

## SOME CLASSIFICATIONS OF AN X FDK-SPACE

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ABSTRACT. In this paper, we introduce definitions of double wedge and double conull FDK-spaces. Also, we give some important corollary for any double sequence space about these definitions.

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### 1. INTRODUCTION

In [4], the first study on double sequences was examined by Bromwich. And than it was investigated by many authors such as Hardy [6], Moricz [7], Tripathy [16], Başarır and Sonalcan [2]. The notion of regular convergence for double sequences was defined by Hardy [6]. After that both the theory of topological double sequence spaces and the theory of summability of double sequences were studied by Zeltser [17]. The statistical and Cauchy convergence for double sequences were examined by Mursaleen and Edely [8] and Tripathy [15] in recent years. Many recent improvements containing the summability by four dimensional matrices might be found in [10].

$\Omega$  denotes the space of all complex valued double sequences which is a vector space with coordinatewise addition and scalar multiplication. Any vector subspace of  $\Omega$  is called as a double sequence space. The space  $\mathcal{M}_u$  of all bounded double sequences is defined by

$$\mathcal{M}_u := \left\{ x \in \Omega : \|x\|_\infty := \sup_{k,l} |x_{kl}| < \infty \right\},$$

which is a Banach space with the norm  $\|\cdot\|_\infty$ . In addition we consider the double sequence spaces

$$\begin{aligned} \Phi &:= \text{span}\{e^{kl} : k, l \in \mathbb{N}\} = \{x \in \Omega : \exists N_0 \in \mathbb{N}, \forall (k, l) \in \mathbb{N}^2/[1, N_0]^2 : x_{kl} = 0\} \\ \Phi_1 &:= \Phi \cup \{e\} \\ \mathcal{L}_u &:= \left\{ x \in \Omega : \sum_{k,l} |x_{kl}| < \infty \right\} \end{aligned}$$

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$$\begin{aligned} \mathcal{BV} &:= \left\{ x \in \Omega : \|x\|_{\mathcal{BV}} := \sum_{k,l} |x_{kl} - x_{k+1,l} - x_{k,l+1} + x_{k+1,l+1}| < \infty \right\} \\ \mathcal{C}_0 &:= \left\{ x \in \mathcal{C} : \forall l \in \mathbb{N} : \lim_k x_{kl} = 0 \right\} \\ \mathcal{C}_c &:= \left\{ x \in \Omega : \forall l \in \mathbb{N} : \lim_{k,l} x_{kl} \text{ exist} \right\}. \end{aligned}$$

Throughout this paper  $e$  denotes the double sequence of ones;  $(\delta^{ij})$ ,  $i, j = 1, 2, \dots$ , with the one in the  $(i, j)$  position.

A subspace  $E$  of the vector space  $\Omega$  is called DK-space, if all the seminorms  $r_{kl} : E \rightarrow \mathbb{R}, x \mapsto |x_{kl}|$  ( $k, l \in \mathbb{N}$ ) are continuous. An FDK space is a DK-space with a complete, metrizable, locally convex topology. A normable FDK-space is called BDK-space.

## 2. MAIN RESULTS

In this section conull (strongly conull) and double wedge (weak double wedge) FDK-spaces are defined and several characterizations are given.

**Definition 2.1.** If  $(E, \tau)$  is a FDK-space containing  $\Phi$ , and  $\delta^{ij} \rightarrow 0$  in  $\tau$ , then  $(E, \tau)$  is called a double wedge space.

**Definition 2.2.** If  $(E, \tau)$  is a FDK-space containing  $\Phi$ , and  $\delta^{ij} \rightarrow 0$  (weakly) in  $\tau$ , then  $(E, \tau)$  is called a weak double wedge space.

**Definition 2.3.** Let  $(E, \tau)$  is an FDK-space containing  $\Phi_1$ . If  $\forall f \in E'$ ,

$$f(e) = \lim_{m,n \rightarrow \infty} \sum_{k,l=1}^{m,n} f(\delta^{kl})$$

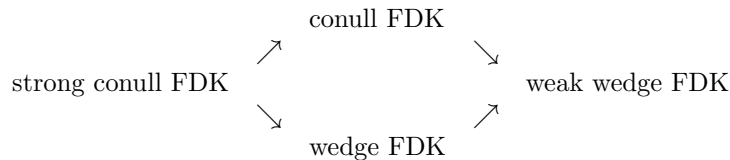
then  $(E, \tau)$  is called a conull FDK-space.

**Definition 2.4.** Let  $(E, \tau)$  is an FDK-space containing  $\Phi_1$ . If  $\forall f \in E'$ ,

$$e = \lim_{m,n \rightarrow \infty} \sum_{k,l=1}^{m,n} \delta^{kl}$$

then  $(E, \tau)$  is called a strong conull FDK-space.

Clearly, each strong conull (wedge) FDK-space is a conull (weak wedge) FDK-space and the following diagram is hold:



Indeed, let  $E$  be a strong conull FDK-space. Then we have  $e^{(kl)} \rightarrow e$ . Hence  $P(e^{(kl)} - e) \rightarrow 0$  ( $k, l \rightarrow \infty$ ) for the seminorm  $P$  in  $\tau$ . In this case, the following

equation is hold

$$\begin{aligned}
\delta^{kl} &= e^{(k,l)} - e^{(k-1,l-1)} - \sum_{i=k,j=1}^{k,l} \delta^{ij} - \sum_{i=1,j=l}^{k,l} \delta^{ij} \\
&= e^{(k,l)} - e^{(k-1,l-1)} - \sum_{i=k,j=1}^{k,l} \delta^{ij} - \sum_{i=1,j=l}^{k,l} \delta^{ij} + e^{(k-1,l-1)} - e^{(k-1,l-1)} \\
&= e^{(k,l)} - e^{(k,l-1)} - e^{(k-1,l)} + e^{(k-1,l-1)} + e - e + e - e .
\end{aligned}$$

So, we have

$$\begin{aligned}
P(\delta^{kl}) &= P\left(e^{(k,l)} - e^{(k,l-1)} - e^{(k-1,l)} + e^{(k-1,l-1)} + e - e + e - e\right) \\
&\leq P\left(e^{(k,l)} - e\right) + P\left(e^{(k,l-1)} - e\right) + P\left(e^{(k-1,l)} - e\right) + P\left(e^{(k-1,l-1)} - e\right).
\end{aligned}$$

It is clearly that since  $P(\delta^{kl}) \rightarrow 0$ ,  $k, l \rightarrow \infty$ ,  $E$  is double wedge space.

We recall that the  $\alpha$ -dual of a subset  $E$  of  $\Omega$  is defined to be ([14])

$$E^\alpha := \left\{ x = (x_{kl}) : \sum_{k,l=1}^{\infty, \infty} |x_{kl} y_{kl}| < \infty, \forall y = (y_{kl}) \in E \right\}.$$

**Lemma 2.1.** *If  $z^{(mn)} \in \mathcal{C}_0$ ,  $m, n = 1, 2, \dots$ , then there exists  $z \in \mathcal{C}_0$  such that*

$$\lim_{i,j \rightarrow \infty} \frac{z_{ij}^{(mn)}}{z_{ij}} = 0 \quad (m, n = 1, 2, \dots).$$

Furthermore, for any such  $z$ , we have  $z^\alpha \subseteq \bigcap_{m,n=1}^{\infty, \infty} \{z^{(mn)}\}^\alpha$ .

*Proof.* Let  $z^{(mn)} \in \mathcal{C}_0$ . We can choose two sequences  $(i_k)$ ,  $(j_l)$  of positive integers such that

$$1 = i_0 < i_1 < i_2 < \dots \quad 1 = j_0 < j_1 < j_2 < \dots$$

and

$$\max_{\substack{1 \leq m \leq k \\ 1 \leq n \leq l}} |z_{ij}^{(mn)}| < \frac{1}{4^{kl}} \quad \left( \begin{array}{l} i \geq i_k, j \geq j_l \\ k, l = 1, 2, \dots \end{array} \right).$$

Define  $z \in \Omega$  as follows:

$$z_{ij} = \frac{1}{2^{kl}} \quad \left( \begin{array}{l} i_k \leq i < i_{k+1}, j_l \leq j < j_{l+1} \\ k, l = 0, 1, 2, \dots \end{array} \right).$$

Clearly,  $z \in \mathcal{C}_0$  and, fixing  $m, n$

$$\left| \frac{z_{ij}^{(mn)}}{z_{ij}} \right| < \frac{1}{2^{kl}} \quad \left( \begin{array}{l} i \geq i_k, j \geq j_l \\ k \geq m, l \geq n \end{array} \right).$$

Thus,  $\lim_{i,j \rightarrow \infty} \frac{z_{ij}^{(mn)}}{z_{ij}} = 0 \quad \forall m, n$ .

Now let  $x \in z^\alpha$ , then

$$\sum_{i,j=1}^{\infty, \infty} |x_{ij} z_{ij}| < \infty.$$

Moreover, for  $m, n = 1, 2, \dots$  we have

$$\begin{aligned} \sum_{i,j=1}^{\infty, \infty} |x_{ij} z_{ij}^{(mn)}| &< \sum_{i,j=1}^{\infty, \infty} |x_{ij} z_{ij}| \frac{1}{2^{kl}} \\ &< \frac{1}{2^{kl}} \sum_{i,j=1}^{\infty, \infty} |x_{ij} z_{ij}| < \infty. \end{aligned}$$

Hence,  $x \in \bigcap_{m,n=1}^{\infty, \infty} \{z^{(mn)}\}^\alpha$ . The proof is completed.  $\square$

$s = (s_m), t = (t_n)$  always denote strictly increasing of nonnegative integers with  $s_1 = 0, t_1 = 0$ . We will be interested in spaces of the form:

$$m|(s, t)| = \left\{ x \in \Omega : \sup_{m,n} \sum_{\substack{k=s_m+1 \\ l=t_n+1}}^{s_{m+1}, t_{n+1}} |x_{kl}| < \infty \right\}$$

which becomes a BDK-space under the norm:

$$x \rightarrow \sup_{m,n} \sum_{\substack{k=s_m+1 \\ l=t_n+1}}^{s_{m+1}, t_{n+1}} |x_{kl}|.$$

**Theorem 2.1.** *Let  $(E, \tau)$  be an FDK-space. These are equivalent:*

- i)  $E$  is a double wedge space,
- ii)  $E$  contains  $z^\alpha$  for some  $z \in \mathcal{C}_0$ ,
- iii)  $E$  contains  $m|(s, t)|$  for some  $s, t$  and the inclusion mapping is compact,
- iv)  $E$  contains  $\mathcal{L}_u$  and the inclusion mapping is compact.

*Proof.* (i  $\Rightarrow$  ii) Let  $\{p_{mn}\}$  be a defining family of seminorms for the topology  $\tau$  and let

$$z_{ij}^{(mn)} = p_{mn}(\delta^{ij}) \quad (m, n, i, j = 1, 2, \dots).$$

Then  $z^{(mn)} \in \mathcal{C}_0$ ,  $m, n = 1, 2, \dots$ , since  $E$  is a double wedge space. Suppose  $y \in \bigcap_{m,n=1}^{\infty, \infty} \{z^{(mn)}\}^\alpha$ , then for each  $m, n$

$$\sum_{i,j} |y_{ij} z_{ij}^{(mn)}| < \infty.$$

Therefore,

$$\sum_{i,j} |y_{ij} p_{mn}(\delta^{ij})| = \sum_{i,j} p_{mn}(y_{ij} \delta^{ij}) < \infty$$

is obtained. Since the space  $E$  is complete  $\sum_{i,j} y_{ij} \delta^{ij}$  converges in  $(E, \tau)$  to, say  $x$ , or  $y^{(mn)} \rightarrow x$ . Thus  $y_{ij}^{(mn)} \rightarrow x_{ij}$  for each  $i, j$ ; also we always have  $y_{ij}^{(mn)} \rightarrow y_{ij}$  for each  $i, j$ . Consequently  $y = x$ ; that is

$$\bigcap_{m,n=1}^{\infty, \infty} \{z^{(mn)}\}^\alpha \subseteq E.$$

Choosing  $z$  as in Lemma 2.1, (ii) follows.

(ii  $\Rightarrow$  iii) Let us choose strictly increasing sequences  $(s_m), (t_n)$  of positive integers such that  $s_1 = 0, t_1 = 0$  and

$$|z_{ij}| \leq \frac{1}{2^{mn}}$$

whenever  $i \geq s_m, j \geq t_n, m, n \geq 2$ .

For  $x \in m|(s, t)|$  and any positive integers  $k, l, u, v$  such that  $l \geq k$  and  $v \geq u$  we have

$$\sum_{\substack{i=s_k+1 \\ j=t_p+1}}^{s_{l+1}, t_{v+1}} |x_{ij}z_{ij}| = \sum_{m=k, n=u}^{l, v} \sum_{\substack{i=s_m+1 \\ j=t_n+1}}^{s_{m+1}, t_{n+1}} |x_{ij}z_{ij}| \leq \|x\| \sum_{m=k, n=u}^{l, v} \frac{1}{2^{mn}}.$$

Hence  $x \in z^\alpha$ . That is;  $m|(s, t)| \subseteq z^\alpha \subseteq E$ . Also, the inclusion theorem  $i : (m|(s, t)|, \|\cdot\|) \rightarrow (E, \tau)$  is compact.

(iii  $\Rightarrow$  iv) Since  $\mathcal{L}_u \subset m|(s, t)|$  always true, the inclusion theorem  $i : \mathcal{L}_u \rightarrow m|(s, t)|$  is continuous. By hypothesis  $\mathcal{L}_u \subset m|(s, t)| \subset E$  and because of  $i : (m|(s, t)|, \|\cdot\|) \rightarrow (E, \tau)$  is compact  $i^* : \mathcal{L}_u \rightarrow E$  is compact.  $\square$

**Corollary 2.1.** *The intersection of all double wedge FDK-spaces is  $\mathcal{L}_u$ .*

*Proof.* Let  $E$  be a double wedge FDK-space. Then we have

$$\bigcap E = \bigcap \{z^\alpha : z \in \mathcal{C}_0\} = \mathcal{C}_0^\alpha.$$

Now we need show that  $\mathcal{C}_0^\alpha = \mathcal{L}_u$ . Since  $\mathcal{C}_0 \subset \mathcal{M}_u, \mathcal{M}_u^\alpha \subset \mathcal{C}_0^\alpha$ . That is,  $\mathcal{L}_u = \mathcal{M}_u^\alpha \subset \mathcal{C}_0^\alpha$  is obtained. Suppose that  $x = (x_{kl}) \in \mathcal{C}_0^\alpha$  but  $x = (x_{kl}) \notin \mathcal{L}_u$ . Then for  $y = (y_{kl}) = \frac{1}{k!l!} \in \mathcal{C}_0$  we have

$$\sum |x_{kl}y_{kl}| = \sum |x_{kl} \frac{1}{k!l!}| = e^2 \sum |x_{kl}| = \infty.$$

This means that  $x = (x_{kl}) \notin \mathcal{C}_0^\alpha$  which contradicts the hypothesis. So,  $x = (x_{kl})$  must be in  $\mathcal{L}_u$ . That is,  $\mathcal{L}_u \subset \mathcal{C}_0^\alpha$ . This completes the proof.  $\square$

The one to one mapping  $S^{(2)}$  of  $\Omega$  to itself defined by

$$S^{(2)}x = \begin{pmatrix} x_{11} & x_{11} + x_{12} & \dots & \dots & \dots \\ x_{11} + x_{21} & x_{11} + x_{12} + x_{21} + x_{22} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \sum_{k,l=1}^{m,n} x_{kl} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

$$(S^{(2)})^{-1}x = \begin{pmatrix} x_{11} & x_{12} - x_{11} & \dots & \dots & \dots \\ x_{21} - x_{11} & x_{22} - x_{12} - x_{21} + x_{11} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & x_{mn} - x_{m,n-1} - x_{m-1,n} + x_{m-1,n-1} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

**Theorem 2.2.** *i)  $(E, \tau)$  is strongly conull FDK-space if and only if the space  $(S^{(2)})^{-1}(E)$  is a double wedge FDK-space.*

*ii)  $(E, \tau)$  is conull FDK-space if and only if the space  $(S^{(2)})^{-1}(E)$  is a weak double wedge FDK-space.*

*Proof.* i) Let  $(E, \tau)$  is strongly conull FDK-space and  $\{P_{mn}\}$  is a set of seminorms on the topology  $\tau$ . Then a topology with the set of seminorms  $\{q_{mn}\}$  make  $(S^{(2)})^{-1}(E)$  is a FDK-space such that

$$q_{mn}(x) := P_{mn}(S^{(2)}(x)).$$

Since  $(E, \tau)$  is strongly conull FDK-space, for all  $m, n \in \mathbb{N}$   $P_{mn}(e - e^{(mn)}) \rightarrow 0$ . Otherwise,

$$S^{(2)}x = e - e^{(mn)} = \sum_{\substack{k=m+1 \\ l=n+1}}^{\infty, \infty} \delta^{kl} + \sum_{\substack{k=1 \\ l=n+1}}^{m, \infty} \delta^{kl} + \sum_{\substack{k=m+1 \\ l=1}}^{\infty, n} \delta^{kl}$$

is hold. Thus we have

$$\begin{aligned} x &= (S^{(2)})^{-1} \left( \sum_{\substack{k=m+1 \\ l=n+1}}^{\infty, \infty} \delta^{kl} + \sum_{\substack{k=1 \\ l=n+1}}^{m, \infty} \delta^{kl} + \sum_{\substack{k=m+1 \\ l=1}}^{\infty, n} \delta^{kl} \right) \\ &= (S^{(2)})^{-1} \begin{pmatrix} 0 & \dots & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 1 & \dots & 1 \\ 1 & \dots & 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & \dots & \dots & \dots & \dots & 1 \end{pmatrix} \\ &= \delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1}. \end{aligned}$$

From the definition of  $(q_{mn})$  the following equation is obtained:

$$q_{mn}(\delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1}) = P_{mn} \left( e - \sum_{k,l=1}^{m,n} \delta^{kl} \right).$$

Since  $P_{mn} \left( e - \sum_{k,l=1}^{m,n} \delta^{kl} \right) \rightarrow 0$  ( $m, n \rightarrow \infty$ ), we can say  $q_{mn}(\delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1}) \rightarrow 0$  ( $m, n \rightarrow \infty$ ).

In this case, we have  $(\delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1}) \rightarrow 0$  ( $m, n \rightarrow \infty$ ) according to the topology of the space  $(S^{(2)})^{-1}(E)$ . Since  $\delta^{m+1,1} \rightarrow 0$  and  $\delta^{1,n+1} \rightarrow 0$ ,  $\delta^{m+1,n+1} \rightarrow 0$  is hold. This gives that  $(S^{(2)})^{-1}(E)$  is a double wedge FDK-space.

Now we suppose that  $(S^{(2)})^{-1}(E)$  is a double wedge FDK-space. Then we have for all  $m, n \in \mathbb{N}$   $q_{mn}(\delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1}) \rightarrow 0$  ( $m, n \rightarrow \infty$ ). Since

$$q_{mn}(\delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1}) = P_{mn} \left( e - \sum_{k,l=1}^{m,n} \delta^{kl} \right),$$

$P_{mn} \left( e - \sum_{k,l=1}^{m,n} \delta^{kl} \right) \rightarrow 0$  ( $m, n \rightarrow \infty$ ) is obtained. So  $E$  strongly conull FDK-space.

ii) Let  $(E, \tau)$  is conull FDK-space and let us define the topology of  $(S^{(2)})^{-1}(E)$  as the proof of (i). Then  $q_{mn}(x) := P_{mn}(S^{(2)}(x))$  and since  $(S^{(2)})^{-1} \left( e - \sum_{k,l=1}^{m,n} \delta^{kl} \right) = \delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1}$ , we have

$$q_{mn}(\delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1}) = P_{mn} \left( e - \sum_{k,l=1}^{m,n} \delta^{kl} \right).$$

Because of  $E$  is conull FDK space,  $P_{mn} \left( e - \sum_{k,l=1}^{m,n} \delta^{kl} \right) \rightarrow 0$  (weak) ( $m, n \rightarrow \infty$ ). Hence  $\delta^{m+1,1} + \delta^{1,n+1} - \delta^{m+1,n+1} \rightarrow 0$  (weak) ( $m, n \rightarrow \infty$ ). Consequently  $\delta^{m+1,n+1} \rightarrow 0$  (weak) ( $m, n \rightarrow \infty$ ) is obtained. That is,  $(S^{(2)})^{-1}(E)$  is a weak double wedge FDK-space. The other hand of the proof is as the proof of (i).  $\square$

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