. It is clear that $A(t)_\alpha$ is a crisp set.

Proposition 2.3. *Let X be a set and $A \in IFS(X)$. If $t \in [0, 1]$ then*

(1) $A(t)_t = \{x : x \in X,\ \mu_A(x) = t \wedge \nu_A(x) = 0\}$
(2) $A(t)_0 = \{x : x \in X,\ \mu_A(x) \geq 0 \wedge \nu_A(x) \leq t\}$

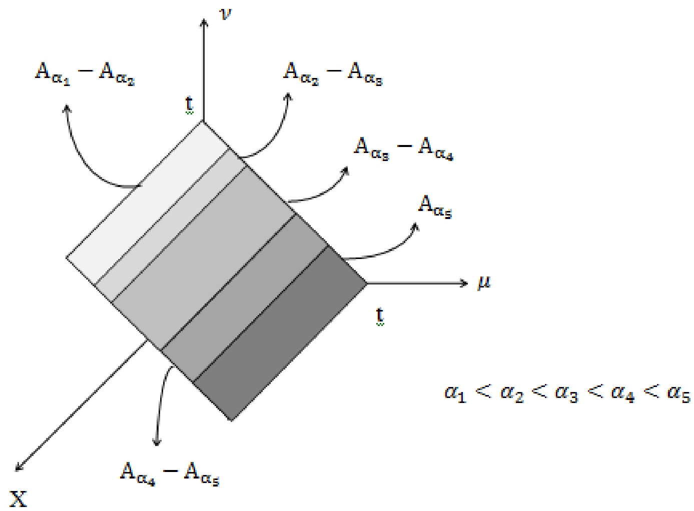

FIGURE 2

(3) $A(0) = U^*$

Example 2.1. Let $X = \{a, b, c, d, e\}$ and
$A = \{(a, 0.5, 0.4), (b, 0.2, 0.3), (c, 0.5, 0.3), (d, 0.4, 0.4), (e, 0.4, 0.1)\}$.

(1) $A(0.5)_{0.3} = \{e\}$ but $C_{0.3,0.5}(A) = \{a, c, d, e\}$ and $C_{0.5,0.3}(A) = \{c\}$.
(2) $A(0.8)_{0.5} = \{c\}$ but $0.8 + 0.5 > 1$ so, we can not obtain $C_{0.5,0.8}(A)$ or $C_{0.8,0.5}(A)$.

Example 2.2. Let $X = \{a, b, c, d, e\}$ and
$A = \{(a, 0.1, 0.2), (b, 0.4, 0.3), (c, 0.6, 0.2), (d, 0.7, 0.1), (e, 0.2, 0.5)\}$.

$A(0.3)_{0.2} = \varnothing$ but $C_{0.2,0.3}(A) = \{b, c, d\}$ and $C_{0.3,0.2}(A) = \{c, d\}$.

That is seen from the examples, $(\alpha, \beta)-$cut of an intuitionistic fuzzy set A and $\alpha - t$ block cut of A are different sets. For all $t \in [0, 1]$ and $\alpha \in [0, t]$, we can determine $\alpha - t$ block cut of A, if $\alpha + t > 1$ then we can not determine $(\alpha, \beta)-$cut of A.Consequently, $\alpha - t$ block cut of an intuitionistic fuzzy set allows a more extensive studying area.

Proposition 2.4. *Let X be a set and $A \in IFS(X)$. If $t \in [0, 1]$ and $\alpha, \beta \in [0, t]$ such that $\alpha \leq \beta$ then $A(t)_\beta \subseteq A(t)_\alpha$.*

Proof. Let $\alpha \leq \beta$. If $x \in A(t)_\beta$ then

$$A(t)(x) \geq (\beta, t - \beta) \geq (\alpha, t - \alpha)$$

Therefore $x \in A(t)_\alpha$. □

Proposition 2.5. *Let X be a set and $A, B \in IFS(X)$. If $t \in [0, 1]$ and $\alpha \in [0, t]$ then*

(1) $A(t)_\alpha \cup B(t)_\alpha = (A(t) \sqcup B(t))_\alpha$
(2) $A(t)_\alpha \cap B(t)_\alpha = (A(t) \sqcap B(t))_\alpha$

(3) $(A(t)^c)_\alpha = t - A(t)_\alpha, (t(x) = t)$
(4) $(A^c(t)^c)_\alpha = t - A(t)_\alpha = t - A^c(t)_\alpha$

Proof. (1)

$$\begin{aligned} x &\in A(t)_\alpha \cup B(t)_\alpha \Leftrightarrow A(t)(x) \geq (\alpha, t-\alpha) \vee B(t)(x) \geq (\alpha, t-\alpha) \\ &\Leftrightarrow (\mu_{A(t)}(x) \geq \alpha \wedge \nu_{A(t)}(x) \leq t-\alpha) \vee (\mu_{B(t)}(x) \geq \alpha \wedge \nu_{B(t)}(x) \leq t-\alpha) \\ &\Leftrightarrow (\mu_{A(t)}(x) \geq \alpha \vee \mu_{B(t)}(x) \geq \alpha) \wedge (\nu_{A(t)}(x) \leq t-\alpha \vee \nu_{B(t)}(x) \leq t-\alpha) \\ &\Leftrightarrow (\mu_{A(t)}(x) \vee \mu_{B(t)}(x)) \geq \alpha \wedge (\nu_{A(t)}(x) \wedge \nu_{B(t)}(x)) \leq t-\alpha \\ &\Leftrightarrow \mu_{A(t)\sqcup B(t)}(x) \geq \alpha \wedge \nu_{A(t)\sqcup B(t)}(x) \leq t-\alpha \\ &\Leftrightarrow x \in (A(t) \sqcup B(t))_\alpha \end{aligned}$$

(2)

$$\begin{aligned} & A(t)_\alpha \cap B(t)_\alpha = \{x \in X : A(t)(x) \geq (\alpha, t-\alpha) \wedge B(t)(x) \geq (\alpha, t-\alpha)\} \\ = & \{x \in X : (\mu_{A(t)}(x) \geq \alpha \wedge \nu_{A(t)}(x) \leq t-\alpha) \wedge (\mu_{B(t)}(x) \geq \alpha \wedge \nu_{B(t)}(x) \leq t-\alpha)\} \\ = & \{x \in X : (\mu_{A(t)}(x) \geq \alpha \wedge \mu_{B(t)}(x) \geq \alpha) \wedge (\nu_{A(t)}(x) \leq t-\alpha \wedge \nu_{B(t)}(x) \leq t-\alpha)\} \\ = & \{x \in X : (\mu_{A(t)}(x) \wedge \mu_{B(t)}(x)) \geq \alpha \wedge (\nu_{A(t)}(x) \vee \nu_{B(t)}(x)) \leq t-\alpha)\} \\ = & \{x \in X : \mu_{A(t)\sqcap B(t)}(x) \geq \alpha \wedge \nu_{A(t)\sqcap B(t)}(x) \leq t-\alpha)\} \\ = & (A(t) \sqcap B(t))_\alpha \end{aligned}$$

(3)

$$\begin{aligned} (A(t)^c)_\alpha &= \{x \in X : A(t)^c(x) \geq (\alpha, t-\alpha)\} \\ &= \{x \in X : \nu_{A(t)}(x) \geq \alpha \wedge \mu_{A(t)}(x) \leq t-\alpha \ \} \\ &= \{x \in X : t - \nu_{A(t)}(x) \leq t-\alpha \wedge t - \mu_{A(t)}(x) \geq \alpha\} \\ &= \{x \in X : (t - A(t))(x) \geq (\alpha, t-\alpha) \ \} \\ &= t - A(t)_\alpha \end{aligned}$$

□

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