IFSCOM2016[1] Proceeding Book
No. 1 pp. 130-148 (2016)
ISBN: 978-975-6900-54-3

DISTANCE MEASURE, SIMILARITY MEASURE, ENTROPY AND INCLUSION MEASURE FOR TEMPORAL INTUITIONISTIC FUZZY SETS

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Abstract. In this study, we proposed distance measures similarity measure, entropy and inclusion measure for temporal intuitionistic fuzzy sets and investigated some properties of these measures. We defined these concepts in two different ways, namely temporal and overall and we examined the relationship between these definitions. Also we gave numerical examples for TIFS. We compared these measures defined with two and three parameters in terms of reliability and applicability.

Received: 30–July–2016 Accepted: 29–August–2016

1. Introduction

The concept of temporal intuitionistic fuzzy sets is defined by Atanassov in 1991. In this theory membership and non-membership degrees are defined depending on the time-moment and element. This idea leads to a rich field to be used in applications on dynamic fields such as weather, economy, image and video processing. On the other hand, the similarity and distance measures on fuzzy and intuitionistic fuzzy sets, as seen in present studies, are used in many different areas and obtained effective results. Temporal intuitionistic fuzzy measures which achieved by combining these two ideas are still not defined in the literature. This is one of major shortcoming of temporal intuitionistic fuzzy set theory. Temporal intuitionistic fuzzy measurement is a natural consequence of idea that making dynamic measurements used in the dynamic areas.

In this study, firstly we give definitions of temporal intuitionistic fuzzy distance and similarity measures. Then, we investigate some major properties of these measures. Also we investigate how to define these measurements. With more clearly, these measures will be examined by defining which parameters need to adhere to. Additionally, the concept of entropy and inclusion which are closely related to aforementioned measures are defined to the temporal intuitionistic fuzzy sets. Finally, some other basic concepts needed in this context will be defined in temporal space intuitionistic fuzzy sets.

[1] 3^{rd} International Intuitionistic Fuzzy Sets and Contemporary Mathemathics Conference

Key words and phrases. temporal intuitionistic fuzzy sets, distance measure, similarity measure, entropy, inclusion measure.

2. Preliminaries

Definition 2.1. [1] An intuitionistic fuzzy set on a non-empty set X given by a set of ordered triples $A = \{(x, \mu_A(x), \eta_A(x)) : x \in X\}$ where $\mu_A(x) : X \to I = [0,1]$, $\eta_A(x) : X \to I$, are functions such that $0 \leq \mu(x) + \eta(x) \leq 1$ for all $x \in X$. For $x \in X$, $\mu_A(x)$ and $\eta_A(x)$ represent the degree of membership and degree of non-membership of x to A respectively. For each $x \in X$; intuitionistic fuzzy index of x in A is defined as follows $\pi_A(x) = 1 - \mu_A(x) - \eta_A(x)$. π_A is the called degree of hesitation or indeterminacy.

Definition 2.2. [1] Let $A, B \in IFS(X)$. Then,
(i) $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\eta_A(x) \geq \eta_B(x)$ for $\forall x \in X$,
(ii) $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$,
(iii) $\bar{A} = \{(x, \eta_A(x), \mu_A(x)) : x \in X\}$,
(iv) $\bigcap A_i = \{(x, \wedge\mu_{A_i}(x), \vee\eta_{A_i}(x)) : x \in X\}$,
(v) $\bigcup A_i = \{(x, \vee\mu_{A_i}(x), \wedge\eta_{A_i}(x)) : x \in X\}$,
(vi) $\underset{\sim}{0} = \{(x, 0, 1) : x \in X\}$ and $\underset{\sim}{1} = \{(x, 1, 0) : x \in X\}$..

Definition 2.3. [2] Let X be an universe and T be a non-empty time set. We call the elements of T as "time moments". Based on the definition of IFS, a temporal intuitionistic fuzzy set (TIFS) is defined as the following:

$$A(T) = \{(x, \mu_A(x,t), \eta_A(x,t)) : X \times T\}$$

where:
a. $A \subseteq X$ is a fixed set
b. $\mu_A(x,t) + \eta_A(x,t) \leq 1$ for every $(x,t) \in X \times T$
c. $\mu_A(x,t)$ and $\eta_A(x,t)$ are the degrees of membership and non-membership, respectively, of the element $x \in X$ at the time moment $t \in T$

For brevity, we write A instead of $A(T)$. The hesitation degree of an TIFS is defined as $\pi_A(x,t) = 1 - \mu_A(x,t) - \eta_A(x,t)$. Obviously, every ordinary IFS can be regarded as TIFS for which T is a singleton set. All operations and operators on IFS can be defined for TIFSs.

By $TIFS^{(X,T)}$, we denote to the set of all temporal intuitionistic fuzzy sets defined on X and time set T. Obviously, each intuitionistic fuzzy sets can be expressed as temporal intuitionistic fuzzy set via a singular time set. In additionally, all operations and operators defined for intuitionistic fuzzy sets can be defined for temporal intuitionistic fuzzy sets.

Definition 2.4. [2] Let

$$A(T') = \{(x, \mu_A(x,t), \eta_A(x,t)) : X \times T'\}$$

and

$$B(T'') = \{(x, \mu_B(x,t), \eta_B(x,t)) : X \times T\prime\prime\}$$

where T' and T'' have finite number of distinct time-elements or they are time intervals. Then;
$A(T') \cap B(T'') =$

$$\{(x, \ min(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)), \max(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t))) : (x,t) \in X \times (T' \cup T'')\}$$

and

$A(T') \cup B(T'') =$

$\{(x,\ \max(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)), \min(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t))):\ (x,t) \in X \times (T' \cup T'')\}$

Also from definition of subset in IFS theory, Subsets of TIFS can be defined as the following:

$$A(T') \subseteq B(T'') \Leftrightarrow \bar{\mu}_A(x,t) \geq \bar{\mu}_B(x,t) \text{ and } \bar{\eta}_A(x,t) \leq \bar{\eta}_B(x,t)$$

for every $[(x,t) \in X \times (T' \cup T'')$ where

$$\bar{\mu}_A(x,t) = \begin{cases} \mu_A(x,t) & , \text{ if } \quad t \in T' \\ 0 & , \text{ if } \quad t \in T'' - T' \end{cases}$$

$$\bar{\mu}_B(x,t) = \begin{cases} \mu_B(x,t) & , \text{ if } \quad t \in T'' \\ 0 & , \text{ if } \quad t \in T' - T'' \end{cases}$$

$$\bar{\eta}_A(x,t) = \begin{cases} \eta_A(x,t) & , \text{ if } \quad t \in T' \\ 1 & , \text{ if } \quad t \in T'' - T' \end{cases}$$

$$\bar{\eta}_B(x,t) = \begin{cases} \eta_B(x,t) & , \text{ if } \quad t \in T'' \\ 1 & , \text{ if } \quad t \in T' - T'' \end{cases}$$

It is obviously seen that if $T' = T''$; $\bar{\mu}_A(x,t) = \mu_A(x,t)$, $\bar{\mu}_B(x,t) = \mu_B(x,t)$, $\bar{\eta}_A(x,t) = \eta_A(x,t)$, $\bar{\eta}_B(x,t) = \eta_B(x,t)$. [2]

Let J be an index set and T_i is a time set for each $i \in J$. Let define that $T = \bigcup_{i \in J} T_i$. Now we extend union and intersection of temporal intuitionistic fuzzy sets to the family $F = \{A_i(T_i) = (x, \mu_{A_i}(x,t), \eta_{A_i}(x,t)) : x \in X \times T_i,\ i \in J\}$ as:

$$\bigcup_{i \in J} A(T_i) = \left\{ \left(x,\ \max_{i \in J} (\bar{\mu}_{A_i}(x,t)), \min_{i \in J} (\bar{\eta}_{A_i}(x,t)):\ (x,t) \in X \times T \right) \right\},$$

$$\bigcap_{i \in J} A(T_i) = \left\{ \left(x,\ \min_{i \in J} (\bar{\mu}_{A_i}(x,t)), \max_{i \in J} (\bar{\eta}_{A_i}(x,t)):\ (x,t) \in X \times T \right) \right\}$$

where

$$\bar{\mu}_{A_i}(x,t) = \begin{cases} \mu_{A_i}(x,t) & , \text{ if } \quad t \in T_i \\ 0 & , \text{ if } \quad t \in T - T_i \end{cases}$$

$$\bar{\eta}_{A_i}(x,t) = \begin{cases} \eta_{A_i}(x,t) & , \text{ if } \quad t \in T_i \\ 1 & , \text{ if } \quad t \in T - T_i \end{cases}$$

3. Distance Measure, Similarity Measure, Entropy And Inclusion Measure For Temporal Intuitionistic Fuzzy Sets

Let X be a universe and T be a non-empty time-moment set. The definition of distance measure defined in [31] can be extended to $TIFS^{(X,T)}$ such as:

Definition 3.1. Let X be a universe, T be a non-empty time-moment set and $d^t : TIFS^{(X,T)} \times TIFS^{(X,T)} \to R^+ \cup \{0\}$ be a mapping for fixed $t \in T$. If d^t satisfies following properties for all $A, B \in TIFS^{(X,T)}$ and fixed time-moment $t \in T$, then d^t is called a temporal intuitionistic fuzzy distance measure on $TIFS^{(X,T)}$ at time-moment t :

D1. $d^t(A,B) = 0 \Leftrightarrow A = B$,

D2. $d^t(A,B) = d^t(B,A)$,

D3. If A is crisp set, $d^t\left(A, \bar{A}\right) = \max_{B,C \in TIFS^{(X,T)}} d^t(B,C)$

D4. If $A \subseteq B \subseteq C$ for $A,\ B,\ C \in\ TIFS^{(X,T)}$, $d^t(A,C) \geq d^t(A,B)$ and $d^t(A,C) \geq d^t(B,C)$.

If there exist a mapping $d: TIFS^{(X,T)} \times TIFS^{(X,T)} \to R^+ \cup \{0\}$ which satisfies these conditions for every time moment $t \in T$, then d is called a overall intuitionistic fuzzy distance measure on $TIFS^{(X,T)}$. In order to distinguish between these two concepts from each other, the first measure is referred as temporal intuitionistic fuzzy distance measure at just only time moment t. But the second one measures overall distance between temporal intuitionistic fuzzy sets in full time range.

Distance measure between temporal intuitionistic fuzzy sets in terms of being discrete time set or interval time set can be defined in the following way (see for more information: [4], [13], [16], [18], [20], [22], [31], [33], [34], [37], [43]):

Theorem 3.1. *Let X be non-empty set and $T = \{t_1, t_2, t_3, ..., t_n\}$ be finite and distinct time set. Let define $A, B \in TIFS^{(X,T)}$ such as*

$$A(T) = \{(x, \mu_A(x,t), \eta_A(x,t)) : (x,t) \in X \times T\}$$

and

$$B(T) = \{(x, \mu_B(x,t), \eta_B(x,t)) : (x,t) \in X \times T\}$$

, respectively. Then we define ${d^i}_t(A,B)$ mappings for $t \in T$ and $i = 1,2,3,4$ as following:

1. ${d^1}_{t_0}(A,B) = \sqrt{\sum\limits_{x\in X}\left((\mu_A(x,t_0) - \mu_B(x,t_0))^2 + (\eta_A(x,t_0) - \eta_B(x,t_0))^2\right)}$,

2. ${d^2}_{t_0}(A,B) = \sum\limits_{x\in X}(|\mu_A(x,t_0) - \mu_B(x,t_0)| + |\eta_A(x,t_0) - \eta_B(x,t_0)|)$,

3. ${d^3}_{t_0}(A,B) = \sum\limits_{x\in E}(\max\{|\mu_A(x,t_0) - \mu_B(x,t_0)|, |\eta_A(x,t_0) - \eta_B(x,t_0)|\})$,

4. ${d^4}_{t_0}(A,B) = \sqrt{\sum\limits_{x\in E}\max\left\{(\mu_A(x,t_0) - \mu_B(x,t_0))^2, \left(\eta_A(x,t_0) - \eta_B(x,t_0)^2\right)\right\}}$.

It is clear that each mapping ${d^i}_t(A,B)$ is a temporal intuitionistic fuzzy distance measures for $t \in T$. On the other hand, the following temporal distance measures are obtained by adding the degree of uncertainty to former ones.

5. ${D^1}_{t_0}(A,B) =$

$$\sqrt{\sum_{x\in X}\left((\mu_A(x,t_0) - \mu_B(x,t_0))^2 + (\eta_A(x,t_0) - \eta_B(x,t_0))^2 + (\pi_A(x,t_0) - \pi_B(x,t_0))^2\right)}$$

6. ${D^2}_{t_0}(A,B) =$

$$\sum_{x\in X}(|\mu_A(x,t_0) - \mu_B(x,t_0)| + |\eta_A(x,t_0) - \eta_B(x,t_0)| + |\pi_A(x,t_0) - \pi_B(x,t_0)|)$$

7. ${D^3}_{t_0}(A,B) =$

$$\sum_{x\in E}\max\{|\mu_A(x,t_0) - \mu_B(x,t_0)|, |\eta_A(x,t_0) - \eta_B(x,t_0)|, |\pi_A(x,t_0) - \pi_B(x,t_0)|\}$$

8. ${D^4}_{t_0}(A,B) =$

$$\sqrt{\sum_{x\in E}\max\left\{(\mu_A(x,t_0) - \mu_B(x,t_0))^2, \left(\eta_A(x,t_0) - \eta_B(x,t_0)^2\right), (\pi_A(x,t_0) - \pi_B(x,t_0))^2\right\}}$$

Additionally, the mappings d_i (or D_i) defined as $d_i(A,B) = \max_{t\in T}\left(d^i{}_t(A,B)\right)$ (or $D_i(A,B) = \max_{t\in T}\left(D^i{}_t(A,B)\right)$) are overall temporal intuitionistic fuzzy distance measures for $i = 1,2,3,4$. It is clear that these temporal and overall intuitionistic fuzzy distance measures are intuitionistic fuzzy distance measure for a singleton time set.

Proof. As it can be seen in studies ([4], [13], [16], [18], [20], [22], [31], [33], [34], [37], [43], etc.) temporal intuitionistic fuzzy distance measures defined above is obtained by the addition of time parameters to the intuitionistic fuzzy distance measures. As stated previously, these measures are also intuitionistic fuzzy distance measure for each individual time moment. Now we prove that $d_1(A,B) = \max_{t\in T}\left(d^1{}_t(A,B)\right)$ is an overall intuitionistic fuzzy distance measure.

D1: Since $d^1{}_t(A,A) = 0$ for all $t \in T$ and $A \in TIFS^{(X,T)}$, it is clearly obtained that $d_1(A,A) = 0$.

D2: Since $d^1{}_t(A,B) = d^1{}_t(B,A)$ for all $t \in T$ and $A, B \in TIFS^{(X,T)}$, Then it is clearly obtained that $d_1(A,B) = \max_{t\in T}\left(d^1{}_t(A,B)\right) = \max_{t\in T}\left(d^1{}_t(B,A)\right) = d_1(B,A)$.

D3: Since $d^1{}_t\left(A,\bar{A}\right) = \max_{B,C\in TIFS^{(X,T)}} d^1{}_t(B,C)$ for all $t \in T$ and A crisp set. Then, we can easily get $d^1\left(A,\bar{A}\right) = \max_{t\in T}\left(d^1{}_t\left(A,\bar{A}\right)\right) = \max_{t\in T}\left\{\max_{B,C\in TIFS^{(X,T)}} d^1{}_t(B,C)\right\} = \max_{B,C\in TIFS^{(X,T)}}\left\{\max_{t\in T} d_t(B,C)\right\} = \max_{B,C\in TIFS^{(X,T)}} d^1(B,C)$

D4: From the definition of being subset in concept of TIFS, When $A \subseteq B \subseteq C$ for A, B, $C \in TIFS^{(X,T)}$, the inequalities $\mu_A(x,t) \geq \mu_B(x,t) \geq \mu_C(x,t)$ and $\eta_A(x,t) \leq \eta_B(x,t) \leq \eta_C(x,t)$ are hold for each $(x,t) \in X \times T$. From the last two inequalities the inequalities $d_1^t(A,C) \geq d_1^t(A,B)$ and $d_1^t(A,C) \geq d_1^t(B,C)$ are obtained for $t \in T$. Then the inequalities $d_1(A,C) \geq d_1(A,B)$ and $d_1(A,C) \geq d_1(B,C)$ are clearly obtained from definition of d_1. The other situations can be proved similarly. □

Each distance measures $d^i{}_t$ indicates temporal distance between the temporal intuitionistic fuzzy sets at time moment $t \in T$. On the other hand, d_i measurements which are obtained from the maximum of $d^i{}_t$ gives a overall distance measurement between temporal intuitionistic fuzzy sets. These two approaches gain different importance degrees depending on the applications. With more open expression, overall distance measure expresses inferential distance in the total situation, while since the temporal distance between the temporal intuitionistic fuzzy sets are sensitive to sudden changes at distance, temporal distance is a measure of instant change between cases represented by the temporal intuitionistic fuzzy set. This situation offers multiple ways to ensure expected benefits from the application.

We give definition of temporal intuitionistic distance measures on infinite and interval time set as follow:

Proposition 3.1. *Let X be a infinite set and $T = \{t_1, t_2, , ..., t_i, ...\}$ be a infinite time set (or time interval). Let suppose that $t_k \leq t_{k+1}$ for each $t_k, t_{k+1} \in T$. On the other hand, let define A, $B \in TIFS^{(X,T)}$ TIFSs as follows:*

$$A(T) = \{(x, \mu_A(x,t), \eta_A(x,t)) : (x,t) \in X \times T\}$$

and

$$B(T) = \{(x, \mu_B(x,t), \eta_B(x,t)) : (x,t) \in X \times T\},$$

respectively. With these definitions, the following statements are overall intuitionistic fuzzy distance measure on $TIFS^{(X,T)}$

1. $d^*{}_1(A,B) =$

$$\sup_{t\in T}\left\{\max\left\{\sup_{x\in X}|\mu_A(x,t)-\mu_B(x,t)|, \sup_{x\in X}|\eta_A(x,t)-\eta_B(x,t)|\right\}\right\}$$

2. $d^*{}_2(A,B) =$

$$\sup_{t\in T}\left\{\sup_{x\in X}(|\mu_A(x,t)-\mu_B(x,t)|+|\eta_A(x,t)-\eta_B(x,t)|)\right\}$$

3. $d^*{}_3(A,B) =$

$$\sup_{t\in T}\sqrt{\sup_{x\in X}\left((\mu_A(x,t)-\mu_A(x,t))^2+(\eta_A(x,t)-\eta_A(x,t))^2\right)}$$

4.$D^*{}_1(A,B) =$

$$\sup_{t\in T}\left\{\max\left\{\sup_{x\in X}|\mu_A(x,t)-\mu_B(x,t)|, \sup_{x\in X}|\eta_A(x,t)-\eta_B(x,t)|, \sup_{x\in X}|\pi_A(x,t)-\pi_B(x,t)|\right\}\right\}$$

5. $D^*{}_2(A,B) =$

$$\sup_{t\in T}\left(\sup_{x\in X}(|\mu_A(x,t)-\mu_B(x,t)|+|\eta_A(x,t)-\eta_B(x,t)|+|\pi_A(x,t)-\pi_B(x,t)|)\right)$$

6. $D^*{}_3(A,B) =$

$$\sup_{t\in T}\sqrt{\sup_{x\in X}\left((\mu_A(x,t)-\mu_B(x,t))^2+(\eta_A(x,t)-\eta_B(x,t))^2+(\pi_A(x,t)-\pi_B(x,t))^2\right)}$$

(see [4], [13], [16], [18], [20], [22], [31], [33], [34], [37], [43], *etc.)*

Proposition 3.2. *Let* X *be an infinite set and* $T = [t_1, t_2]$ *for* t_1, $t_2 \in R^+$ *and* $t_1 < t_2$. *Then the following statements are overall intuitionistic c fuzzy distance measures on* $TIFS^{(X,T)}$

1. $D^{***}{}_1(A,B) = \sup_{t\in T}\left\{\int_X(\mu_A(x,t)-\mu_B(x,t)).dx + \left|\int_X(\eta_A(x,t)-\eta_A(x,t)).dx\right|\right\}$

2.$D^{***}{}_2(A,B) =$

$$\sup_{t\in T}\left\{\left|\int_X(\mu_A(x,t)-\mu_B(x,t)).dx\right| + \left|\int_X(\eta_A(x,t)-\eta_B(x,t)).dx\right| + \left|\int_X(\pi_A(x,t)-\pi_B(x,t)).dt\right|\right\}$$

for $A, B \in TIFS^{(X,T)}$ *and* $t \in T = [t_1, t_2]$. *(see* [4], [13], [16], [18], [20], [22], [31], [33], [34], [37], [43], *etc.)*

Example 3.1. Let suppose that $X = [0,4]$ and $T = [0,5]$. Let define that

$$A(T) = \{((x,t), \mu_A(x,t), \eta_A(x,t)); (x,t) \in X \times T\}$$

and

$$B(T) = \{((x,t), \mu_B(x,t), \eta_B(x,t)); (x,t) \in X \times T\}$$

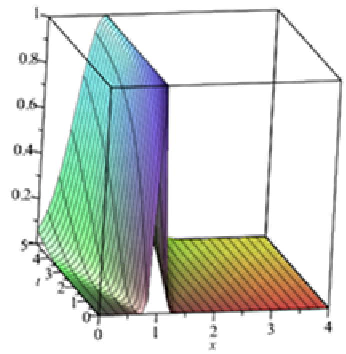

FIGURE 1

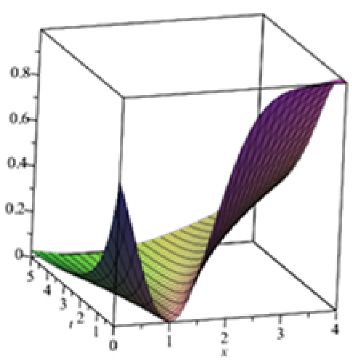

FIGURE 2

where degrees of membership and non-membership are defined as follows respectively $\mu_A(x,t) = e^{-10\frac{(x-1)^2}{t+1}}$ and $\eta_A(x,t) = 1 - e^{-\frac{(x-1)^2}{t+1}}$; $\mu_B(x,t) = \frac{1}{1+e^{-t.(x-2)^2}}$ and $\eta_B(x,t) = \frac{e^{-t.(x-2)^2}}{2+e^{-t.(x-2)}}$ for all $(x,t) \in X \times T$. 3D- graphics of μ_A, η_A, μ_B and η_B are given in Fig.1,2,3,4, respectively.

Fig 1. Graphic of temporal intuitionistic fuzzy membership function μ_A

Fig 2. Graphic of temporal intuitionistic fuzzy non-membership function η_A
Fig 3. Graphic of temporal intuitionistic fuzzy membership function μ_B
Fig 4. Graphic of temporal intuitionistic fuzzy non-membership function η_B

In the following figures, we give changing of distance between A and B obtained by D_1^{***} and D_2^{***} over time in Fig 5. and Fig. 6.

Fig. 5. Changing of distance between A and B obtained by D_1^{***}

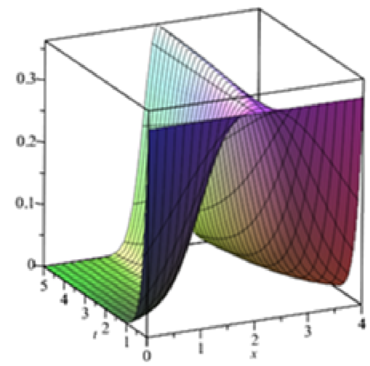

Figure 3

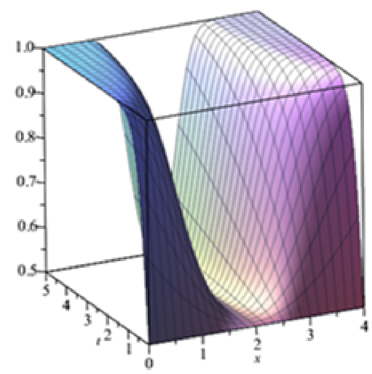

Figure 4

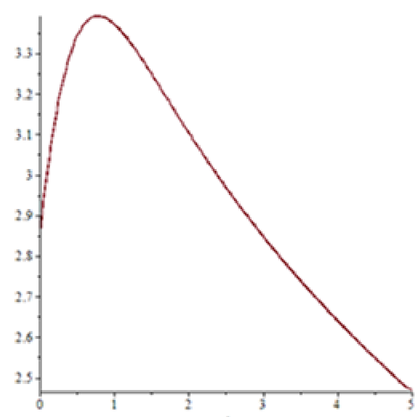

Figure 5

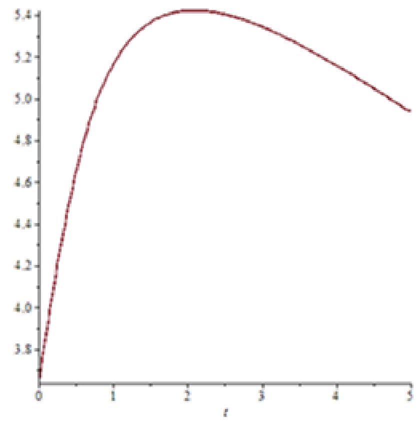

FIGURE 6

Fig. 6. Changing of distance between A and B obtained by D_2^{***}

From these figures, the following overall distances are obtained

$$d_{D_1^{***}}(A,B)=\sup_{t\in T}\{D_1^{***}(A,B)\}=3,391$$

and

$$d_{D_2^{***}}(A,B)=\sup_{t\in T}\{D_2^{***}(A,B)\}=5,415$$

respectively. A key issue at this point is D_2^{***} temporal distance measure is more durable and more reliable than D_1^{***}in high degrees of uncertainty. As previously stated in various studies of Szmidt and Kacprzyk for intuitionistic fuzzy sets, in the cases which contains data with high degree of uncertainty, it is obvious that temporal (or overall) distance which obtained with three parameters between temporal intuitionistic fuzzy sets is more accurate than temporal (or overall) distance which obtained with two parameters .

We use aggregation function to generalize the correlation between temporal and overall intuitionistic fuzzy distance as used in [11].

Theorem 3.2. *Let X be a non-empty set and $T=\{t_1,t_2,...,t_n\}$ be a finite time set. Let suppose that $d_t : TIFS^{(X,T)}\times TIFS^{(X,T)}\to[0,1]$ is a normal temporal intuitionistic fuzzy sets for $t\in T$ and f is a n-aggregation function without zero divisor. Then, the mapping $d : TIFS^{(X,T)}\times TIFS^{(X,T)}\to[0,1]$ which defined as:*

$$d(A,B)=f\left(d_{t_1}(A,B),d_{t_2}(A,B),...,d_{t_n}(A,B)\right)$$

for $A,B\in TIFS^{(X,T)}$ is a overall intuitionistic fuzzy distance measure.

Proof. Since $d_t : TIFS^{(X,T)}\times TIFS^{(X,T)}\to[0,1]$ is a intuitionistic fuzzy distance measure on TIFSs for each $t\in T$. $d(A,B)=f\left(d_{t_1}(A,B),d_{t_2}(A,B),...,d_{t_n}(A,B)\right)$ is a intuitionistic fuzzy distance measure (see [11]). Due to d_t expresses temporal distance between A and B for each $t \in T$,$d$which is obtained by d_t expresses overall distance between A and B. $\square$

Now we give definitions of temporal and overall intuitionistic fuzzy similarity measure in sense of [31]

Definition 3.2. Let X be a non-empty set, T be time set and $s^t : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ be a mapping. If s^t satisfies following conditions for all $A, B, C \in TIFS^{(X,T)}$ and fixed time moment $t \in T$, s_t is called temporal intuitionistic fuzzy similarity measure on $TIFS^{(X,T)}$ at time moment t.

S1. If A is a crisp set, $s^t\left(A, \bar{A}\right) = 0$

S2. $A = B \Leftrightarrow S(A, B) = 1$ for all $A, B \in TIFS^{(X,T)}$

S3. $s^t(A, B) = s^t(B, A)$ for all $A, B \in TIFS^{(X,T)}$

S4. The inequalties $s^t(A, C) \leq s^t(A, B)$ and $s^t(A, C) \leq s^t(B, C)$ are satisfied for all $A, B, C \in TIFS^{(X,T)}$ which are satisfied $A \subseteq B \subseteq C$

As in the concept of temporal distance measure, similarity measure can be examined in two parts as named temporal and overall. Let us give examples of temporal intuitionistic fuzzy similarity measure can be defined in accordance with this approach. It is clear that these measures are obtained by changing domain set of intuitionistic fuzzy similarity measure which are defined in the literature (see [4, 5, 6, 8, 10, 13, 23, 26, 27, 28, 29, 30, 31, 33, 34, 36, 37, 41, 44, 45]) as $TIFS^{(X,T)}$:

1. $S_1^{t_0}(A, B) =$

$$1 - \frac{\sum_{i=1}^{n} \left(|\mu_A(x_i, t_0) - \mu_B(x_i, t_0)|^{t_0} + |\eta_A(x_i, t_0) - \eta_B(x_i, t_0)|^{t_0} \right)^{\frac{1}{t_0}}}{2n}$$

where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., t_m\}$, $t_0 \in T$ and $A, B \in TIFS^{(X,T)}$,

2. $S_2^{t_{j_0}}(A, B) =$

$$1 - \frac{\sum_{i=1}^{n} \omega_{(i,j_0)} \left| \left(|\mu_A(x_i, t_{j_0}) - \mu_B(x_i, t_{j_0})| + |\eta_A(x_i, t_{j_0}) - \eta_B(x_i, t_{j_0})| \right) \right|}{2n}$$

where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., t_m\}$, $t_{j_0} \in T$, $A, B \in TIFS^{(X,T)}$ and $\sum_{i=1}^{n} \omega_{(i,j_0)} = 1$ for each $j_0 \in \{1, 2, ..., m\}$ where $\omega_{(i,j)} \in [0,1]$ for each $(i,j) \in \{1, 2, ..., n\} \times \{1, 2, ..., m\}$.

3. $S_3^{t_{j_0}}(A, B) =$

$$1 - \frac{\sum_{i=1}^{n} \omega_{(i,j_0)} \left(|\mu_A(x_i, t_{j_0}) - \mu_B(x_i, t_{j_0})| + |\eta_A(x_i, t_{j_0}) - \eta_B(x_i, t_{j_0})| \right)}{2n}$$

where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., x_m\}$, $t_{j_0} \in T$, $A, B \in TIFS^{(X,T)}$ and $\sum_{i=1}^{n} \omega_{(i,j_0)} = 1$ for each $j_0 \in \{1, 2, ..., m\}$ where $\omega_{(i,j)} \in [0,1]$ for each $(i,j) \in \{1, 2, ..., n\} \times \{1, 2, ..., m\}$.

4. $S_4^{t_{j_0}}(A,B) =$

$$1-\frac{\sum\limits_{i=1}^{n}\omega_{(i,j_0)}\left(\left|\mu_A(x_i,t_{j_0})-\mu_B(x_i,t_{j_0})\right|+\left|\eta_A(x_i,t_{j_0})-\eta_B(x_i,t_{j_0})\right|+\left|\pi_A(x_i,t_{j_0})-\pi_B(x_i,t_{j_0})\right|\right)}{2n}$$

where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., x_m\}$, $t_{j_0} \in T$, $A, B \in TIFS^{(X,T)}$ and $\sum\limits_{i=1}^{n}\omega_{(i,j_0)} = 1$ for each $j_0 \in \{1, 2, ..., m\}$ where $\omega_{(i,j)} \in [0, 1]$ for each $(i, j) \in \{1, 2, ..., n\} \times \{1, 2, ..., m\}$.

5. $S_5^{t_0}(A,B) = \frac{1}{n}\sum\limits_{i=1}^{n}\frac{\min\{\mu_A(x_i,t_0),\mu_B(x_i,t_0)\}+\min\{\eta_A(x_i,t_0),\eta_B(x_i,t_0)\}}{\max\{\mu_A(x_i,t_0),\mu_B(x_i,t_0)\}+\max\{\eta_A(x_i,t_0),\eta_B(x_i,t_0)\}}$
where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., x_m\}$, $t_0 \in T$, $A, B \in TIFS^{(X,T)}$

6. $S_6^{t_0}(A,B) = \frac{\sum\limits_{i=1}^{n} 1-\frac{1}{2}\left(\left|\mu_A(x_i,t_0)-\mu_B(x_i,t_0)\right|+\left|\eta_A(x_i,t_0)-\eta_B(x_i,t_0)\right|\right)}{n}$
where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., x_m\}$, $t_0 \in T$, $A, B \in TIFS^{(X,T)}$

7. $S_7^{t_0}(A,B) = \frac{1}{n}\frac{\sum\limits_{i=1}^{n}\min\{\mu_A(x_i,t_0),\mu_B(x_i,t_0)\}+\min\{\eta_A(x_i,t_0),\eta_B(x_i,t_0)\}}{\sum\limits_{i=1}^{n}\max\{\mu_A(x_i,t_0),\mu_B(x_i,t_0)\}+\max\{\eta_A(x_i,t_0),\eta_B(x_i,t_0)\}}$
where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., x_m\}$, $t_0 \in T$, $A, B \in TIFS^{(X,T)}$

8. $S_8^{t_0}(A,B) = 1-\frac{1}{2}\left(\max\left|\mu_A(x_i,t_0)-\mu_B(x_i,t_0)\right|+\max\left|\eta_A(x_i,t_0)-\eta_B(x_i,t_0)\right|\right)$
where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., x_m\}$, $t_0 \in T$, $A, B \in TIFS^{(X,T)}$

9. $S_9^{t_0}(A,B) = 1-\frac{\sum\limits_{i=1}^{n}\left(\left|\mu_A(x_i,t_0)-\mu_B(x_i,t_0)\right|+\left|\eta_A(x_i,t_0)-\eta_B(x_i,t_0)\right|\right)}{\sum\limits_{i=1}^{n}\left(\left|\mu_A(x_i,t_0)+\mu_B(x_i,t_0)\right|+\left|\eta_A(x_i,t_0)+\eta_B(x_i,t_0)\right|\right)}$
where $X = \{x_1, x_2, ..., x_n\}$, $T = \{t_1, t_2, ..., x_m\}$, $t_0 \in T$, $A, B \in TIFS^{(X,T)}$

In these definitions, it is seen that the temporal similarity measures are dependent on t the moment with selected temporal intuitionistic fuzzy sets. Since these TIFSs change over time, similarity measures on $TIFS^{(X,T)}$ inevitably change over time. This approach elicits a more spacious work area for applications changed in comparison mechanism. there are many different methods to achieve the overall similarity measure defined from a temporal intuitionistic fuzzy similarity measures. Among these the most significant are defined as follows:

Theorem 3.3. *Let X be non-empty set, T be time set and s^t be a temporal intuitionistic fuzzy similarity measure for each $t \in T$. Then the mapping s defined as $s(A,B) = \max\limits_{t\in T}\{s^t(A,B)\}$ is a overall intuitionistic fuzzy similarity measure.*

More general version of this theorem with Du and Xu's approach [11] can be given as follows:

Theorem 3.4. *: Let X be non-empty set and $T = \{t_1, t_2, ..., t_n\}$ be a time set. Let suppose that the mappings $s^t : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0, 1]$ are temporal intuitionistic fuzzy similarity measure for each $t \in T$ and f is a n-aggregation function without zero divisor . Then, the mapping $s : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0, 1]$ defined as:*

$$s(A,B) = f\left(s^{t_1}(A,B), s^{t_2}(A,B), ..., s^{t_n}(A,B)\right)$$

for all $A, B \in TIFS^{(X,T)}$ is a overall intuitionistic fuzzy similarity measure.

Proof. It can be proven as Theorem 1. □

As noted for temporal intuitionistic fuzzy distance measure, all similarity measures defined for intuitionistic fuzzy sets can be accepted as temporal intuitionistic fuzzy similarity measure for singular time set and they also can be converted into temporal intuitionistic fuzzy similarity measure by selecting the domain set $TIFS^{(X,T)}$. After this process, overall intuitionistic fuzzy similarity measurements indicating the general jurisdiction can be obtained by using some methods such as aggregation function. The most common feature of data used in real-world applications is that they can change over time. In this context, the most remarkable feature is that the uncertainty of the situation will may increase sometimes. As in fuzzy and intuitionistic fuzzy sets, temporal (or overall) intuitionistic fuzzy distance and similarity measures are dual concepts. So, there are several ways to obtain other one from another. In [11], Du and Xu have generalized this relationship by fuzzy negation and aggregation function for intuitionistic fuzzy distance and similarity measures. Their feature is also available in the temporal intuitionistic fuzzy sets. Having not generalized some basic concepts for temporal intuitionistic fuzzy sets is shortcoming in the literature. Some of these concepts which given in [11] are generalized for the temporal intuitionistic fuzzy sets as follows:

Definition 3.3. Let T be a time set. If the mapping $N_t : [0,1] \to [0,1]$ is satisfied following condition for $t \in T$, it is called temporal fuzzy negation at time moment t :
N1. $N_t(0) = 1$, $N_t(1) = 0$
N2. $N_t(b) \leq N_t(a)$ for all $a \leq b$
if N_t is satisfied
a. $N_t(N_t(a)) = a$ for $t \in T$ and all $a \in [0,1]$, it is called temporal fuzzy strong negation at time moment t,
b. $x = 0 \Leftrightarrow N_t(x) = 1$ for $t \in T$ and all $a \in [0,1]$, it is called temporal fuzzy non-filling negation at time moment t,
c. $x = 1 \Leftrightarrow N_t(x) = 0$ for $t \in T$ and all $a \in [0,1]$, it is called temporal fuzzy non-vanishing negation at time moment t.

The novelty of this definition is that negation will change over time. Adding the time parameters and changing temporal fuzzy negation over time offers unlimited options for obtaining similarity measure from distance measure (or conversely). the temporal fuzzy strong negations can be obtained by adding time parameters to fuzzy negations as follows:

1. $N_{1,\lambda_t}(x) = \frac{1-x}{1+\lambda_t x}$ for $\lambda_t \in (-1,+\infty)$ and $t \in T$,
2. $N_{2,\delta_t}(x) = \sqrt[\delta_t]{1-x^{\delta_t}}$ for $\delta_t \in (0,+\infty)$ and $t \in T$,
3. $N_{3,\varphi_t}(x) = \begin{cases} 1, & x \leq \varphi_t \\ 0, & otherwise \end{cases}$ for $\varphi_t \in (0,1)$ and $t \in T$.

Following theorems are obtained by adding time parameter to theorems which given by Du and Xu [11].

Theorem 3.5. *Let X be non-empty set, T be time set and $A, B \in TIFS^{(X,T)}$. Let suppose that d^t is a temporal distance measure and N_t is a temporal fuzzy* *non-filling negation. Then, the mapping* $s_{N_t} : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ *defined as* $s_{N_t}(A,B) = N_t(d^t(A,B))$*is a temporal intuitionistic fuzzy similarity measure. Conversely, Let suppose that* s^t *is a temporal intuitionistic fuzzy similarity measure and* N_t *is a temporal fuzzy non-vanishing negation, then the mapping* $d_{N_t} : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ *defined as* $d_{N_t}(A,B) = N_t(s^t(A,B))$ *is a temporal intuitionistic fuzzy distance measure. If* N_t *is a temporal fuzzy strong negation, the equations* $d^t(A,B) = N^t(s_{N^t}(A,B))$ *and* $s^t(A,B) = N_t(d_{N_t}(A,B))$ *are satisfied.*

This theorem is preserved with Du and Xu‘s approach [11] for overall temporal measures as follows.

Theorem 3.6. *Let* X *be non-empty set,* T *be time set and* $A, B \in TIFS^{(X,T)}$*. Let suppose that* d *is overall intuitionistic fuzzy distance measure and* N *is a fuzzy non-filling negation. Then, the mapping* $s_N : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ *defined as* $s_N(A,B) = N(d(A,B))$ *is an overall intuitionistic fuzzy similarity measure. Conversely, let suppose that* s *is an overall intuitionistic fuzzy similarity measure and* N *is a fuzzy non-vanishing negation, then the mapping* $d_N : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ *defined as* $d_N(A,B) = N(s(A,B))$ *is a overall intuitionistic fuzzy similarity measure. If* N *is a fuzzy strong negation, the equations* $d(A,B) = N(s_N(A,B))$ *and* $s(A,B) = N(d_N(A,B))$ *are satisfied.*

Another theorem which is given in [11] can be generalized to temporal intuitionistic fuzzy sets as below.

Theorem 3.7. *: Let* X *be a non empty set and* $T = \{t_1, t_2, ..., t_n\}$ *be a finite time set. Let suppose that* $d^t : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ *is a temporal intuitionistic fuzzy distance measure for each* $t \in T$*,* f *is a aggregation function without zero divisor and* N_t *is a temporal fuzzy non-filling negation, then the mapping* $s : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$*defined as*

$$s(A,B) = f\left(s_{N_{t_1}}(A,B), s_{N_{t_2}}(A,B), ..., s_{N_{t_n}}(A,B)\right)$$

for all $A, B \in TIFS^{(X,T)}$ *is a overall intuitionistic fuzzy similarity measure.*

Proof. It is clear that each s_{N_t} is a temporal intuitionistic similarity measure for $t \in T$ from Theorem 5. Then, it is obtained that s is a overall intuitionistic fuzzy similarity measure from Theorem 4. □

Now, entropy which is a measure of difference between intuitionistic fuzzy set (or fuzzy sets) and crisp set will be defined for temporal intuitionistic fuzzy sets. the temporal variability encountered in real-world problems changes this difference in a continuous manner. The definition of entropy (as temporal entropy and overall entropy) with Szmidt and Kacprzyk's approach [35] is defined for temporal intuitionistic fuzzy sets as follow:

Definition 3.4. Let X be a non-empty set and T be a time set. If the mapping $e^t : TIFS^{(X,T)} \to [0,1]$ is satisfied the following conditions for $A \in TIFS^{(X,T)}$ and fixed $t \in T$, e^t is called temporal intuitionistic fuzzy entropy on $TIFS^{(X,T)}$.

E1. A is a crisp set for $\Leftrightarrow e^t(A) = 0$,

E2. $e^t(A) = 1 \Leftrightarrow \mu_A(x,t) = \eta_A(x,t)$ for all $x \in X$ and fixed $t \in T$

E3. $e^t(A) \leq e^t(B) \Leftrightarrow$
i. $\mu_A(x,t) \geq \mu_B(x,t)$ and $\eta_A(x,t) \leq \eta_B(x,t)$ for $\mu_B(x,t) \leq \eta_B(x,t)$
ii. $\mu_A(x,t) \leq \mu_B(x,t)$ and $\eta_A(x,t) \geq \eta_B(x,t)$ for $\mu_B(x,t) \geq \eta_B(x,t)$
for all $x \in X$ and fixed $t \in T$
E4. $e^t(A) = e^t(\overline{A})$.

With this definition, the value $e^t(A)$ represents how much is far from being crisp sets at time moment t. If we change the condition E1 with

E1*. A is a fuzzy set $\Leftrightarrow e^t(A) = 0$

, the value $e^t(A)$ represents how much is far from being fuzzy sets at time moment t. Let's call this second measure as type-2 temporal entropy. On the other hand, If the mapping $e : TIFS^{(X,T)} \to [0,1]$ is satisfied the conditions (E1, E2, E3, E4) for each $t \in T$, e is called overall entropy on $TIFS^{(X,T)}$. The mapping $e : TIFS^{(X,T)} \to [0,1]$ defined as $e(A) = \sup_{t \in T} \{e^t(A)\}$ is an overall intuitionistic fuzzy entropy.

Theorem 3.8. *Let X be a non-empty set and $T = \{t_1, t_2, ..., t_n\}$ be a finite time set. Let suppose that $e^t : TIFS^{(X,T)} \to [0,1]$ is temporal intuitionistic fuzzy entropy for $t \in T$ and f is a aggregation function without zero divisor. Then the mapping $e : TIFS^{(X,T)} \to [0,1]$ defined as*

$$e(A) = f\left(e^{t_1}(A), e^{t_2}(A), ..., e^{t_n}(A)\right)$$

for $A \in TIFS^{(X,T)}$ is an overall intuitionistic fuzzy entropy.

Proof. It can be proven as Theorem 1. □

The intuitionistic fuzzy entropies which are defined in [*12, 14, 19, 21, 24, 25, 28, 29, 30, 31, 35, 36, 38, 41, 42, 43, 45*] can be converted into the temporal intuitionistic fuzzy entropy with some minor modifications as follows:

Proposition 3.3. *Let $X = \{x_1, x_2, ..., x_n\}$ be a non-empty set and T is a time set, Then the following mappings are temporal intuitionistic fuzzy entropy on $TIFS^{(X,T)}$*

1. $e^t_{SK}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n} |\mu_A(x_i,t) - \eta_A(x_i,t)|$
2. $e^t_{VS-1} = \frac{\sum_{i=1}^{n} \min\{\mu_A(x_i,t),\eta_A(x_i,t)\}+\min\{1-\mu_A(x_i,t),1-\eta_A(x_i,t)\}}{\sum_{i=1}^{n} \max\{\mu_A(x_i,t),\eta_A(x_i,t)\}+\max\{1-\mu_A(x_i,t),1-\eta_A(x_i,t)\}}$
3. $e^t_{VS-2}(A) = \sum_{i=1}^{n} \frac{2\mu_A(x_i,t)\eta_A(x_i,t)+\pi_A(x_i,t)^2}{\mu_A(x_i,t)^2+\eta_A(x_i,t)^2+\pi_A(x_i,t)^2}$
4. $e^t_{Li-1}(A) = 1 - \frac{1}{2n}\sum_{i=1}^{n}\left(|\mu_A(x_i,t) - \eta_A(x_i,t)|^3 + |\mu_A(x_i,t) - \eta_A(x_i,t)|\right)$
5. ${e^t}_{GS-1}(A) = \frac{1}{n}\sum_{i=1}^{n}(1 - |\mu_A(x_i,t) - \eta_A(x_i,t)|)\,\frac{1+\pi_A(x_i,t)}{2}$
6. ${e^t}_{Huang-1}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left|\mu_A(x_i,t)^2 - \eta_A(x_i,t)^2\right|$
7. ${e^t}_{Huang-2}(A) = 1 - \sqrt{[p]}\frac{1}{n}\sum_{i=1}^{n}\left|\mu_A(x_i,t)^2 - \eta_A(x_i,t)^2\right|^p$
8. ${e^t}_{Huang-3}(A) = 1 - \sqrt{[p]}\frac{1}{n}\sum_{i=1}^{n}|\mu_A(x_i,t) - \eta_A(x_i,t)|^p$

9. $e^t{}_{Huang-4}(A) = \frac{1}{n}\sum\limits_{i=1}^{n} \frac{1-|\mu_A(x_i,t)-\eta_A(x_i,t)|+\pi_A(x_i,t)}{1+|\mu_A(x_i,t)-\eta_A(x_i,t)|+\pi_A(x_i,t)}$

10. $e^t{}_{Huang-5}(A) =$

$$\frac{1}{n}\sum_{i=1}^{n} 1-|\mu_A(x_i,t)-\eta_A(x_i,t)|+\pi_A(x_i,t)\,|\mu_A(x_i,t)-\eta_A(x_i,t)|$$

11. $e^t{}_{Hung-1}(A) = 1-\sum\limits_{i=1}^{n}\frac{1}{n}|\mu_A(x_i,t)-\eta_A(x_i,t)|$

12. $e^t{}_{Hung-2}(A) = 1-\sqrt{\sum\limits_{i=1}^{n}\frac{1}{n}\left|\mu_A(x_i,t)-\eta_A(x_i,t)^2\right|}$

13. $e^t{}_{Ye-1}(A) =$

$$\frac{1}{n}\sum_{i=1}^{n}\left(\sin\left(\frac{\pi[1+\mu_A(x_i,t)-\eta_A(x_i,t)]}{4}\right)+\sin\left(\frac{\pi[1-\mu_A(x_i,t)+\eta_A(x_i,t)]}{4}-1\right)\frac{1}{\sqrt{2}-1}\right)$$

14. $e^t{}_{Ye-2}(A) =$

$$\frac{1}{n}\sum_{i=1}^{n}\left(\cos\left(\frac{\pi[1+\mu_A(x_i,t)-\eta_A(x_i,t)]}{4}\right)+\cos\left(\frac{\pi[1-\mu_A(x_i,t)+\eta_A(x_i,t)]}{4}-1\right)\frac{1}{\sqrt{2}-1}\right)$$

15. $e^t_{ZL-1} = 1-\frac{1}{b-a}\int\limits_a^b|\mu_A(x,t)-\eta_A(x,t)|\,dx$ *for* $X=[a,b]$

16. $e^t{}_{ZL-1} = \frac{\int_a^b \mu_A(x,t)\wedge\eta_A(x,t)dx}{\int_a^b \mu_A(x,t)\vee\eta_A(x,t)dx}$ *for* $X=[a,b]$

The other measure which is closely related with distance measure, similarity measure and entropy in intuitionistic fuzzy sets (and fuzzy set) is inclusion measure (named subsethood measure in some studies). Inclusion measure has been defined for intuitionistic fuzzy sets by Cornelis and Kerre in [9]. This concept is defined for temporal intuitionistic fuzzy sets as follows:

Definition 3.5. Let X be a non-empty set and T be a time set. If the mapping $I_t : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ is satisfied following conditions for fixed $t\in T$, it is named temporal intuitionistic fuzzy inclusion measure on $t \in T$.

I1. $I_t\left(\widetilde{1},\tilde{0}\right)=0,$

I2. $I_t(A,B)=1 \Leftrightarrow A\subseteq B$

I3. $I_t(C,A) \le I_t(C,B)$and $I_t(B,C)\le I_t(A,C)$ when $A\subseteq B$ and $C\in TIFS^{(X,T)}$

On the other hand, if the mapping $I : TIFS^{(X,T)}\times TIFS^{(X,T)} \to [0,1]$ is satisfied the conditions (I1, I2, I3) for each $t\in T$, I is called overall intuitionistic fuzzy inclusion measure on $TIFS^{(X,T)}$. The mapping $I : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ defined as $I(A,B)=\sup\limits_{t\in T}\{I_t(A,B)\}$ for all $A,B\in TIFS^{(X,T)}$ and $t\in T$ is an overall intuitionistic fuzzy inclusion measure.

The intuitionistic fuzzy inclusion measures which are defined in [44], [45] can be converted into the temporal intuitionistic fuzzy inclusion measures with some minor modifications as well as for other measures.

Proposition 3.4. *Let $X = \{x_1, x_2, ..., x_n\}$ be a non-empty set and T be a time set. Then the following mappings are temporal intuitionistic fuzzy inclusion measure on $TIFS^{(X,T)}$.*

1. $I^t_{ZHXL-1}(A,B) =$

$$1-\frac{1}{2n}\sum_{i=1}^{n}\{|\mu_A(x_i,t)-\min\{\mu_A(x_i,t),\mu_B(x_i,t)\}|+|\max\{\eta_A(x_i,t),\eta_B(x_i,t)\}-\eta_A(x_i,t)|\}$$

2. $I^t_{ZHXL-2}(A,B) =$

$$1-\sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left\{|\mu_A(x_i,t)-\min\{\mu_A(x_i,t),\mu_B(x_i,t)\}|^2+|\max\{\eta_A(x_i,t),\eta_B(x_i,t)\}-\eta_A(x_i,t)|^2\right\}}$$

3. $I^t_{ZDZS-3}(A,B) =$

$$\begin{cases} 1 & , \quad \begin{matrix}\mu_B(x_i,t)=\mu_A(x_i,t)\\ \eta_B(x_i,t)=\eta_A(x_i,t)\end{matrix} \\ \sum\limits_{i=1}^{n}\frac{1}{2}\left[\frac{\mu_B(x_i,t)-\mu_A(x_i,t)+\eta_B(x_i,t)-\eta_A(x_i,t)}{|\mu_B(x_i,t)-\mu_A(x_i,t)|+|\eta_B(x_i,t)-\eta_A(x_i,t)|}\right]+1 & , \quad otherwise \end{cases}$$

The following theorem which is given for the others measures can be given for temporal intuitionistic fuzzy inclusion measure and overall intuitionistic fuzzy inclusion measure as follows:

Theorem 3.9. *Let X be a non-empty set and $T = \{t_1, t_2, ..., t_n\}$ be a finite time set. Let suppose that the mappings $I_t : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ is a temporal intuitionistic fuzzy inclusion measure for each $t \in T$ and f is a aggregation function without zero divisor. Then the mapping $I : TIFS^{(X,T)} \times TIFS^{(X,T)} \to [0,1]$ defined as*

$$I(A,B) = f(I_{t_1}(A), I_{t_2}(A), ..., I_{t_n}(A))$$

for all $A, B \in TIFS^{(X,T)}$ is a overall intuitionistic fuzzy inclusion measure.

The relationship between these measures are protected as described for the fuzzy and intuitionistic fuzzy sets. Some of these relationships can be generalized for the temporal intuitionistic fuzzy as follows. In this context, these relationships can be proved as in the studies which are given in the reference. Therefore, we will give the following theorems without proof (see more information: *[3, 4, 5, 6, 8, 9, 10,12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 40, 41, 42, 43, 44, 45, etc.]*) .

Theorem 3.10. *Let X be a non-empty set and T be time set. Let suppose that s^t is a temporal intuitionistic fuzzy similarity measure for $t \in T$. Then the mapping $e^t : TIFS^{(X,T)} \to [0,1]$ defined as $e^t(A) = s^t(A, \bar{A})$ for all $A \in TIFS^{(X,T)}$ is a temporal intuitionistic fuzzy entropy*

Theorem 3.11. *Let X be a non-empty set and $T = \{t_1, t_2, ..., t_n\}$ be finite time set. Let suppose that s^t is a temporal intuitionistic fuzzy similarity measure for*

$t \in T$ ve f is a aggregation function without zero divisor. Then the mapping $e : TIFS^{(X,T)} \to [0,1]$ defined as $e(A) = f\left(s^{t_1}\left(A,\bar{A}\right), s^{t_2}\left(A,\bar{A}\right), ..., s^{t_n}\left(A,\bar{A}\right)\right)$ for all $A \in TIFS^{(X,T)}$ is a overall intuitionistic fuzzy entropy.

Theorem 3.12. *Let X be a non-empty set and T be time set. Let suppose that d^t is a temporal intuitionistic fuzzy distance measure for $t \in T$ and N_t is a temporal fuzzy non-filling negation. Then the mapping $e^t : TIFS^{(X,T)} \to [0,1]$ defined as $e^t(A) = N_t\left(d^t\left(A,\bar{A}\right)\right)$ for all $A \in TIFS^{(X,T)}$ is a temporal intuitionistic fuzzy entropy.*

Theorem 3.13. *Let X be a non-empty set and $T = \{t_1, t_2, ..., t_n\}$ be finite time set. Let suppose that s^t is a temporal intuitionistic fuzzy similarity measure for $t \in T$, f is a aggregation function without zero divisor and N_t is a temporal fuzzy non-filling negation. The mapping $e : TIFS^{(X,T)} \to [0,1]$ defined as*

$$e(A) = f\left(N_{t_1}\left(d^{t_1}\left(A,\bar{A}\right)\right), N_{t_2}\left(d^{t_2}\left(A,\bar{A}\right)\right), ..., N_{t_n}\left(d^{t_n}\left(A,\bar{A}\right)\right)\right)$$

for all $A \in TIFS^{(X,T)}$ is a overall intuitionistic fuzzy entropy.

Theorem 3.14. *Let X be a non-empty set and T be time set. Let suppose that I_t is a temporal intuitionistic fuzzy inclusion measure. Then the mapping $E_I^t : TIFS^{(X,T)} \to [0,1]$ defined as $E_I^t(A) = \left(I_t\left(A \cup \overline{A}, A \cap \overline{A}\right)\right)$ for all $A \in TIFS^{(X,T)}$ is a temporal intuitionistic fuzzy entropy.*

Theorem 3.15. *Let X be a non-empty set and T be time set. . Let suppose that $*$ is a $t-$ norm için I^t is a temporal intuitionistic fuzzy inclusion measure for $t \in T$. Then the mapping $E_I^t : TIFS^{(X,T)} \to [0,1]$ defined as $S_I^t(A,B) = *(I_t(A,B), I_t(B,A))$ for all $A, B \in TIFS^{(X,T)}$ is a temporal intuitionistic fuzzy similarity measure.*

As seen from the above theorem, the basic relationship between distance measure, similarity measure, entropy and coverage measure are protected for temporal intuitionistic fuzzy sets as provided in for fuzzy and intuitionistic fuzzy sets. Furthermore, these measures can be separate two groups which are named as temporal and overall for temporal intuitionistic fuzzy sets. It can be done the first instant evaluation and later general evaluation for all data with time-varying nature by this method. Thus, the optimal results can be obtained by various way in the application.

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