NEW INTUITIONISTIC FUZZY LEVEL SETS

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Abstract. The concept of Intuitionistic Fuzzy Sheet $t$–Cut Set and Intuitionistic Fuzzy $\alpha$–$t$ Block Cut Set are introduced. The differences between $C_{\alpha,\beta}$ level set and new intuitionistic fuzzy sets is shown.

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1. Introduction

The function $\mu : X \to [0, 1]$ is called a fuzzy set over $X(FS(X))$. For $x \in X$, $\mu(x)$ is the membership degree of $x$ and the non-membership degree is $1 - \mu(x)$. Intuitionistic fuzzy sets have been introduced by Atanassov [2], as an extension of fuzzy sets. If $X$ is a universal then a intuitionistic fuzzy set $A$, the membership and non-membership degree for each $x \in X$ respectively, $\mu_A(x)$ and $\nu_A(x)$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

The class of intuitionistic fuzzy sets on $X$ is denoted by $IFS(X)$.

Definition 1.1. [2] An intuitionistic fuzzy set (shortly IFS) on a set $X$ is an object of the form

$$A = \{ < x, \mu_A(x), \nu_A(x) > : x \in X \}$$

where $\mu_A(x), (\mu_A : X \to [0, 1])$ is called the “degree of membership of $x$ in $A$”, $\nu_A(x), (\nu_A : X \to [0, 1])$ is called the “degree of non-membership of $x$ in $A$”, and where $\mu_A$ and $\nu_A$ satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \leq 1, \text{ for all } x \in X.$$

Definition 1.2. [1] An intuitionistic fuzzy set $A$ is said to be contained in an intuitionistic fuzzy set $B$ if and only if, for all $x \in X$ : $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$. If fuzzy set $B$ contains fuzz set $A$ then it is shown by $A \subseteq B$.

It is clear that $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Definition 1.3. [2] Let $A \in IFS(X)$ and let $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in X \}$ then the set

$$A^c = \{ < x, \nu_A(x), \mu_A(x) > : x \in X \}$$

is called the complement of $A$. 
The intersection and the union of two IFSs $A$ and $B$ on $X$ are defined by
\[
A \cap B = \{<x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x)> : x \in X\}
\]
\[
A \cup B = \{<x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x)> : x \in X\}
\]

Some special Intuitionistic Fuzzy Sets on $X$ are defined as following:
\[
O^* = \{(x, 0, 1) : x \in X\}
\]
\[
U^* = \{(x, 0, 0) : x \in X\}
\]

Definition 1.4. [4] Let $A \in IFS(X)$. Then $(\alpha, \beta)$–cut of $A$ is a crisp subset $C_{\alpha,\beta}(A)$ of the IFS $A$ is given by
\[
C_{\alpha,\beta}(A) = \{x : x \in X \text{ such that } \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}
\]
where $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$.

2. Sheet and Block Cut Intuitionistic Fuzzy Level Sets

Definition 2.1. Let $X$ be a set and $A = \{<x, \mu_A(x), \nu_A(x)> : x \in X\} \in IFS(X)$. If $t \in [0, 1]$ then sheet $t$–cut of $A$ defined as following
\[
A(t) = \{(x, \mu_A(x), \nu_A(x)) : \mu_A(x) + \nu_A(x) = t, x \in X\}
\]

Proposition 2.1. Let $X$ be a set and $A, B \in IFS(X)$. For every $t \in [0, 1]$,
\[
(1) \quad (A \cup B)(t) = A(t) \cup B(t)
\]
\[
(2) \quad A(t) \cap B(t) = (A \cap B)(t)
\]
\[
(3) \quad (A^c(t))^c = A(t)
\]
Proof. (1)  
\[
A(t) \sqcup B(t) = \left\{ \langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle : \nu_A(x) + \nu_A(x) = t \land \mu_B(x) + \nu_B(x) = t, x \in X \right\}
\]

If \( \mu_A(x) \geq \mu_B(x) \) then from \( \mu_A(x) + \nu_A(x) = t = \mu_B(x) + \nu_B(x) \) we obtain \( \nu_A(x) \leq \nu_B(x) \).

\[
\max(\mu_A(x), \mu_B(x)) + \min(\nu_A(x), \nu_B(x)) = \mu_A(x) + \nu_A(x) = t
\]

If \( \mu_A(x) \leq \mu_B(x) \) then from \( \mu_A(x) + \nu_A(x) = t = \mu_B(x) + \nu_B(x) \) we obtain \( \nu_A(x) \leq \nu_B(x) \).

\[
\max(\mu_A(x), \mu_B(x)) + \min(\nu_A(x), \nu_B(x)) = \mu_B(x) + \nu_B(x) = t
\]

Thence, \( (A \sqcup B)(t) = A(t) \sqcup B(t) \).

(2)  
\[
A(t) \cap B(t) = \left\{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle : \nu_A(x) + \nu_A(x) = t \land \mu_B(x) + \nu_B(x) = t, x \in X \right\}
\]

If \( \mu_A(x) \geq \mu_B(x) \) then from \( \mu_A(x) + \nu_A(x) = t = \mu_B(x) + \nu_B(x) \) we obtain \( \nu_A(x) \leq \nu_B(x) \).

\[
\min(\mu_A(x), \mu_B(x)) + \max(\nu_A(x), \nu_B(x)) = \mu_B(x) + \nu_B(x) = t
\]

If \( \mu_A(x) \leq \mu_B(x) \) then from \( \mu_A(x) + \nu_A(x) = t = \mu_B(x) + \nu_B(x) \) we obtain \( \nu_A(x) \leq \nu_B(x) \).

\[
\min(\mu_A(x), \mu_B(x)) + \max(\nu_A(x), \nu_B(x)) = \mu_A(x) + \nu_A(x) = t
\]

Therefore, we obtain that \( A(t) \cap B(t) = (A \cap B)(t) \).

(3) It is clear. \(\square\)

Remark 2.1. Let \( X \) be a set and \( A \in IFS(X) \). \( A(t) \) is a fuzzy set on \([0, t]\).

Proposition 2.2. Let \( X \) be a set and \( A \in IFS(X) \). If \( t, s \in [0, 1] \) then

Either \( A(t) \cap A(s) = O^* \) or \( t = s \)

Proof. If \( A(t) \cap A(s) \neq O^* \) and \( t \neq s \) then there exists \( x \in X \),

\[
\mu_A(x) + \nu_A(x) = t \quad \text{and} \quad \mu_A(x) + \nu_A(x) = s
\]

\[
\Rightarrow \quad t = s
\]

\(\square\)

Corollary 2.1. There exist an equivalence relation on \( X \) such that the sheet \( t \)-cuts are equivalence class of that relation.

Definition 2.2. Let \( X \) be a set and \( A \in IFS(X) \). If \( t \in [0, 1] \) and \( \alpha \in [0, t] \) then

\[
A(t)_\alpha = \{ x : x \in X, A(t)(x) \geq (\alpha, t - \alpha) \}
\]

is called \( \alpha - t \) block cut of \( A \).

From definitions, it is easily seen that for every \( t \in [0, 1] \), \( A(t) \in FS(X) \). Because \( A(t) : X \rightarrow [0, t] \) and \([0, t] \sim [0, 1] \). For short notation, if \( A(t) : X \rightarrow [0, t] \) then \( A(t) \) will be called \( t \)-fuzzy set on \( X(A(t) \in F_{S1}(X)) \). It is clear that \( A(t)_\alpha \) is a crisp set.

Proposition 2.3. Let \( X \) be a set and \( A \in IFS(X) \). If \( t \in [0, 1] \) then

1. \( A(t)_1 = \{ x : x \in X, \mu_A(x) = t \land \nu_A(x) = 0 \} \)
2. \( A(t)_0 = \{ x : x \in X, \mu_A(x) \geq 0 \land \nu_A(x) \leq t \} \)
Example 2.1. Let $X = \{a, b, c, d, e\}$ and $A = \{(a, 0.5, 0.4), (b, 0.2, 0.3), (c, 0.5, 0.3), (d, 0.4, 0.4), (e, 0.4, 0.1)\}.$

1. $A(0.5) = \{c\}$ but $C_{0.3,0.5}(A) = \{a, c, d, e\}$ and $C_{0.5,0.3}(A) = \{c\}.$
2. $A(0.8) = \{c\}$ but $0.8 + 0.5 > 1$ so, we can not obtain $C_{0.5,0.8}(A)$ or $C_{0.8,0.5}(A)$.

Example 2.2. Let $X = \{a, b, c, d, e\}$ and $A = \{(a, 0.1, 0.2), (b, 0.4, 0.3), (c, 0.6, 0.2), (d, 0.7, 0.1), (e, 0.2, 0.5)\}.$

$A(0.3) = \emptyset$ but $C_{0.2,0.3}(A) = \{b, c, d\}$ and $C_{0.3,0.2}(A) = \{c, d\}.$

That is seen from the examples, $(\alpha, \beta)$–cut of an intuitionistic fuzzy set $A$ and $\alpha - t$ block cut of $A$ are different sets. For all $t \in [0, 1]$ and $\alpha \in [0, t]$, we can determine $\alpha - t$ block cut of $A$, if $\alpha + t > 1$ then we can not determine $(\alpha, \beta)$–cut of $A$. Consequently, $\alpha - t$ block cut of an intuitionistic fuzzy set allows a more extensive studying area.

Proposition 2.4. Let $X$ be a set and $A \in IFS(X)$. If $t \in [0, 1]$ and $\alpha, \beta \in [0, t]$ such that $\alpha \leq \beta$ then $A(t)_{\beta} \subseteq A(t)_{\alpha}$.

Proof. Let $\alpha \leq \beta$. If $x \in A(t)_{\beta}$ then

$$A(t)(x) \geq (\beta, t - \beta) \geq (\alpha, t - \alpha)$$

Therefore $x \in A(t)_{\alpha}$. \hfill $\square$

Proposition 2.5. Let $X$ be a set and $A, B \in IFS(X)$. If $t \in [0, 1]$ and $\alpha \in [0, t]$ then

1. $A(t)_{\alpha} \cup B(t)_{\alpha} = (A(t) \cup B(t))_{\alpha}$
2. $A(t)_{\alpha} \cap B(t)_{\alpha} = (A(t) \cap B(t))_{\alpha}$
Proof. (1)

\[ x \in A(t)_\alpha \cup B(t)_\alpha \Leftrightarrow A(t)(x) \geq (\alpha, t - \alpha) \lor B(t)(x) \geq (\alpha, t - \alpha) \]
\[ \Leftrightarrow (\mu_{A(t)}(x) \geq \alpha \land \nu_{A(t)}(x) \leq t - \alpha) \lor (\mu_{B(t)}(x) \geq \alpha \land \nu_{B(t)}(x) \leq t - \alpha) \]
\[ \Leftrightarrow (\mu_{A(t)}(x) \geq \alpha \lor \mu_{B(t)}(x) \geq \alpha) \land (\nu_{A(t)}(x) \leq t - \alpha \lor \nu_{B(t)}(x) \leq t - \alpha) \]
\[ \Leftrightarrow (\mu_{A(t)}(x) \lor \mu_{B(t)}(x)) \geq \alpha \land (\nu_{A(t)}(x) \lor \nu_{B(t)}(x)) \leq t - \alpha \]
\[ \Rightarrow \mu_{A(t) \cup B(t)}(x) \geq \alpha \land \nu_{A(t) \cup B(t)}(x) \leq t - \alpha \]
\[ \Rightarrow x \in (A(t) \cup B(t))_\alpha \]

(2)

\[ A(t)_\alpha \cap B(t)_\alpha = \{ x \in X : A(t)(x) \geq (\alpha, t - \alpha) \land B(t)(x) \geq (\alpha, t - \alpha) \} \]
\[ = \{ x \in X : (\mu_{A(t)}(x) \geq \alpha \land \nu_{A(t)}(x) \leq t - \alpha) \land (\mu_{B(t)}(x) \geq \alpha \land \nu_{B(t)}(x) \leq t - \alpha) \} \]
\[ = \{ x \in X : (\mu_{A(t)}(x) \land \mu_{B(t)}(x)) \geq \alpha \land (\nu_{A(t)}(x) \lor \nu_{B(t)}(x)) \leq t - \alpha \} \]
\[ = \{ x \in X : \mu_{A(t) \cap B(t)}(x) \geq \alpha \land \nu_{A(t) \cap B(t)}(x) \leq t - \alpha \} \]
\[ = (A(t) \cap B(t))_\alpha \]

(3)

\[ (A(t)^c)_\alpha = \{ x \in X : A(t)^c(x) \geq (\alpha, t - \alpha) \} \]
\[ = \{ x \in X : \nu_{A(t)}(x) \geq \alpha \land \mu_{A(t)}(x) \leq t - \alpha \} \]
\[ = \{ x \in X : x - \nu_{A(t)}(x) \leq t - \alpha \land \mu_{A(t)}(x) \geq \alpha \} \]
\[ = \{ x \in X : (t - A(t))(x) \geq (\alpha, t - \alpha) \} \]
\[ = t - A(t)_\alpha \]

\[ \square \]

REFERENCES


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