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SOME RESULTS ON $S_{\alpha,\beta}$ AND $T_{\alpha,\beta}$ INTUITIONISTIC FUZZY MODAL OPERATORS

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Abstract. In 1999, first Intuitionistic Fuzzy Modal Operators introduced in[2]. Expansion of these operators and new operators defined by different authors[3, 5, 6, 7, 8, 9]. Characteristics of these operators has been studied by several researchers.

In this study, we obtained new results on modal operators which are called $S_{\alpha,\beta}$ and $T_{\alpha,\beta}$.

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1. Introduction

The concept of Intuitionistic fuzzy sets was introduced by Atanassov in 1986 [1], form an extension of fuzzy sets[10] by expanding the truth value set to the lattice $[0,1]\times[0,1]$.

Intuitionistic fuzzy modal operators defined firstly by Atanassov[1, 2]. Then severel extensions of these operators introduced by different authors[2, 8, 5, 6]. Some algebraic and characteristic properties of these operators were studied by several authors.

Definition 1.1. [1] An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$$

where $\mu_A(x),(\mu_A : X\to[0,1])$ is called the "degree of membership of x in A ", $\nu_A(x),(\nu_A : X\to[0,1])$is called the " degree of non- membership of x in A ",and where μ_A and ν_A satisfy the following condition:

$$\mu_A(x)+\nu_A(x)\leq 1, \text{ for all } x\in X.$$

The class of intuitionistic fuzzy sets on X is denoted by $IFS(X)$.

The hesitation degree of x is defined by $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$

Definition 1.2. [1]An IFS A is said to be contained in an IFS B (notation $A\sqsubseteq B$) if and only if, for all $x\in X:\mu_A(x)\leq\mu_B(x)$ and $\nu_A(x)\geq\nu_B(x)$.

It is clear that $A=B$ if and only if $A\sqsubseteq B$ and $B\sqsubseteq A$.

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Definition 1.3. [1]Let $A \in IFS$ and let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ then the above set is callede the complement of A

$$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$$

The intersection and the union of two IFSs A and B on X is defined by

$$A \sqcap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$$

$$A \sqcup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$$

The notion of Second Type Intuitionistic Fuzzy Modal Operators was firstly introduced by Atanassov as following:

Definition 1.4. [1]Let X be universal and $A \in IFS(X)$ then

(1) $\Box(A) = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$
(2) $\Diamond(A) = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle : x \in X\}$

Definition 1.5. [2]Let X be universal and $A \in IFS(X)$, $\alpha \in [0, 1]$ then

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1 - \alpha)\pi_A(x)\rangle : x \in X\}$$

Definition 1.6. [2]Let X be universal and $A \in IFS(X)$, $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$ then

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x)\rangle : x \in X\}$$

Definition 1.7. [2]Let X be universal and $A \in IFS(X)$, $\alpha, \beta \in [0, 1]$ then

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha\mu_A(x), \beta\nu_A(x)\rangle : x \in X\}$$

Definition 1.8. [2]Let X be universal and $A \in IFS(X)$, $\alpha, \beta \in [0, 1]$ then

(1) $H_{\alpha,\beta}(A) = \{\langle x, \alpha\mu_A(x), \nu_A(x) + \beta\pi_A(x)\rangle : x \in X\}$
(2) $H^*_{\alpha,\beta}(A) = \{\langle x, \alpha\mu_A(x), \nu_A(x) + \beta(1 - \alpha\mu_A(x) - \nu_A(x))\rangle : x \in X\}$
(3) $J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha\pi_A(x), \beta\nu_A(x)\rangle : x \in X\}$
(4) $J^*_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha(1 - \mu_A(x) - \beta\nu_A(x)), \beta\nu_A(x)\rangle : x \in X\}$

The simplest One Type Intuitionistic Fuzzy Modal Operators defined in 1999.

Definition 1.9. [2] Let X be a set and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \in IFS(X), \alpha, \beta \in [0, 1]$.

(1) $\boxplus A = \left\{\left\langle x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2}\right\rangle : x \in X\right\}$
(2) $\boxtimes A = \left\{\left\langle x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2}\right\rangle : x \in X\right\}$

After this definition, in 2001, the extension of these operators were defined as following:

Definition 1.10. [3] Let X be a set and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \in IFS(X)$, $\alpha, \beta \in [0, 1]$.

(1) $\boxplus_\alpha A = \{\langle x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha\rangle : x \in X\}$
(2) $\boxtimes_\alpha A = \{\langle x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x)\rangle : x \in X\}$

The operators $\boxplus_\alpha$and $\boxtimes_\alpha$ are the extensions of the operators $\boxplus$, $\boxtimes$, resp.
In 2004, Dencheva introduced the second extension of $\boxplus_\alpha$and $\boxtimes_\alpha$.

Definition 1.11. [8] Let X be a set and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \in IFS(X)$, $\alpha, \beta \in [0, 1]$.

(1) $\boxplus_{\alpha,\beta}A = \{\langle x, \alpha\mu_A(x), \alpha\nu_A(x)+\beta\rangle : x \in X\}$ where $\alpha+\beta \in [0,1]$.
(2) $\boxtimes_{\alpha,\beta}A = \{\langle x, \alpha\mu_A(x)+\beta, \alpha\nu_A(x)\rangle : x \in X\}$where $\alpha+\beta \in [0,1]$.

In 2006, the third extension of the above operators was studied by author . He defined the following operators;

Definition 1.12. [3]Let X be a set and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \in IFS(X)$.

(1) $\boxplus_{\alpha,\beta,\gamma}(A) = \{\langle x, \alpha\mu_A(x), \beta\nu_A(x)+\gamma\rangle : x \in X\}$
where $\alpha, \beta, \gamma \in [0,1], \max\{\alpha,\beta\}+\gamma \leqslant 1$.
(2) $\boxtimes_{\alpha,\beta,\gamma}(A) = \{\langle x, \alpha\mu_A(x)+\gamma, \beta\nu_A(x)\rangle : x \in X\}$
where $\alpha, \beta, \gamma \in [0,1], \max\{\alpha,\beta\}+\gamma \leqslant 1$.

In 2007, author[5] defined a new operator named $E_{\alpha,\beta}$ and studied some of its properties. This operator as following:

Definition 1.13. [5]Let X be a set and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \in IFS(X)$, $\alpha, \beta \in [0,1]$. We define the following operator:

$$E_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x)+1-\alpha), \alpha(\beta\nu_A(x)+1-\beta)\rangle : x \in X\}$$

At the same year, Atanassov introduced the operator $\boxdot_{\alpha,\beta,\gamma,\delta}$ which is a natural extension of all these operators in [3].

Definition 1.14. [3]Let Xbe a set, $A \in IFS(X)$, $\alpha, \beta, \gamma, \delta \in [0,1]$ such that

$$\max(\alpha,\beta)+\gamma+\delta \leqslant 1$$

then the operator $\boxdot_{\alpha,\beta,\gamma,\delta}$ defined by

$$\boxdot_{\alpha,\beta,\gamma,\delta}(A) = \{\langle x, \alpha\mu_A(x)+\gamma, \beta\nu_A(x)+\delta\rangle : x \in X\}$$

In 2008, most general operator $\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$defined as following:

Definition 1.15. [3]Let X be a set,$A \in IFS(X)$, $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0,1]$ such that

$$\max(\alpha-\zeta, \beta-\varepsilon)+\gamma+\delta \leqslant 1$$

and

$$\min(\alpha-\zeta, \beta-\varepsilon)+\gamma+\delta \geq 0$$

then the operator $\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$ defined by

$$\circledcirc_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(A) = \{\langle x, \alpha\mu_A(x)-\varepsilon\nu_A(x)+\gamma, \beta\nu_A(x)-\zeta\mu_A(x)+\delta\rangle : x \in X\}$$

In 2010, Çuvalcıoğlu[6] defined a new operator which is a generalization of $E_{\alpha,\beta}$.

Definition 1.16. [6]Let X be a set and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \in IFS(X)$, $\alpha, \beta, \omega \in [0,1]$ then

$$Z^{\omega}_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x)+\omega-\omega.\alpha), \alpha(\beta\nu_A(x)+\omega-\omega.\beta)\rangle : x \in X\}$$

Definition 1.17. [6]Let X be a set and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\} \in IFS(X)$, $\alpha, \beta, \omega, \theta \in [0,1]$ then

$$Z^{\omega,\theta}_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x)+\omega-\omega.\alpha), \alpha(\beta\nu_A(x)+\theta-\theta.\beta)\rangle : x \in X\}$$

The operator $Z^{\omega,\theta}_{\alpha,\beta}$ is a generalization of $Z^{\omega}_{\alpha,\beta}$,and also, $E_{\alpha,\beta}, \boxplus_{\alpha,\beta}, \boxtimes_{\alpha,\beta}$.
Uni-type intuitionistic fuzzy modal operators introduced by author as following;

Definition 1.18. [7]Let X be a universal, $A \in IFS(X)$ and $\alpha, \beta, \omega \in [0,1]$ then

(1) $\boxplus^{\omega}_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x)), \alpha(\beta\nu_A(x) + \omega - \omega\beta)\rangle : x \in X\}$
(2) $\boxtimes^{\omega}_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x) + \omega - \omega\alpha), \alpha((1-\beta)\mu_A(x) + \nu_A(x))\rangle : x \in X\}$

Definition 1.19. [7]Let X be a set and $A \in IFS(X)$, $\alpha, \beta, \omega, \theta \in [0,1]$ then

$$E^{\omega,\theta}_{\alpha,\beta}(A) = \left\{ \left\langle \begin{array}{c} x, \beta((1-(1-\alpha)(1-\theta))\mu_A(x) + (1-\alpha)\theta\nu_A(x) + (1-\alpha)(1-\theta)\omega), \\ \alpha((1-\beta)\theta\mu_A(x) + (1-(1-\beta)(1-\theta))\nu_A(x) + (1-\beta)(1-\theta)\omega) \end{array} \right\rangle : x \in X \right\}$$

Definition 1.20. [7]Let X be a set, $A \in IFS(X)$ and $\alpha, \beta \in [0,1]$ then

(1) $B_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x)), \alpha((1-\beta)\mu_A(x) + \nu_A(x))\rangle : x \in X\}$
(2) $\boxminus_{\alpha,\beta}(A) = \{\langle x, \beta(\mu_A(x) + (1-\beta)\nu_A(x)), \alpha((1-\alpha)\mu_A(x) + \nu_A(x))\rangle : x \in X\}$

In 2014, new one type intuitionistic fuzzy modal operators were defined in [9].

Definition 1.21. [9]Let X be a set and $A \in IFS(X), \alpha, \beta, \omega \in [0,1]$ and $\alpha + \beta \leq 1$

(1) $L^{\omega}_{\alpha,\beta}(A) = \{\langle x, \alpha\mu_A(x) + \omega(1-\alpha), \alpha(1-\beta)\nu_A(x) + \alpha\beta(1-\omega)\rangle \; x \in X\}$
(2) $K^{\omega}_{\alpha,\beta}(A) = \{\langle x, \alpha(1-\beta)\mu_A(x) + \alpha\beta(1-\omega), \alpha\nu_A(x) + \omega(1-\alpha)\rangle \; x \in X\}$

As above, we get the following diagram;

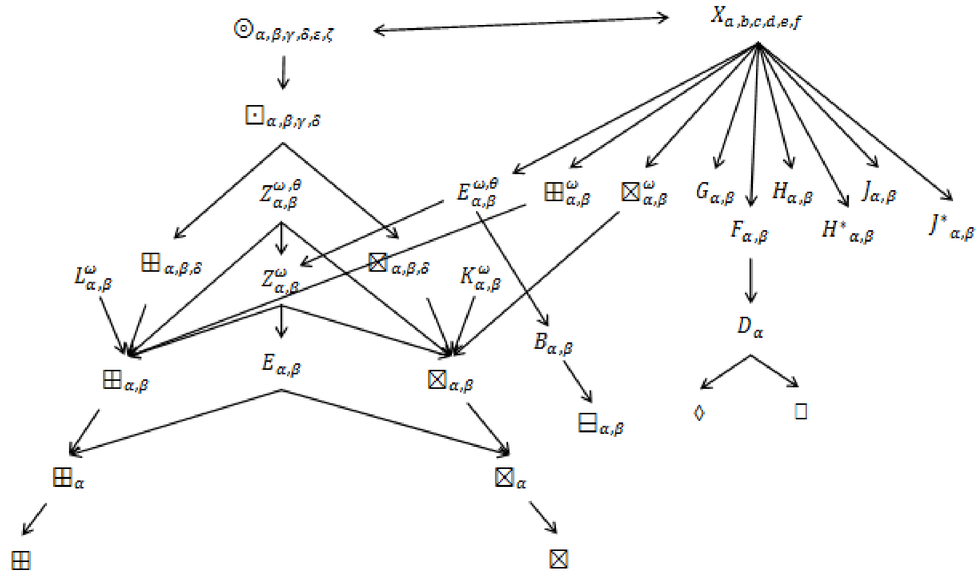

FIGURE 1

The intuitionistic fuzzy modal operator, represented by $\otimes_{\alpha,\beta,\gamma,\delta}$, introduced in 2014 as following;

Definition 1.22. [4]Let X be a set and $A \in IFS(X), \alpha, \beta, \gamma, \delta \in [0,1]$ and $\alpha+\beta \leq 1, \gamma+\delta \leq 1$ then

$$\otimes_{\alpha,\beta,\gamma,\delta}(A) = \{\langle x, \alpha\mu_A(x) + \gamma\nu_A(x), \beta\mu_A(x) + \delta\nu_A(x)\rangle\}$$

2. Some Properties of New Intuitionistic Fuzzy Modal Operators

Definition 2.1. Let X be a set and $A \in IFS(X), \alpha, \beta, \alpha+\beta \in [0,1]$.

(1) $T_{\alpha,\beta}(A) = \{< x, \beta(\mu_A(x) + (1-\alpha)\nu_A(x) + \alpha), \alpha(\nu_A(x) + (1-\beta)\mu_A(x)) >: x \in X\}$ where $\alpha + \beta \in [0,1]$.
(2) $S_{\alpha,\beta}(A) = \{< x, \alpha(\mu_A(x) + (1-\beta)\nu_A(x)), \beta(\nu_A(x) + (1-\alpha)\mu_A(x) + \alpha) >: x \in X\}$ where $\alpha + \beta \in [0,1]$.

It is clear that;

$$\begin{aligned} \beta(\mu_A(x) + (1-\alpha)\nu_A(x) + \alpha) + \alpha(\nu_A(x) + (1-\beta)\mu_A(x)) &= (\mu_A(x) + \nu_A(x))(\alpha + \beta - \alpha\beta) + \alpha\beta \\ &\leq \alpha + \beta - \alpha\beta + \alpha\beta \leq 1 \end{aligned}$$

Theorem 2.1. *Let X be a set and $A \in IFS(X)$. If $\alpha, \beta, \alpha+\beta \in [0,1]$ then $T_{\alpha,\beta}(A)^c = S_{\alpha,\beta}(A^c)$.*

Proof. It is clear from definition. □

Proposition 2.1. *Let X be a set and $A \in IFS(X)$. If $\alpha, \beta, \alpha+\beta \in [0,1]$ then*

(1) $T_{\beta,\alpha}(A)^c \sqsubseteq T_{\alpha,\beta}(A^c)$

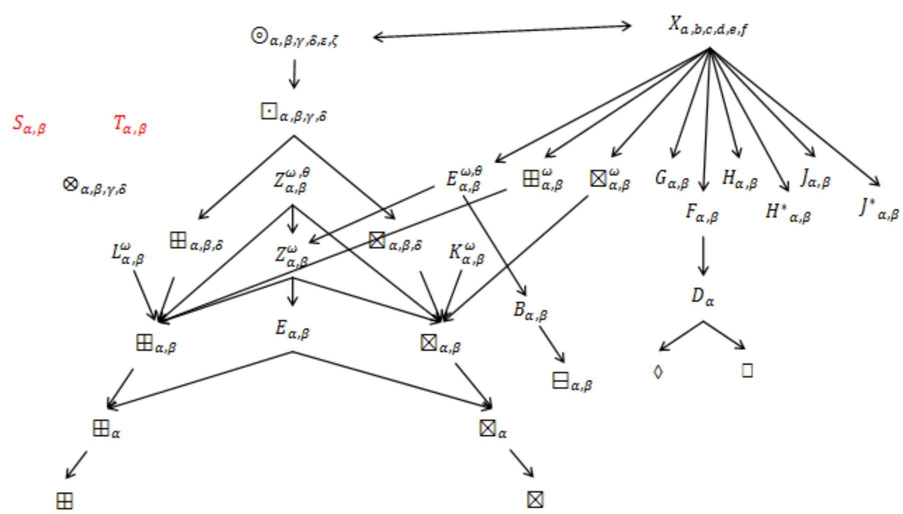

FIGURE 2

(2) $S_{\alpha,\beta}(A^c) \sqsubseteq S_{\beta,\alpha}(A)^c$

Proof. (1)From definition of this operators and complement of an intuitionistic fuzzy set we get that,

$$\beta(\nu_A(x) + (1-\alpha)\mu_A(x)) \leq \beta(\nu_A(x) + (1-\alpha)\mu_A(x) + \alpha)$$

and

$$\alpha(\mu_A(x) + (1-\beta)\nu_A(x) + \beta) \geq \alpha(\mu_A(x) + (1-\beta)\nu_A(x))$$

So, we can say $T_{\beta,\alpha}(A)^c \sqsubseteq T_{\alpha,\beta}(A^c)$.

(2)We can show this inclusion same way. □

Theorem 2.2. *Let X be a set and $A \in IFS(X)$. If $\alpha, \beta, \alpha+\beta \in [0,1]$ and $\beta \leq \alpha$ then*

(1) $T_{\alpha,\beta}(A) \sqsubseteq T_{\beta,\alpha}(A)$
(2) $S_{\beta,\alpha}(A) \sqsubseteq S_{\alpha,\beta}(A)$

Proof. It is clear. □

Theorem 2.3. *Let X be a set and $A, B \in IFS(X)$. If $\alpha, \beta, \alpha+\beta \in [0,1]$ then*

(1) $T_{\alpha,\beta}(A) \sqcap T_{\alpha,\beta}(B) \sqsubseteq T_{\alpha,\beta}(A \sqcap B)$
(2) $T_{\alpha,\beta}(A \sqcup B) \sqsubseteq T_{\alpha,\beta}(A) \sqcup T_{\alpha,\beta}(B)$

Proof. (1) Let $\alpha, \beta \in [0,1]$,

$$\begin{aligned}
\beta(1-\alpha)\min(\nu_A(x),\nu_B(x)) &\leq \beta(1-\alpha)\max(\nu_A(x),\nu_B(x)) \\
&\Rightarrow \beta\left(\min\left(\mu_A(x),\mu_B(x)\right) + (1-\alpha)\min\left(\nu_A(x),\nu_B(x)\right) + \alpha\right) \\
&\leq \beta\left(\min\left(\mu_A(x),\mu_B(x)\right) + (1-\alpha)\max\left(\nu_A(x),\nu_B(x)\right) + \alpha\right)
\end{aligned}$$

and

$$\begin{aligned}
\alpha(1-\beta)\max(\mu_A(x),\mu_B(x)) &\geq \alpha(1-\beta)\min(\mu_A(x),\mu_B(x)) \\
&\Rightarrow \alpha\left(\max\left(\nu_A(x),\nu_B(x)\right)+(1-\beta)\max\left(\mu_A(x),\mu_B(x)\right)\right) \\
&\geq \alpha\left(\max(\nu_A(x),\nu_B(x))+(1-\beta)\min(\mu_A(x),\mu_B(x))\right)
\end{aligned}$$

It is appear from here that $T_{\alpha,\beta}(A)\sqcap T_{\alpha,\beta}(B)\sqsubseteq T_{\alpha,\beta}(A\sqcap B)$.

(2) It can be shown easily. □

Theorem 2.4. *Let X be a set and $A,B\in IFS(X)$. If $\alpha,\beta,\alpha+\beta\in[0,1]$ then*

(1) $S_{\alpha,\beta}(A\sqcup B)\sqsubseteq S_{\alpha,\beta}(A)\sqcup S_{\alpha,\beta}(B)$

(2) $S_{\alpha,\beta}(A)\sqcap S_{\alpha,\beta}(B)\sqsubseteq S_{\alpha,\beta}(A\sqcap B)$

Proof. (1) Let $\alpha,\beta\in[0,1]$,

$$\begin{aligned}
\alpha(1-\beta)\min(\nu_A(x),\nu_B(x)) &\leq \alpha(1-\beta)\max(\nu_A(x),\nu_B(x)) \\
&\Rightarrow \alpha\left(\max\left(\mu_A(x),\mu_B(x)\right)+(1-\beta)\min\left(\nu_A(x),\nu_B(x)\right)\right) \\
&\leq \alpha\left(\max\left(\mu_A(x),\mu_B(x)\right)+(1-\beta)\max\left(\nu_A(x),\nu_B(x)\right)\right)
\end{aligned}$$

and

$$\begin{aligned}
\beta(1-\alpha)\max(\mu_A(x),\mu_B(x)) &\geq \beta(1-\alpha)\min(\mu_A(x),\mu_B(x)) \\
&\Rightarrow \beta\left(\min\left(\nu_A(x),\nu_B(x)\right)+(1-\alpha)\max\left(\mu_A(x),\mu_B(x)\right)+\alpha\right) \\
&\geq \beta\left(\min\left(\nu_A(x),\nu_B(x)\right)+(1-\alpha)\min\left(\mu_A(x),\mu_B(x)\right)+\alpha\right)
\end{aligned}$$

Thus, $S_{\alpha,\beta}(A\sqcup B)\sqsubseteq S_{\alpha,\beta}(A)\sqcup S_{\alpha,\beta}(B)$.

(2) Can be proved similarly. □

References

[1] Atanassov K.T., Intuitionistic Fuzzy Sets, VII ITKR's Session, Sofia, June (1983).
[2] Atanassov K.T., Intuitionistic Fuzzy Sets, Phiysica-Verlag, Heidelberg, NewYork, (1999).
[3] Atanassov K.T., Studies in Fuzziness and Soft Computing-On Intuitionistic Fuzzy Sets Theory, ISBN 978-3-642-29126-5, Springer Heidelberg, New York, 2012.
[4] Atanassov K.T., Çuvalcıoğlu, G., Atanassova V. K., A new modal operator over intuitionistic fuzzy sets, Notes on IFS, 20(5), 2014, 1-8.
[5] Çuvalcıoğlu, G., On the diagram of One Type Modal Operators on Intuitionistic Fuzzy Sets: Last Expanding with $Z^{\omega,\theta}_{\alpha,\beta}$, Iranian Journal of Fuzzy Systems Vol. 10, No. 1, (2013) pp. 89-106.
[6] Çuvalcıoğlu G., New Tools Based on Intuitionistic Fuzzy Sets and Generalized Nets, ISBN 978-3-319-26301-4, Springer International Publishing Switzerland, 2016, 55-71.
[7] Çuvalcıoğlu G., Yılmaz S. On New Intuitionistic Fuzzy Operators: $S_{\alpha,\beta}$ and $T_{\alpha,\beta}$,Kasmera, 43(2), 2015, 317-327.
[8] Dencheva K., Extension of intuitionistic fuzzy modal operators ⊞ and ⊠,Proc.of the Second Int. IEEE Symp. Intelligent systems, Varna, June 22-24, (2004), Vol. 3, 21-22.
[9] Yılmaz, S., Bal, A., Extentsion of Intuitionistic Fuzzy Modal Operators Diagram with New Operators, "Notes on IFS", Vol. 20, 2014, Number 5, pp. 26-35.
[10] Zadeh L.A., Fuzzy Sets, Information and Control, 8, (1965) , p. 338-353.

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